Module G1

Electric Power Generation and Machine Controls

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Module G1: HW Probs 1, 4, 8, 9, 10, 11. Due: 3/14. (To turn in).
Overview

• Energy transformation into electrical form

• Generation operation
  – Revolving magnetic field
  – Phasor diagram
  – Equivalent Circuit
  – Power relationships
  – Generator pull-out power

• Excitation control

• Turbine speed control
Energy Transformation

• Transformation processes:
  – Chemical
  – photovoltaic
  – electromechanical

• Electromechanical: conversion of energy from coal, petroleum, natural gas, uranium, water flow, geothermal, and wind into electrical energy

• Turbine-synchronous AC generator conversion process most common in industry today
Repairs to the overhang insulation of this 200MW generator rotor were carried out by experienced tradesmen working on shift to ensure the earliest possible return to service.
Feedback Control Systems for Synchronous Generators

• Turbine-generator basic form

![Diagram of turbine-generator system]

• Governor and excitation systems are known as *feedback control systems*; they control the speed and voltage respectively.
The negative terminal for each phase is 180 degrees from the corresponding positive terminal.
Synchronous Machine Structure

Two-pole
Salient pole construction

Four-pole
Salient pole construction
Salient Pole Construction

Smooth rotor Construction
Also known as round rotor or cylindrical rotor construction
A Four Pole Machine (p=4)  
(Salient Pole Structure)
Generation Operation

- The generator is classified as a synchronous machine because it is only at synchronous speed that it can develop electromagnetic torque.

\[ \omega_m = \frac{2}{p}(\omega_e) \]

- \( \omega_m \) = mechanical speed in rad/sec
  - \( p=2 \): 1 mechanical rotation gives 1 electrical rotation.
  - \( p=4 \): 1 mechanical rotation gives 2 electrical rotations.
  - \( p=6 \): 1 mechanical rotation gives 3 electrical rotations.
  - \( \ldots \)

\[ \omega_e = \frac{p}{2}(\omega_m) \]

- \( \omega_e = 2\pi f \) = electrical speed (frequency) in rad/sec

\[ N_s = (\omega_m) \frac{rad}{sec} \times \frac{1\text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ sec}}{\text{min}} = (\omega_m) \frac{30}{\pi} \left( \frac{2}{p} \omega_e \right) \frac{30}{\pi} = \left( \frac{2}{p} 2\pi f \right) \frac{30}{\pi} = \frac{120}{p} f \]

- \( \omega_e \) = machine speed in RPM
For 60 Hz operation (f=60)

- Synchronous generator

Rotor construction

- **Round Rotor**
  - Two pole
    - \( N_s = 3600 \text{ rpm} \)
  - Four Pole
    - \( N_s = 1800 \text{ rpm} \)
  - Eight Pole
    - \( N_s = 900 \text{ rpm} \)

- **Salient Pole**
For 60 Hz operation (f=60)

<table>
<thead>
<tr>
<th>No. of Poles (p)</th>
<th>Synchronous speed (Ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3600</td>
</tr>
<tr>
<td>4</td>
<td>1800</td>
</tr>
<tr>
<td>6</td>
<td>1200</td>
</tr>
<tr>
<td>8</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>$N_s = \frac{120}{p} f$</td>
</tr>
<tr>
<td>10</td>
<td>720</td>
</tr>
<tr>
<td>12</td>
<td>600</td>
</tr>
<tr>
<td>14</td>
<td>514</td>
</tr>
<tr>
<td>16</td>
<td>450</td>
</tr>
<tr>
<td>18</td>
<td>400</td>
</tr>
<tr>
<td>20</td>
<td>360</td>
</tr>
</tbody>
</table>

Few poles $\Rightarrow$ High speed  
Many poles $\Rightarrow$ Slow speed  

$(p)$ represents the number of poles, $(f)$ is the frequency, and $(Ns)$ is the synchronous speed.
Fact: hydro turbines are slow speed, steam turbines are high speed.

Do hydro-turbine generators have few poles or many?
Many, because they rotate slowly.

Do steam-turbine generators have few poles or many?
Few, because they rotate very fast.

