Module B3

3.1 Sinusoidal steady-state analysis (single-phase), a review

3.2 Three-phase analysis

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Chapter 2: AC Voltage, Current and Power
2.1 Sources and Power
2.2 Resistors, Inductors, and Capacitors

Chapter 4: Polyphase systems
4.1 Three-phase systems
4.2 Line-Line Voltages
Three-phase power

All of what we have done in the previous slides is for “single phase” circuits. However, almost all transmission systems in the US are 3-phase AC systems (the only exceptions are a few DC transmission lines). Three-phase AC is preferred over single-phase AC because the investment and operating costs per MW of transmission capacity are more attractive, and because a 3-phase system provides constant power (not pulsating as we saw before).

You can see this in the next slide.
Three single phase systems? Or one three-phase system?

<table>
<thead>
<tr>
<th>Three single phase systems</th>
<th>One three-phase system</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 wires</td>
<td>4 wires; capital savings!</td>
</tr>
<tr>
<td>Each neutral carries full load current</td>
<td>Neutral carries little or no current and can therefore be much smaller; capital savings!</td>
</tr>
<tr>
<td>Each neutral carries full load current</td>
<td>Neutral carries little or no current, therefore has little losses; operational savings!</td>
</tr>
<tr>
<td>Each single phase circuit delivers instantaneous power that varies at $2\omega$. Large generators &amp; motor loads vibrate.</td>
<td>We will show that three phase circuits deliver constant instantaneous power; large generators and motors run smoothly.</td>
</tr>
</tbody>
</table>
Three-phase power

AC generators on the grid supply 3-phase power. A circuit diagram for the stator of a typical 3-phase generator is provided in the next two slides.
The identified voltages are referred to as “line-to-neutral voltages,” or “phase voltages.”
The identified voltages are referred to as “line-to-line voltages,” or just “line voltages.”
Phasor diagram for line-neutral (phase) voltages

What is rotating?

The peak value of the sinusoid; this peak value is projected onto the positive horizontal axis to obtain the instantaneous value of the quantity, a concept equivalent to writing $v_{an}(t) = V_{peak} \sin \omega t$.

$$\hat{V}_{bn} = \hat{V}_{an} \angle -120^\circ$$

$$\hat{V}_{cn} = \hat{V}_{an} \angle +120^\circ$$

www.animations.physics.unsw.edu.au/jw/phasor-addition.html
Phasor diagram for line-line (line) voltages

\[ \hat{V}_{ca} \rightarrow \hat{V}_{ab} \rightarrow \hat{V}_{bc} \]

Rotation

\[ \hat{V}_{ab} \angle -120^\circ \]

\[ \hat{V}_{ca} = \hat{V}_{ab} \angle +120^\circ \]
Relating phase and line voltages

\[
\hat{V}_{ab} - \hat{V}_{an} + \hat{V}_{bn} = 0 \Rightarrow \hat{V}_{ab} - \hat{V}_{an} + \hat{V}_{an} \angle -120^\circ = 0
\]

\[
\Rightarrow \hat{V}_{ab} - \hat{V}_{an} (1-1\angle -120^\circ) = 0
\]

\[
\Rightarrow \hat{V}_{ab} = \hat{V}_{an} (1-1\angle -120^\circ)
\]

\[
\Rightarrow \hat{V}_{ab} = \hat{V}_{an} \sqrt{3} \angle 30^\circ
\]
Relating phase and line voltages

\[ \hat{V}_{ab} = \sqrt{3} \hat{V}_{an} \angle 30^\circ \]
\[ \hat{V}_{bc} = \sqrt{3} \hat{V}_{bn} \angle 30^\circ \]
\[ \hat{V}_{ca} = \sqrt{3} \hat{V}_{cn} \angle 30^\circ \]
Wye-connected sources and loads
Balanced conditions

Balanced 3-phase conditions have:

- Line and phase voltages related as in previous slides.
- $Z_a = Z_b = Z_c$

This results in: $\hat{I}_b = \hat{I}_a \angle -120^\circ$, $\hat{I}_c = \hat{I}_a \angle +120^\circ$, $\hat{I}_n = 0$

Note: In Wye-connected loads, the line current and the phase current (current through $Z_a$) are identical.
**Per-phase analysis**

*Under balanced conditions*, we may perform single-phase analysis on a “lifted-out” a-phase and neutral circuit, as shown below.

![Diagram of per-phase analysis](chart)

- $\hat{I}_a$ is the phase current for phase a.
- $\hat{I}_b$ is the phase current for phase b.
- $\hat{I}_c$ is the phase current for phase c.
- $\hat{V}_{an}$ is the voltage across the a-phase and neutral circuit.
- $Z_a$, $Z_b$, and $Z_c$ are the impedances for phases a, b, and c, respectively.
Per-phase analysis

Now it is clear that:

\[ \hat{I}_a = \frac{\hat{V}_{an}}{Z_a} \]

\[ S_{1\phi} = \hat{V}_{an} \hat{I}_a^* = P_{1\phi} + jQ_{1\phi} \]

Also, we still have:

\[ P_{1\phi} = V_{an} I_a \cos \theta, \quad Q_{1\phi} = V_{an} I_a \sin \theta \]
After we perform the single-phase analysis, we may then compute the 3-phase quantities according to:

\[ S_{3\phi} = 3S_{1\phi} \Rightarrow P_{3\phi} = 3P_{1\phi}, \quad Q_{3\phi} = 3Q_{1\phi} \]
Three phase power relations

The previous power relations utilize line-to-neutral voltages and line currents. Power may also be computed using line voltages, as developed in what follows:

\[ P_{1\phi} = V_{an} I_a \cos \theta \]

\[ \hat{V}_{ab} = \sqrt{3}\hat{V}_{an} \angle 30^\circ \Rightarrow V_{ab} = \sqrt{3}V_{an} \Rightarrow V_{an} = \frac{V_{ab}}{\sqrt{3}} \]

\[ P_{1\phi} = \frac{V_{ab}}{\sqrt{3}} I_a \cos \theta = \frac{V_{ab}}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} I_a \cos \theta = \frac{V_{ab}}{3} I_a \cos \theta \]

\[ P_{3\phi} = 3P_{1\phi} = 3 \frac{V_{ab}}{3} \sqrt{3} I_a \cos \theta = \sqrt{3}V_{ab} I_a \cos \theta \]

Likewise, we may develop that

\[ Q_{3\phi} = \sqrt{3}V_{ab} I_a \sin \theta \]
Three phase power relations

In summary:

\[ S_{3\phi} = 3S_{1\phi} \Rightarrow P_{3\phi} = 3P_{1\phi}, \quad Q_{3\phi} = 3Q_{1\phi} \]

\[ P_{1\phi} = V_{an} I_a \cos \theta \]

\[ P_{3\phi} = \sqrt{3} V_{ab} I_a \cos \theta \]

\[ Q_{1\phi} = V_{an} I_a \sin \theta \]

\[ Q_{3\phi} = \sqrt{3} V_{ab} I_a \sin \theta \]

**Note 1:** In Wye-connections, the power factor angle \( \theta \) is the angle by which the line-to-neutral voltage \( \hat{V}_{an} \) leads the phase current \( \hat{I}_a \). It is not the angle by which the line-to-line voltage \( \hat{V}_{ab} \) leads the phase current. More generally, the power factor angle at two terminals is the angle by which the voltage across those terminals leads the current into the positive terminal.

**Note 2:** You may see notation \( V_L \) or \( V_{LL} \) for \( V_{ab} \).