Module B3

3.1 Sinusoidal steady-state analysis (single-phase), a review
3.2 Three-phase analysis

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Chapter 2: AC Voltage, Current and Power
2.1 Sources and Power
2.2 Resistors, Inductors, and Capacitors

Chapter 4: Polyphase systems
4.1 Three-phase systems
4.2 Line-Line Voltages
Three-phase power

All of what we have done in the previous slides is for “single phase” circuits. However, almost all transmission systems in the US are 3-phase AC systems (the only exceptions are a few DC transmission lines). Three-phase AC is preferred over single-phase AC because a 3-phase system provides constant power (not pulsating as we saw before) and because the cost/MW of transmission capacity is more attractive.

A wind generator also supplies 3-phase power. A circuit diagram for the stator of a typical 3-phase wind generator is provided in the next two slides.
The identified voltages are referred to as “line-to-neutral voltages,” or “phase voltages.”
The identified voltages are referred to as “line-to-line voltages,” or just “line voltages.”
Phasor diagram for line-neutral voltages

What is rotating?
- The peak value of the sinusoid, which is projected onto one of the axes to obtain the instantaneous value of the quantity.

\[
\hat{V}_{bn} = \hat{V}_{an} \angle -120^\circ \\
\hat{V}_{cn} = \hat{V}_{an} \angle +120^\circ 
\]

www.animations.physics.unsw.edu.au/jw/phasor-addition.html
Phasor diagram for line-line voltages

\[ V_{ab} = V_{ab} \angle -120^\circ \]
\[ V_{ca} = V_{ab} \angle +120^\circ \]
Relating phase and line voltages

\[
\begin{align*}
\hat{V}_{ab} &= \sqrt{3}\hat{V}_{an} \angle 30^\circ \\
\hat{V}_{bc} &= \sqrt{3}\hat{V}_{bn} \angle 30^\circ \\
\hat{V}_{ca} &= \sqrt{3}\hat{V}_{cn} \angle 30^\circ 
\end{align*}
\]
Balanced conditions

Balanced 3-phase conditions have:

- Line and phase voltages related as in previous slides.
- \( Z_a = Z_b = Z_c \)

This results in:

\[
\begin{align*}
\hat{I}_b &= \hat{I}_a \angle -120^\circ, \\
\hat{I}_c &= \hat{I}_a \angle +120^\circ, \\
\hat{I}_n &= 0
\end{align*}
\]

Note: In Wye-connected loads, the line current and the phase current (current through \( Z_a \)) are identical.
Under balanced conditions, we may perform single-phase analysis on a “lifted-out” a-phase and neutral circuit, as shown below.

\[ \hat{I}_a \rightarrow \hat{I}_b \rightarrow \hat{I}_n \rightarrow \hat{I}_c \]

\[ \hat{V}_{an} \]

\[ Z_a, Z_b, Z_c \]
Per-phase analysis

Now it is clear that:

$$\hat{I}_a = \frac{\hat{V}_{an}}{Z_a} \quad S_{1\phi} = \hat{V}_{an} \hat{I}_a^* = P_{1\phi} + jQ_{1\phi}$$

Also, we still have:

$$P_{1\phi} = V_{an} I_a \cos \theta, \quad Q_{1\phi} = V_{an} I_a \sin \theta$$
Following the single-phase analysis, one may then compute the 3-phase quantities according to:

\[ S_{3\phi} = 3S_{1\phi} \Rightarrow P_{3\phi} = 3P_{1\phi}, \quad Q_{3\phi} = 3Q_{1\phi} \]
Three phase power relations

The previous power relations utilize line-to-neutral voltages and line currents. Power may also be computed using line voltages, as developed in what follows:

\[ P_{1\phi} = V_{an} I_a \cos \theta \]

\[ \hat{V}_{ab} = \sqrt{3} \hat{V}_{an} \angle 30^\circ \Rightarrow V_{ab} = \sqrt{3} V_{an} \Rightarrow V_{an} = \frac{V_{ab}}{\sqrt{3}} \]

\[ P_{1\phi} = \frac{V_{ab}}{\sqrt{3}} I_a \cos \theta = \frac{V_{ab}}{\sqrt{3}} \sqrt{3} I_a \cos \theta = \frac{V_{ab} \sqrt{3}}{3} I_a \cos \theta \]

\[ P_{3\phi} = 3P_{1\phi} = 3 \frac{V_{ab} \sqrt{3}}{3} I_a \cos \theta = \sqrt{3} V_{ab} I_a \cos \theta \]

Likewise, we may develop that

\[ Q_{3\phi} = \sqrt{3} V_{ab} I_a \sin \theta \]
Three phase power relations

In summary:

\[
S_{3\phi} = 3S_{1\phi} \implies P_{3\phi} = 3P_{1\phi}, \quad Q_{3\phi} = 3Q_{1\phi}
\]

\[
P_{1\phi} = V_{an}I_a \cos \theta \quad Q_{1\phi} = V_{an}I_a \sin \theta
\]

\[
P_{3\phi} = \sqrt{3}V_{ab}I_a \cos \theta \quad Q_{3\phi} = \sqrt{3}V_{ab}I_a \sin \theta
\]

**Note 1:** In Wye-connections, the power factor angle \( \theta \) is the angle by which the line-to-neutral voltage \( \hat{V}_{an} \) leads the phase current \( \hat{I}_a \). It is not the angle by which the line-to-line voltage \( \hat{V}_{ab} \) leads the phase current. More generally, the power factor angle at two terminals is the angle by which the voltage across those terminals leads the current into the positive terminal.

**Note 2:** The text uses notation \( V_{LL} \) for \( V_{ab} \).