AGC 2

1.0 Introduction

In the last set of notes, we developed a model of the speed governing mechanism, which is given below:

$$\Delta\hat{x}_E = \frac{K_G}{1+T_G s}(\Delta\hat{P}_C - \frac{1}{R}\Delta\hat{\omega}) \qquad (1)$$

In these notes, we want to extend this model so that it relates the actual mechanical power into the machine (instead of Δx_E), so that we can then examine the relation between the mechanical power into the machine and frequency deviation.

What lies between Δx_E, which represents the steam valve, and ΔP_M, which is the mechanical power into the synchronous machine?

2.0 Extended model

Your text (p. 381) does not go into great detail in regards to the turbine model but rather argues that it responds much like the speed governing system, which is a single time-constant system.

Being a single time-constant system implies that there is only one pole (the characteristic equation has only one root). Thinking in terms of inverse LaPlace transforms, this means that the response to a step change in valve opening will be exponential (as opposed to oscillatory). In these notes, we simply confirm that this is the case.

Analytically, this means that the relation between the change in valve opening Δx_E and the change in mechanical power into the generator ΔP_M is given by:

$$\Delta P_M = \frac{K_T}{1+T_T s} \Delta \hat{x}_E \tag{2}$$

Substituting (1) into (2) results in

$$\Delta P_M = \frac{K_T}{1+T_T s}\left[\frac{K_G}{1+T_G s}(\Delta \hat{P}_C - \frac{1}{R}\Delta\hat{\omega})\right] \tag{3}$$

which is

$$\Delta P_M = \frac{K_T K_G}{(1+T_T s)(1+T_G s)}\left[\Delta \hat{P}_C - \frac{1}{R}\Delta\hat{\omega}\right] \tag{4}$$

Let's assume that K_T and K_G are chosen so that $K_T K_G = 1$, then eq. (4) becomes:

$$\Delta P_M = \frac{1}{(1+T_T s)(1+T_G s)}\left[\Delta \hat{P}_C - \frac{1}{R}\Delta\hat{\omega}\right] \qquad (5)$$

A block diagram representing eq. (5) is given in Fig. 1 (Fig. 11.4 in text).

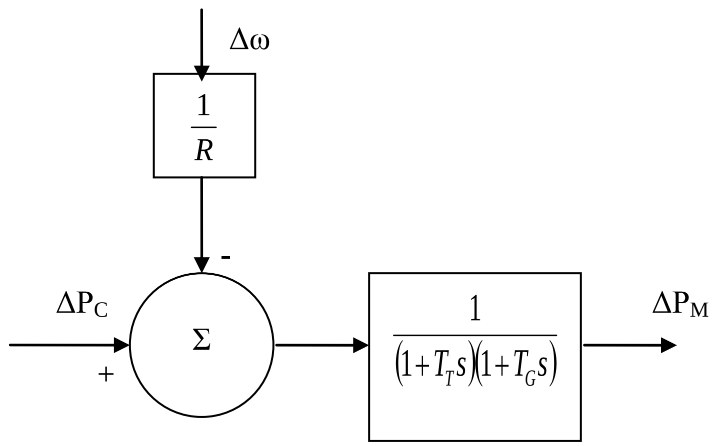

Fig. 1

<u>3.0 Mechanical power and frequency</u>

Let's expand (5) so that

$$\Delta \hat{P}_M = \frac{\Delta \hat{P}_C}{(1+T_T s)(1+T_G s)} - \frac{1}{(1+T_T s)(1+T_G s)}\frac{\Delta\hat{\omega}}{R}$$

(6)

Consider a step-change in power of ΔP_C and in frequency of $\Delta\omega$, which in the LaPlace domain is:

$$\Delta\hat{\omega} = \frac{\Delta\omega}{s} \tag{7a}$$

$$\Delta\hat{P}_C = \frac{\Delta P_C}{s} \tag{7b}$$

Substitution of (7a) and (7b) into (6) results in:

$$\Delta\hat{P}_M = \frac{\Delta P_C}{s(1+T_T s)(1+T_G s)} - \frac{\Delta\omega}{s(1+T_T s)(1+T_G s)}\frac{1}{R} \tag{8}$$

One easy way to examine eq. (8) is to consider $\Delta P_M(t)$ for very large values of t, i.e., for the steady-state.