Fact: salient pole incurs significant mechanical stress at high speed.

Do steam-turbine generators have salient poles or smooth?
Smooth, because salient pole creates too much mechanical stress at high speed.

Fact: Salient pole rotors are cheaper to build than smooth.

Do hydro-turbine generators have salient poles or smooth?
Salient, because it is cheaper.
Generation Operation

• A magnetic field is provided by the DC-current carrying field winding which induces the desired AC voltage in the armature winding.

• Field winding is always located on the rotor where it is connected to an external DC source via slip rings and brushes or to a revolving DC source via a special brushless configuration.

• Armature winding is located on the stator where there is no rotation.

• The armature consists of three windings all of which are wound on the stator, physically displaced from each other by 120 degrees.
Synchronous Machine Structure

- Voltage induced in phase wdgs by flux from rotating field wdg
- Current in phase wdgs produces flux that also induces voltage in phase wdgs.
Rotating magnetic field

- There are 3 stator windings, separated in space by 120°, with each carrying AC, separated in time by $\omega_0 t=120°$.
- Each of these three currents creates a magnetic field in the air gap of the machine. Let’s look at only the a-phase:

$$B_a(\alpha, t) = B'_{\text{max}} \cos(\omega_0 t + \angle I_a) \cos \alpha$$

- $B_a$, in webers/m$^2$, is flux density from the a-phase current (We could also use $H_a$, which is magnetic field strength in amp-turns/m or Oersteds, related to $B_a$ by $B_a = \mu H_a$).
- $\alpha$ is the spatial angle along the air gap.
- For any time $t$, $\alpha=0,180$ are spatial maxima (absolute value of flux is maximum at these points).
Rotating magnetic field – temporal characterization

\[ B_a(\alpha,t) = B'_\text{max} \cos(\omega_0 t + \angle I_a) \cos \alpha \]

Let’s fix \( \alpha = 0 \) and see what happens at \( \omega_0 t_1 \), such that \( \omega_0 t_1 + \angle I_a \) is just less than \( \pi/2 \).
Rotating magnetic field – spatial characterization

Now let’s fix $t=t_4$ ($\omega_0 t_4 = \omega_0 t_1 + 270$):

$$B_a(\alpha, t) = B'_\text{max} \cos(\omega_0 t_4 + \angle I_a)\cos \alpha$$

and see what happens at

$\alpha=0, \alpha=45, \alpha=90, \alpha=135, \alpha=180, \alpha=225, \alpha=270, \alpha=315$.

One observes that the magnetic field is sinusoidally distributed around the airgap.
Rotating magnetic field

- Now consider the magnetic field from all windings simultaneously.

\[ B_a(\alpha, t) = B_{\text{max}}' \cos(\omega_0 t + \angle I_a) \cos \alpha \quad (1) \]
\[ B_b(\alpha, t) = B_{\text{max}}' \cos \left( \omega_0 t + \angle I_a - \frac{2\pi}{3} + \cos \left( \alpha - \frac{2\pi}{3} \right) \right) \quad (2) \]
\[ B_c(\alpha, t) = B_{\text{max}}' \cos \left( \omega_0 t + \angle I_a + \frac{2\pi}{3} \right) \cos \left( \alpha + \frac{2\pi}{3} \right) \quad (3) \]

- Add them up, then perform trig manipulation to obtain:

\[ B_{abc}(\alpha, t) = \frac{3B_{\text{max}}'}{2} \cos(\omega_0 t + \angle I_a - \alpha) \quad (4) \]

Notice that locations of the spatial maxima in (1), (2), and (3) do not vary w/time (i.e., although the value of the spatial maxima changes, their locations do not), indicated by:

\[ B_a(\alpha, t) = B_{\text{max}}' \cos(\omega_0 t + \angle I_a) \cos \alpha \Rightarrow \alpha = \{0, \pi\} \]
\[ B_b(\alpha, t) = B_{\text{max}}' \cos \left( \omega_0 t + \angle I_a - \frac{2\pi}{3} + \cos \left( \alpha - \frac{2\pi}{3} \right) \right) \Rightarrow \alpha - \frac{2\pi}{3} = \{0, \pi\} \Rightarrow \alpha = \left\{ \frac{2\pi}{3}, \frac{5\pi}{3} \right\} \]
\[ B_c(\alpha, t) = B_{\text{max}}' \cos \left( \omega_0 t + \angle I_a + \frac{2\pi}{3} \right) \cos \left( \alpha + \frac{2\pi}{3} \right) \Rightarrow \alpha + \frac{2\pi}{3} = \{0, \pi\} \Rightarrow \alpha = \left\{ -\frac{2\pi}{3}, \frac{\pi}{3} \right\} \]

