A Bayesian Approach for Short-Term Transmission Line Thermal Overload Risk Assessment

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***Abstract*—An on-line conductor thermal overload risk assessment method is presented in this paper. Bayesian time series models are used to model weather conditions along the transmission lines. An estimate of the thermal overload risk is obtained by Monte Carlo (MC) simulation. We predict the thermal overload risk for the next hour based on the current weather conditions and power system operating conditions. The predicted risk of thermal overload is useful for on-line decision making in a stressed operational environment.**

***Index Terms*—Bayesian analysis, Markov Chain Monte Carlo (MCMC), security assessment, transmission line thermal overload risk assessment.**

I. Introduction

THE power industry is experiencing deregulation worldwide. To survive in this environment, a successful competitor must offer high-quality service to customers and keep costs low. The market environment has changed the way power systems are planned and operated. Competition in the market requires more long distance deliveries than before and a deregulated market introduces high uncertainty into power systems. Current deregulated power systems are often operated in highly stressed operating conditions. Traditionally, engineers use deterministic methods with large safety margins to maintain relatively high security, which requires high costs for the competitors in the market. Improved measures of risk are needed for appropriate decision-making in the market environment [1]–[3].

Since the security of power systems depends on the availability of transmission lines, on-line tools for predicting the short-term thermal overload risk are useful in making decisions involving tradeoffs between economy and security. We present a framework to predict the next hour's transmission line thermal overload risk. This framework includes stochastic weather models and models pertaining to the thermal behaviors of transmission lines. An estimate of the thermal risk is obtained by Monte Carlo (MC) simulation.

Power engineers traditionally use a deterministic method to compute the thermal ratings for conductors [4], [5]. A set of severe weather conditions are assumed, and conservative thermal ratings are obtained. An alternative that has drawn much attention during the past few years is dynamic thermal circuit rating (DTCR) technologies, which require a real-time monitoring system. These monitoring systems may be weather-based, temperature-based, and/or sag/tension-based [6]. We do not attempt here to compare the merits of these monitoring systems, recognizing only that a company's decision to deploy one of them for a particular line is influenced by the investment and maintenance cost together with the expected benefit.

Deployment of weather-based monitors, as in [11], have been hampered by the need to monitor the weather conditions at many locations along a line. If the weather monitoring and communicating equipment cost is high, then this approach may not be cost effective, particularly for long lines. However, there is evidence that these costs are declining, as indicated in [7] and also at several web sites [8]. For example, one particularly interesting wind monitor is the sonic anemometer [9]; unlike cup anemometers with moving parts, the sonic anemometer has no stall speed and can therefore measure very low wind velocities. In addition, it is more accurate, and it can simultaneously measure wind speed and direction. It is highly likely that new developments in weather sensing technologies will continue [10] as high-quality weather information is useful in operating many kinds of power system equipment.

In this paper, we describe and illustrate an approach for utilizing weather-based monitoring systems to aid in making operational decisions that affect line loading. A key feature of this approach is that it is predictive, i.e., it utilizes a novel probabilistic weather prediction method to indicate transmission line risk for a future time period. Probabilistic methods for assessing weather conditions associated with transmission line ratings have been used in the past [12], [13], but these methods have been mainly used for providing seasonal ratings; in constrast, we focus on providing information based on very recent measurements for predicting near-term weather and thus relevant to near-term operating conditions.

We assume that hourly recordings of wind speed, wind direction, and ambient temperature are available for the last six days at multiple sites along the transmission line of interest. Telemetering data from weather meter stations capable of monitoring temperature, wind speed, and wind direction along the line is the most accurate means of gathering this information. An alternative, if appropriate real-time data are not available, is to use a very conservative, deterministic, estimate for wind speed, and to use temperature data obtained from the National Weather Service (NWS) observations available from the Internet. We approach the problem assuming that weather station measurements at appropriate locations are available as this results in the

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most general solution and can be applied in a degenerate form to the simpler alternative approach of using a conservative wind speed estimate with NWS temperature data. We have used NWS temperature and wind data to illustrate, recognizing the limitations of the latter,[1] as other data were not available to us.