To do this, recall that the variable ΔP_M in eq. (8) is a LaPlace variable. To consider the corresponding time-domain variable under the steady-state, we may employ the *final value theorem*, which is:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\hat{f}(s) \tag{9}$$

Applying eq. (9) to eq. (8), we get:

$$\Delta P_M = \lim_{t\to\infty} \Delta P_M(t) = \lim_{s\to 0} s\Delta \hat{P}_M$$

$$= \lim_{s\to 0}\left\{ \frac{s\Delta P_C}{s(1+T_T s)(1+T_G s)} - \frac{s\Delta\omega}{s(1+T_T s)(1+T_G s)}\frac{1}{R} \right\}$$

$$= \lim_{s\to 0}\left\{ \frac{\Delta P_C}{(1+T_T s)(1+T_G s)} - \frac{\Delta\omega}{(1+T_T s)(1+T_G s)}\frac{1}{R} \right\} \quad (10)$$

$$= \Delta P_C - \frac{\Delta\omega}{R}$$

Therefore, when considering the relation between steady-state changes,

$$\Delta P_M = \Delta P_C - \frac{\Delta\omega}{R} \quad (11)$$

This is eq. (11.6) in your text.

Make sure that you understand that in eq. (11), ΔP_M, ΔP_C, and $\Delta\omega$ in eq. (11) are

- Time-domain variables (not LaPlace variables)
- Steady-state values of the time-domain variables (the values after you wait along time)

Because we developed eq. (11) assuming a step-change in frequency, you might be mislead into thinking that the frequency change is the initiating change that causes the change in mechanical power ΔP_M.

However, recall Fig. 3 of AGC1 notes, repeated below for convenience as Fig. 2 in these notes.

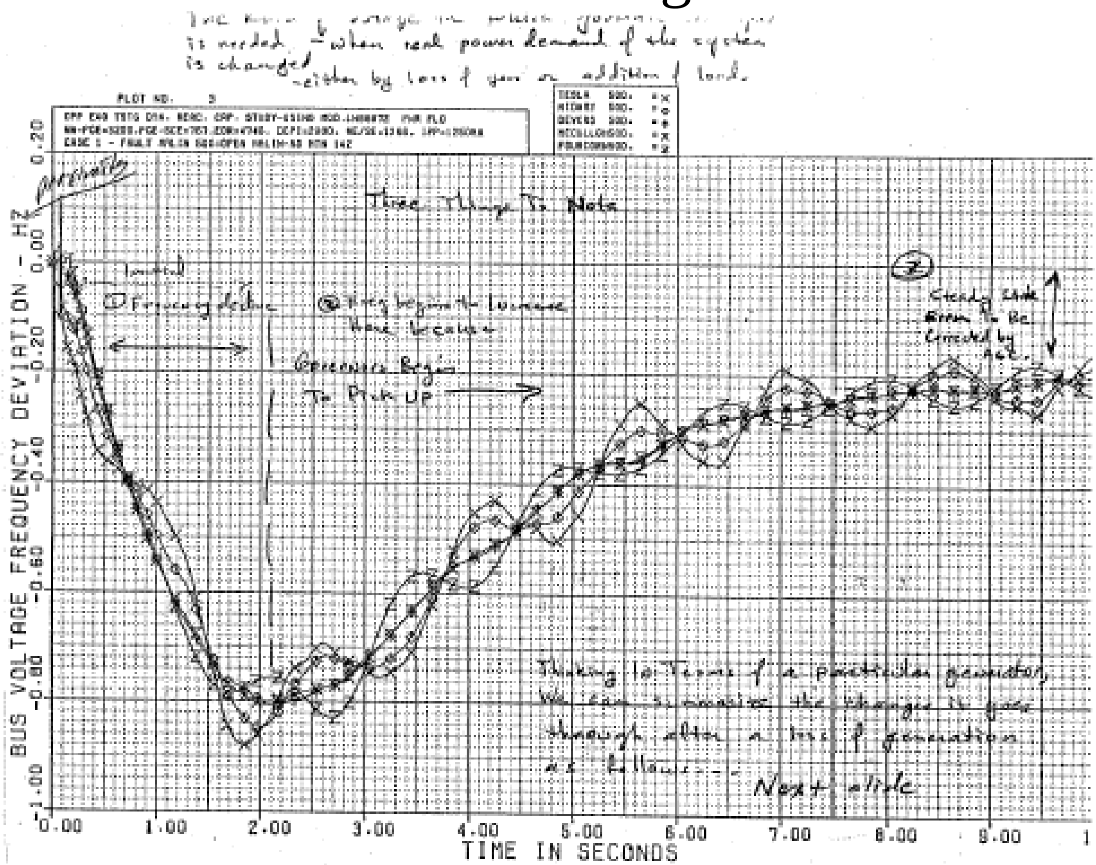

Fig. 2

The frequency change expressed by $\Delta\omega$ in eq. (11) is the frequency deviation at the end of the simulation. The ΔP_M in eq. (11), associated with Fig. 2, is

- not the amount of generation that was outaged,

- but rather the amount of generation increased at a certain generator in response to the generation outage.