The location around the air gap (specified by \( \alpha \)), at any given time, for which the field is max, is not a function of t.

But the spatial maxima of (4) has spatial location which does vary w/time, This is a characteristic of a rotating magnetic field.

\[ B_{abc}(\alpha, t) = \frac{3B_{\text{max}}'}{2} \cos(\omega_0 t + \angle I_a - \alpha) \]
\[ \Rightarrow \omega_0 t + \angle I_a - \alpha = \{0, \pi\} \Rightarrow \alpha = \{\omega_0 t + \angle I_a, \omega_0 t + \angle I_a - \pi\} \]

The location around the air gap (specified by \( \alpha \)), at any given time, for which the field is max, is a function of t.
Rotating magnetic field

One observes this using the following:

http://educypedia.karadimov.info/library/rotating_field.swf

The shape of the individual winding fields $B_a, B_b, B_c$, throughout the air gap are spatially fixed, but their amplitudes pulsate up and down.

In contrast, the amplitude of the composite is fixed in time, but it rotates in space. What you see in the visualization are just the variation of the maximum flux point.

The plot on the middle right, is misleading. It should show a single period of a sinusoidal waveform rather than a square wave.
Equivalent circuit model for synchronous machine

• Each stator winding a, b, c will have a voltage induced in it proportional to the speed of rotation of the rotor, the number of turns of the winding N, and the flux produced by the field winding ϕ.

• Since the speed of rotation of the rotor must equal the synchronous speed, and since the synchronous speed is set by the grid frequency f according to $N_S = \frac{120f}{p}$ where p is number of poles, the induced rms voltage will be:

$$E_f = 4.44 K_w fN \phi$$

Here, $K_w$, called the winding factor, is a reduction factor between 0.85 to 0.95 that accounts for the distribution of the armature coils.

We call $E_f$ the excitation voltage because it is produced by the field which is also known as the machine’s excitation. It is also sometimes called the “internal voltage” because it is the voltage measured when the machine is unloaded (open-circuited).
Equivalent circuit model for synchronous machine

• The line-to-neutral rms terminal voltage of the a-phase winding is given by $V_t$. We will assume this is the reference, so that:

$$\bar{V}_t = V_t \angle 0^\circ$$

• The excitation voltage is also a phasor, with magnitude and angle given by:

$$\bar{E}_f = E_f \angle \delta$$

• When the machine is unloaded, the terminal voltage equals the excitation voltage, i.e.,

$$\bar{V}_t = \bar{E}_f \Rightarrow E_f = V_t, \quad \delta = 0^\circ$$
Equivalent circuit model for synchronous machine

- However, when the machine is loaded, i.e., when there is a current flowing through the a-phase winding, then the terminal voltage will differ from the excitation voltage due to voltage drops caused by:

1. Resistance of the phase winding; we will ignore this.
2. Armature reaction: This is the interaction of the flux from the (rotating) field winding and the flux from the a-phase winding current. It tends to decrease the terminal voltage. It is represented by a reactance $X_{ar}$.
3. Flux leakage: There is some flux developed by the field winding which does not link with the armature winding. This leakage is captured by a reactance $X_l$.

$$\bar{V}_t = \bar{E}_f - j\left( X_{ar} + X_l \right) \bar{I}_a$$

- We define the synchronous reactance as $X_s = X_{ar} + X_l$, so that

$$\bar{V}_t = \bar{E}_f - jX_s \bar{I}_a$$
Equivalent circuit model for synchronous machine

\[
\vec{V}_t = \vec{E}_f - j(X_{ar} + X_l)\vec{I}_a
\]

\[
\vec{V}_t = \vec{E}_f - jX_s\vec{I}_a
\]

All voltages and currents on the above diagram are phasors.
Equivalent circuit model for synchronous machine

All voltages and currents on the above diagram are phasors.
Equivalent circuit model for synchronous machine

You can perform per-phase equivalent analysis or you can perform per-unit analysis.

