Module B3
Three Phase Analysis
B3.2

Balanced Three Phase Circuits
Delta Connection

\[ V_{ab} \quad V_{cb} \quad V_{ca} \]

\[ I_{AA} \quad I_{BB} \quad I_{CC} \]

\[ z_\Delta \quad z_\Delta \quad z_\Delta \]

\[ I_{AB} \quad I_{AB} \quad I_{CA} \]

\[ V_{AB} \quad V_{BC} \quad V_{CA} \]
\[ V_{AB} = V_\phi \]
\[ I_{AB} = I_\phi \angle 0^\circ \]
\[ I_{BC} = I_\phi \angle -120^\circ \]
\[ I_{CA} = I_\phi \angle +120^\circ \]

\[ I_{aA} = I_{AB} - I_{CA} = I_\phi \angle 0^\circ - I_\phi \angle 120^\circ = \sqrt{3} I_\phi \angle -30^\circ \]

\[ I_{bB} = I_{BC} - I_{AB} = I_\phi \angle -120^\circ - I_\phi \angle 0^\circ = \sqrt{3} I_\phi \angle -150^\circ \]

\[ I_{cC} = I_{CA} - I_{BC} = I_\phi \angle 120^\circ - I_\phi \angle -120^\circ = \sqrt{3} I_\phi \angle 90^\circ \]
Delta connection:

Line-line voltages equal phase voltages

Line currents are $\sqrt{3}$ times phase currents in magnitude and lag them by 30 degrees in angle.

**Delta**: Line **Currents** **Lag**: Delta-LC-LAG
WYE Connection

\[ a \rightarrow I_{aA} \]

\[ V_{ab} \]

\[ V_{ca} \]

\[ V_{bc} \]

\[ b \rightarrow I_{bB} \]

\[ B \]

\[ I_{BN} \]

\[ V_{BN} \]

\[ V_{AN} \]

\[ Z_Y \]

\[ A \]

\[ N \]

\[ C \]

\[ I_{AN} \]

\[ I_{CN} \]

\[ I_{cC} \]
$I_{aA}$ is equal to the phase current $I_{AN} = I_\phi$

\[ V_{AN} = V_\phi \angle 0^\circ \]
\[ V_{BN} = V_\phi \angle -120^\circ \]
\[ V_{CN} = V_\phi \angle +120^\circ \]

\[ V_{AB} = V_{AN} - V_{BN} = V_\phi - V_\phi \angle -120^\circ = \sqrt{3} V_\phi \angle 30^\circ \]

\[ V_{BC} = V_{BN} - V_{CN} = V_\phi \angle -120^\circ - V_\phi \angle 120^\circ = \sqrt{3} V_\phi \angle -90^\circ \]

\[ V_{CA} = V_{CN} - V_{AN} = V_\phi \angle 120^\circ - V_\phi \angle 0^\circ = \sqrt{3} V_\phi \angle 150^\circ \]
Wye connection:

Line-Line voltages are $\sqrt{3}$ times phase voltages in magnitude and lead them by 30 degrees in angle.

Y: Line-Line Voltages Lead: Y-LV-Lead-

Line currents equal phase currents
Power relations for three phase circuits

\[ P = 3V_\phi I_\phi \cos \theta \]

\[ P = \sqrt{3}V_L I_L \cos \theta \]

\[ Q = 3V_\phi I_\phi \sin \theta \]

\[ Q = \sqrt{3}V_L I_L \sin \theta \]

\[ S = P + jQ \]

\[ |S| = \sqrt{P^2 + Q^2} = \sqrt{3}V_L I_L \]

- can be used for Wye or Delta
- all quantities are magnitudes
- \( \theta \) is the angle for which leads \( \theta \)
- Positive Q is for power flowing into L load
- Negative Q is for power flowing “into” C load
Per-phase analysis of 3 phase circuits

- Convert all delta connections to Y connections using
  \[ Z_Y = \frac{Z_\Delta}{3} \]

- “Lift out” the a-phase to neutral circuit
- Perform single phase analysis using phase quantities and per phase powers
- Multiply all powers by 3 to get solution in terms of three phase powers.
Example B3.2

Three balanced three-phase loads are connected in parallel. Load 1 is Y-connected with an impedance of $150 + j50$; load 2 is delta-connected with an impedance of $900 + j600$; and load 3 is 95.04 kVA at 0.6 pf leading. The loads are fed from a distribution line with an impedance of $3 + j24$. The magnitude of the line-to-neutral voltage at the load end of the line is 4.8 kV.

a) Calculate the total complex power at the sending end of the line.

b) What percent of the average power at the sending end of the line is delivered to the load?
Solution:

Load 1: \( Z_1 = 150 + j50 \)
Load 2: \( Z_2 = \frac{(900 + j600)}{3} = 300 + j200 \)
Load 3:

\[
S_{3/\phi} = \frac{95040}{3} (0.6 - j0.8)
\]

\[
= 19,008 - j25,344 \text{ VA}
\]

\[
V_{\phi} = 4800 \text{ volts}
\]
Per phase equivalent circuit
Compute current from source

\[
I_\ell = \frac{4800}{150 + j50} + \frac{4800}{300 + j200} + \frac{19008 + j25344}{4800}
\]

\[
= 28.8 - j9.6 + 11.0769 - j7.3846 + 3.96 + j5.28
\]

\[
= I_1 + I_2 + I_3
\]

\[
= 43.8369 - j11.7046 A(rms) = 45.3725 \angle -14.949^\circ A(rms)
\]
Compute losses in the transmission line

\[ P_{loss} = 3 |I_{eff}|^2 \]
\[ Q_{loss} = 3 |I_{eff}|^2 \]
\[ R = 3(45.3725)^2(3) = 18,528.04 \text{ W} \]
\[ X = 3(45.3725)^2(24) = 148,224.34 \text{ VAR} \]

Compute power consumed by load 1.

\[ P_1 = 3 \left| 28.8 - j9.6 \right|^2 \]
\[ Q_1 = 3 \left| 28.8 - j9.6 \right|^2 \]
\[ (150) = 414,720 \text{ W} \]
\[ (50) = 138,240 \text{ VAR} \text{ (abs)} \]
Compute power consumed by load 2:

\[ P_2 = 3 \left| 11.0769 - j7.3846 \right|^2 \left(300\right) = 159,507.02 \text{ W} \]

\[ Q_2 = 3 \left| 11.0769 - j7.3846 \right|^2 \left(200\right) = 106,338.02 \text{ VAR(\text{abs})} \]

Compute power consumed by load 3:

\[ P_3 = 95,040(0.6) = 57,024 \text{ W} \]

\[ Q_3 = -95,040(0.8) = -76,032 \text{ VAR} \]
Add the powers to the three loads

\[ S_{total\,load\,,\,3\phi} = 631,251 + j168,546 \text{ VA (load end)} \]

We could have also obtained this from 3\(VI^*\) (see the “check” in the text)

Add in the losses to get sending end power:

\[ S_{sending\,,\,3\phi} = 631,251 + j168,546 + 18,528.04 + j148,224.34 \text{ VA} \]

\[ = 649779.04 + j316,770.34 \text{ VA} \]
Part b of the problem wants the percent of the power from the sending end that is actually delivered at the load. This is a measure of efficiency. Why is 100% of the power not delivered?

\[
\% P \text{ delivered} = \frac{631,251}{649,779.04} \times 100 = 97.148
\]