So ΔP_M and $\Delta\omega$ are the conditions that can be observed at the end of a transient initiated by a load-generation imbalance. They are conditions that result from the action of the primary governing control.

In other words, the primary governing control will operate (in response to some frequency deviation caused by a load-generation imbalance) to change the generation level by ΔP_M and leave a steady-state frequency deviation of $\Delta\omega$.

Although we have not developed relations for ω, P_M, and P_C (but rather $\Delta\omega$, ΔP_M, and ΔP_C), lets assume we have at our disposal a plot of P_M vs. ω for a certain setting of $P_C=P_{C1}$. Such a plot appears in Fig. 3. (The text makes the following assumption on pg 383 (I added the italicized text): "…the local behavior *(as characterized by* *eq. (11))* can be extrapolated to a larger domain.")

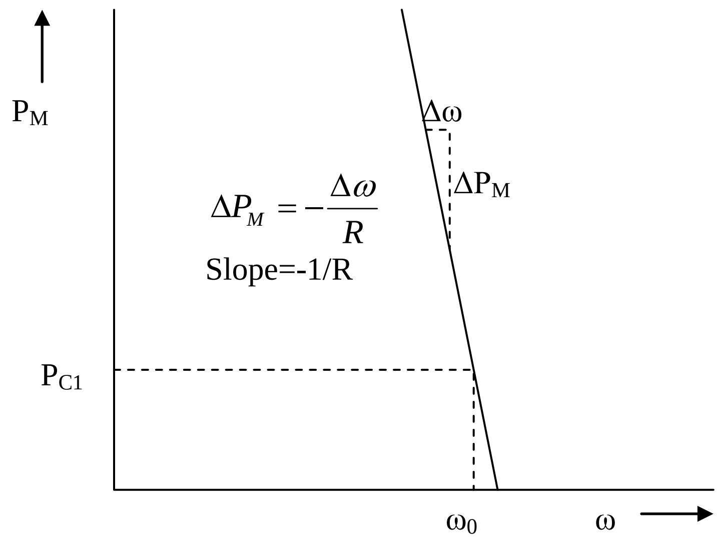

Fig. 3

An important assumption behind Fig. 3 is that the adjustment to the generator set point, designated by $P_C=P_{C1}$, is done by a control system (as yet unstudied), called the secondary or supplementary control system, which results in $\omega=\omega_0$. The plot, therefore, provides an indication of what happens to the mechanical power P_M, and the frequency ω, following a disturbance from this pre-disturbance condition for which $P_M=P_{C1}$ and $\omega= \omega_0$.

It is clear from Fig. 3 that the "local" behavior is characterized by $\Delta P_M = -\frac{\Delta\omega}{R}$.

If we were to change the generation set point to $P_C = P_{C2}$, under the assumption that the secondary control that actuates such a change maintains ω_0, then the entire characteristic moves to the right, as shown in Fig. 4.

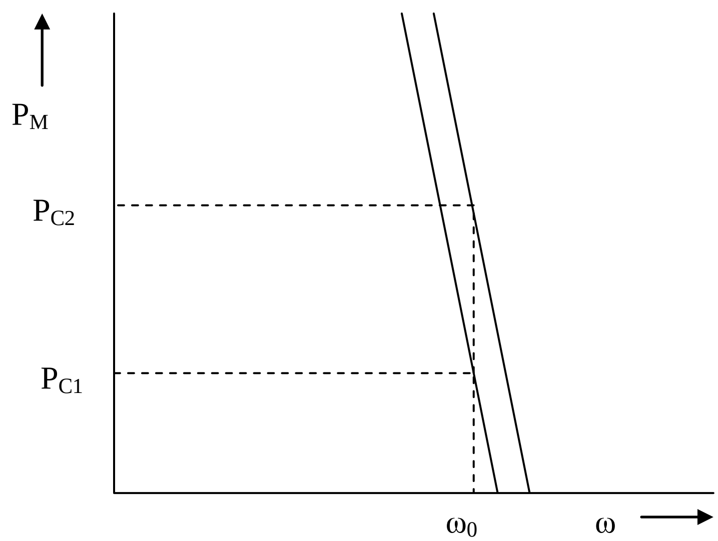

Fig. 4

We may invert Fig. 3, so that the power axis is on the vertical and the frequency axis is on the horizontal, as shown in Fig. 5.