In per-phase, \( \overline{E}_f \) and \( \overline{V}_t \) are both line to neutral voltages, \( \overline{I}_a \) is the line current, and \( Z \) is the impedance of the equivalent Y-connected load.

In per-unit, \( \overline{E}_f \) and \( \overline{V}_t \) are per-unit voltages, \( \overline{I}_a \) is the per unit current, and \( Z \) is the per unit load impedance.
Leading and Lagging Generator Operation

Let \( Z = R + jX = |Z| \angle \theta \), \( \bar{V}_t = V_t \angle \theta_v \)

From the equivalent circuit,

\[
\bar{I}_a = \frac{\bar{V}_t}{Z} = \frac{V_t \angle \theta_v}{|Z| \angle \theta} = \frac{V_t}{|Z|} \angle (\theta_v - \theta)
\]

So here we see that \( \theta_i = \theta_v - \theta \) \( \Rightarrow \) \( \theta = \theta_v - \theta_i \)
Assign $\overline{V}_t$ as the reference: $\theta_v = 0$

Then,

$$I_a = \frac{\overline{V}_t}{Z} = \left| \frac{V_t}{Z} \right| \angle (\theta_v - \theta) = \left| \frac{V_t}{Z} \right| \angle (-\theta)$$

So here we see that $\theta_i = -\theta$

This gives an easy way to remember the relation between load, sign of current angle, leading/lagging, and sign of power angle.
### Leading and Lagging Generator Operation

Circle the correct answer in each of the six rows for each column.

<table>
<thead>
<tr>
<th>Inductive load</th>
<th>Capacitive load</th>
</tr>
</thead>
<tbody>
<tr>
<td>load absorbs/supplies $Q$</td>
<td>load absorbs/supplies $Q$</td>
</tr>
<tr>
<td>gen absorbs/supplies $Q$</td>
<td>gen supplies/absorbs $Q$</td>
</tr>
<tr>
<td>$X &gt; ? &lt; 0$</td>
<td>$X &gt; ? &lt; 0$</td>
</tr>
<tr>
<td>$\theta &gt; ? &lt; 0$</td>
<td>$\theta &gt; ? &lt; 0$</td>
</tr>
<tr>
<td>$\theta_i &gt; ? &lt; 0$</td>
<td>$\theta_i &gt; ? &lt; 0$</td>
</tr>
<tr>
<td>current is leading/lagging</td>
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## Leading and Lagging Generator Operation

### Answers

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Equivalent circuit model for synchronous machine

From KVL: \[ \bar{E}_f = \bar{V}_t + jX_s \bar{I}_a \]
Phasor Diagram for Equivalent Circuit

\[ \vec{E}_f = \vec{V}_t + jX_s \vec{I}_a \]

This equation gives directions for constructing the phasor diagram.

1. Draw \( \vec{V}_t \) phasor
2. Draw \( \vec{I}_a \) phasor
3. Scale \( \vec{I}_a \) phasor magnitude by \( X_s \) and rotate it by 90 degrees.
4. Add scaled and rotated vector to \( \vec{V}_t \)

Try it for lagging case.
This equation gives directions for constructing the phasor diagram.

1. Draw \( V_t \) phasor
2. Draw \( I_a \) phasor
3. Scale \( I_a \) phasor magnitude by \( X_s \) and rotate it by 90 degrees.
4. Add scaled and rotated vector to \( V_t \)

You do it for leading case.

Question: What does the condition (leading or lagging) imply about the magnitude of the internal voltage \( E_f \).
Let's define the angle that $E_f$ makes with $V_t$ as $\delta$

$$\bar{E}_f = \bar{V}_t + jX_s \bar{I}_a$$

For generator operation (power supplied by machine), this angle is always positive.

For motor operation, this angle is negative.