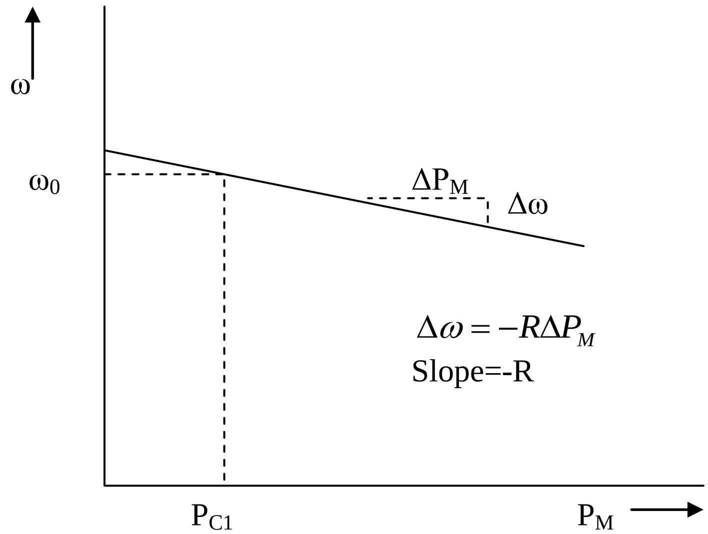

Fig. 5

Fig. 6 illustrates what happens when we change the generation set point from $P_C=P_{C1}$ to $P_C=P_{C2}$,

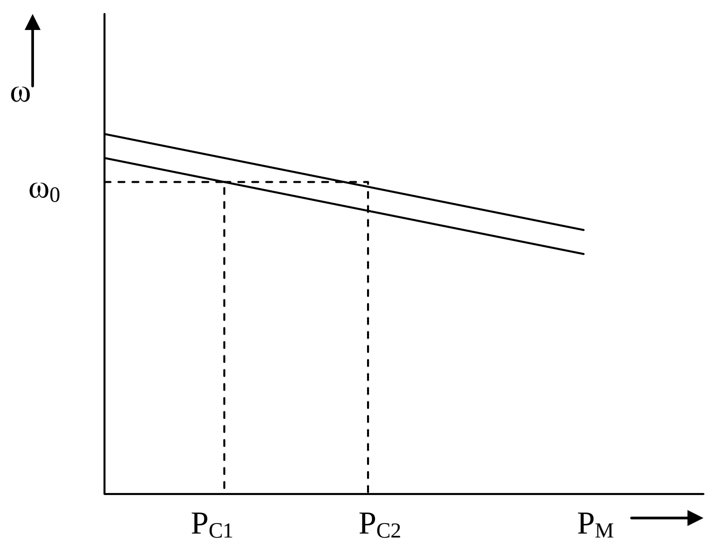

Fig. 6

It is conventional to illustrate the relationship of frequency ω and mechanical power P_M as in Figs. 5 and 6, rather than Figs. 3 and 4. (Regardless, however, be careful not to fall into the trap that it is showing P_M as the "cause" and ω as the "effect." As repeated now in different ways, they are both "effects" of the primary control system response to a frequency deviation caused by a load-generation imbalance).

From such a picture as Figs. 5 and 6, we obtain the terminology "droop," in that the primary control system acts in such a way so that the resulting frequency "droops" with increasing mechanical power.

The R constant, previously called the regulation constant, is also referred to as the droop setting.

4.0 Units

Recall eq. (11), repeated here for convenience.

$$\Delta P_M = \Delta P_C - \frac{\Delta\omega}{R} \tag{11}$$

With no change to the generation set point, i.e., $\Delta P_C=0$, then

$$\Delta P_M = -\frac{\Delta\omega}{R} \tag{12}$$

where we see that

$$R = -\frac{\Delta\omega}{\Delta P_M} \tag{13}$$

We see then that the units of R must be (rad/sec)/MW.

A more common way of specifying R is in per-unit, where we per-unitize top and bottom of eq. (13), so that:

$$R_{pu} = -\frac{\Delta\omega/\omega_0}{\Delta P_M/S_r} = -\frac{\Delta\omega_{pu}}{\Delta P_{Mpu}} \tag{14}$$

where ω_0=377 and S_r is the three-phase MVA rating of the machine. When specified this way, R relates fractional changes in ω to fractional changes in P_M.

It is also useful to note that

$$\Delta\omega_{pu} = \frac{\Delta\omega}{\omega_0} = \frac{2\pi\Delta f}{2\pi f_0} = \frac{\Delta f}{f_0} = \Delta f_{pu} \tag{15}$$

Thus, we see that per-unit frequency is the same independent of whether it is computed using rad/sec or Hz, as long as the proper base is used.

Therefore, eq. (14) can be expressed as

$$R_{pu} = -\frac{\Delta\omega_{pu}}{\Delta P_{Mpu}} = -\frac{\Delta f_{pu}}{\Delta P_{Mpu}} \tag{16}$$

5.0 Example

Consider a 2-unit system, with data as follows:

Gen A: S_{RA}=100 MVA, R_{puA}=0.05

Gen B: S_{RB}=200 MVA, R_{puB}=0.05

The load increases, with appropriate primary speed control (but no secondary control) so that the steady-state frequency deviation is 0.01 Hz. What are ΔP_A and ΔP_B?

Solution:

$$\Delta f_{pu} = \frac{-0.01}{60} = -0.000167\,pu$$

$$\Delta P_{puA} = \frac{-\Delta f_{pu}}{R_{puA}} = \frac{-(-0.000167)}{0.05} = 0.0033\,pu$$

$$\Delta P_{puB} = \frac{-\Delta f_{pu}}{R_{puB}} = \frac{-(-0.000167)}{0.05} = 0.0033\,pu$$

Note!!! Since $R_{puA}=R_{puB}$ (and since the steady-state frequency is the same everywhere in the system), we get $\Delta P_{puA}=\Delta P_{puB}$, i.e., the generators "pick up" the same amount of per-unit power (given on their own base).

But let's look at it in MW:

$$\Delta P_A = \Delta P_{puA} S_{RA} = 0.0033(100) = 0.33MW$$

$$\Delta P_B = \Delta P_{puB} S_{RB} = 0.0033(200) = 0.66MW$$

Conclusion: When two generators have the same per-unit droop, they "pick-up" (compensate for load-gen imbalance) in proportion to their MVA rating.

In North America, droop constants for most units are set at about 0.05 (5%).

6.0 Multimachine case

Now let's consider a general multimachine system having K generators. From eq. (16), for a load change of ΔP MW, the ith generator will respond according to:

$$R_{pui} = -\frac{\Delta f/60}{\Delta P_{Mi}/S_{Ri}} \Rightarrow \Delta P_{Mi} = \frac{-S_{Ri}}{R_{pui}}\frac{\Delta f}{60} \qquad (17)$$

The total change in generation will equal ΔP, so:

$$\Delta P = -\left[\frac{S_{R1}}{R_{1pu}} + \ldots + \frac{S_{RK}}{R_{Kpu}}\right]\frac{\Delta f}{60} \qquad (18)$$

Solving for Δf results in

$$\frac{\Delta f}{60} = \frac{-\Delta P}{\left[\frac{S_{R1}}{R_{1pu}} + \ldots + \frac{S_{RK}}{R_{Kpu}}\right]} \qquad (19)$$

Substitute eq. (19) back into eq. (17) to get:

$$\Delta P_{Mi} = \frac{-S_{Ri}}{R_{pui}} \frac{\Delta f}{60} = \frac{S_{Ri}}{R_{pui}} \frac{\Delta P}{\left[\frac{S_{R1}}{R_{1pu}} + \ldots + \frac{S_{RK}}{R_{Kpu}} \right]} \qquad (20)$$

If all units have the same per-unit droop constant, i.e., $R_{pui}=R_{1pu}=\ldots=R_{Kpu}$, then eq. (20) becomes:

$$\Delta P_{Mi} = \frac{-S_{Ri}}{R_{pui}} \frac{\Delta f}{60} = \frac{S_{Ri} \Delta P}{\left[S_{R1} + \ldots + S_{RK} \right]} \qquad (21)$$

which generalizes our earlier conclusion for the two-machine system that units "pick up" in proportion to their MVA ratings. This conclusion should drive the way an engineer performs contingency analysis of generator outages, i.e., one should redistribute the lost generation to the remaining generators in proportion to their MVA rating, as given by eq. (21).