This general solution approach uses Bayesian time series models to model the weather conditions. The models related the current weather conditions to those in the previous hour. We compute the parameters of the weather models from the most recent six days' (144 h) weather data. Then, using the current hour's weather data, we predict the next hour's weather conditions. With the predicted weather conditions, we compute the thermal overload risk for the next hour with MC simulation.

We describe the weather models in Section II. The next hour's ambient temperature, wind speed, and direction may be predicted with time series models. Solar radiation is computed with a deterministic model. A current–temperature model for overhead bare transmission lines is presented in Section III.

The spatial problem of relating temperature data from observing stations to transmission line environments is addressed in Section IV. An example in Section V illustrates how the weather models and current–temperature model are used in a MC simulation to identify the transmission line risk.

II. Weather Models

A transmission line's temperature depends on ambient temperature, wind speed, wind direction, and solar radiation. Time series models for wind direction, wind speed, and ambient temperature are developed to predict the next hour's weather conditions. Bayesian analysis is used to obtain the posterior distribution of the parameters in the wind direction model and the ambient temperature model. The wind speed model is a mixture model for which Bayesian analysis can be difficult. For that model, we obtain maximum-likelihood estimates (MLE) via the expectation–maximization (EM) algorithm.

We do not build a statistical model for solar radiation. Because solar radiation depends on the sizes, positions, and types of the clouds in the sky, a stochastic model for solar radiation would be very complex. Another reason for not building such a model is that detailed cloud data are not typically available. Therefore, a conservative deterministic model for solar radiation is used. Fortunately, the conductor temperature is not very sensitive to solar radiation at elevated temperatures [14].

A. Statistical Methods

Our basic approach is to construct a probability model relating current weather conditions to earlier conditions and unknown parameters. Depending on the model, we obtain inference for the unknown parameters using either the method of maximum-likelihood or Bayesian methods. Here, these approaches are reviewed briefly.

Let Y be the observed variables, and θ be model parameters. Let $P(Y|\theta)$ denote the probability model for the observed data. When viewed as a function of the unknown parameters θ, $P(Y|\theta)$ is known as the likelihood function. The MLE is the value $\hat{\theta}_{mle}$ that maximizes $P(Y|\theta)$. MLEs have a number of nice statistical properties in large samples. The maximum-likelihood approach is used for the wind speed model.

For the ambient temperature and wind direction models, we use the Bayesian approach to analyze the data Y. For the Bayesian approach, we introduce a prior distribution $P(\theta)$ on the unknown parameters. Combined with the likelihood function $P(Y|\theta)$, this yields a joint distribution

$$P(Y,\theta) = P(Y|\theta)P(\theta) \tag{1}$$

and then using Bayes' rule, the distribution of θ given Y

$$P(\theta|Y) = \frac{P(Y|\theta)P(\theta)}{\int P(Y|\theta)P(\theta)d\theta} = \frac{P(Y|\theta)P(\theta)}{P(Y)}. \tag{2}$$

We call $P(\theta|Y)$ the posterior distribution of θ. This describes uncertainty about θ given the observed data Y. In most applications, an analytic expression for $\int P(Y|\theta)P(\theta)d\theta$ is very difficult to obtain, i.e., we cannot obtain the posterior distribution in closed form in most cases. However, noticing in (2) that the factor $P(Y)$ does not depend on θ and Y is fixed, we can consider $P(Y)$ as a constant and focus on the unnormalized posterior distribution $Q(\theta|Y)$

$$P(\theta|Y) \propto P(Y|\theta)P(\theta) = Q(\theta|Y). \tag{3}$$

The unnormalized posterior distribution is sufficient for developing algorithms to study the normalized posterior distribution.

Working with the unnormalized posterior distribution $Q(\theta|Y)$, we can use Markov Chain Monte Carlo (MCMC) simulation to obtain a numerical approximation to the posterior distribution $P(\theta|Y)$. In the MCMC approach, a Markov chain with stationary distribution equal to the posterior distribution is constructed. The Markov chain is simulated until the simulated values are judged to be representative of the stationary distribution (that is the target posterior distribution). At that point the simulated values are treated as a collection of samples from the desired posterior distribution. These samples can be used to estimate any summary of the posterior distribution, e.g., the posterior mean. The Metropolis-Hasting (MH) algorithm is one algorithm for constructing Markov chains to draw samples from the Bayesian posterior distribution. It proceeds as follows [15]:

- Generate starting value θ^0.
- For $t = 1, 2, \ldots$
  - Sample a candidate θ^* from the jumping distribution $J_t(\theta^*|\theta^{t-1})$[2]
  - Compute the importance ratio: $r = \min(1, Q(\theta^*|Y)/J_t(\theta^*|\theta^{t-1})/Q(\theta^{t-1}|Y)/J_t(\theta^{t-1}|\theta^*))$
  - Generate U from the Uniform distribution Unif(0,1)
  - If $U \le r$, then $\theta^t = \theta^*$, else $\theta^t = \theta^{t-1}$

[1]The Internet provides access to NWS surface observations, which thoroughly cover the United States, with a typical station spacing of 50 km–100 km, but more dense in urbanized areas. Use of the temperature data with this station spacing is appropriate, as spatial variation of temperature can be captured using this kind of station spacing, indeed, meterologists do it regularly. However, wind speeds and directions incur spatial variation too great to be captured from data obtained at this level of station spacing.

[2]$J_t(\theta^*|\theta^{t-1})$ means the parameters of J_t are defined by θ^{t-1}. Details about jumping distribution can be found in [15].

- Stop when recent values of θ appear to represent draws from the stationary distribution.[3]

The Gibbs sampler, also known as alternating conditional sampling, is a special case of the MH algorithm with the importance ratio always 1 [15]. Denoting the parameter vector $\theta = (\theta_1, \ldots, \theta_k)$, the Gibbs sampling algorithm can be described by the following pseudocode [15]:

- Generate starting value $\theta^0 = (\theta_1^0, \ldots, \theta_k^0)$.
- For $t = 1, 2, \ldots$
  - generate θ_1^t from $P(\theta_1|\theta_2^{t-1}, \theta_3^{t-1}, \ldots, \theta_k^{t-1}, Y)$.
  - generate θ_2^t from $P(\theta_2|\theta_1^{t}, \theta_3^{t-1}, \ldots, \theta_k^{t-1}, Y)$.
  - $\ldots$
  - generate θ_k^t from $P(\theta_k|\theta_1^{t}, \theta_2^{t}, \ldots, \theta_{k-1}^{t}, \theta_k^{t-1}, Y)$.
- Stop when recent values of θ appear to represent draws from the stationary distribution.

When sampling from all of the conditional posterior distributions in the second step of the Gibbs sampler is impossible, we can embed MH algorithm steps to sample from some of conditional distributions.[4] The Gibbs sampler can be used in the ambient temperature model. The Gibbs sampler with embedded MH steps is required for the wind direction model.

The Bayesian approach has several attractive features. First, it yields reasonable inference without requiring large sample theory for justification. Second, the use of simulation based inference makes it easy to compute a number of summaries, including a predictive distribution for future weather conditions. We prefer the Bayesian approach but have used maximum-likelihood for the wind speed model because the likelihood function there can be difficult to study with Bayesian methods.

B. Wind Direction Model

Conventional distributions can only be used to fit data on the real line. For wind direction data, the Von Mises (VM) distribution is more suitable since it has a range of 0–2π. Let ϕ denote the random variable representing wind direction. The probability density function (pdf) of the VM for ϕ is [16]

$$f(\phi) = [2\pi * I_0(\kappa)]^{-1} \exp[\kappa \cos(\phi - \mu)] \tag{4}$$

where $0 \le \phi \le 2\pi$, $0 \le \kappa \le \infty$, and $I_0(\kappa)$ is

$$I_0(\kappa) = (2\pi)^{-1} \int_0^{2\pi} \exp[\kappa \cos(\omega - \mu)] d\omega. \tag{5}$$

[3]Some methods in [15] can be used to monitor the convergence. For example, we can run several Markov chains and monitor them to see if they all converge to the same stationary distribution.

[4]MH algorithm is not restricted to known distributions. Therefore, we can use MH to sample from the unknown conditional posterior distribution.

$I_0(\kappa)$ is the normalizing constant that makes the VM integrate to 1; it is a modified Bessel function of order 0 and can be expressed as

$$I_0(\kappa) = \sum_{r=0}^{\infty} (r!)^{-2} (0.5\kappa)^{2r}. \tag{6}$$

The parameters of this distribution are μ and κ.

Here, the wind direction at time t ϕ_t is to be related to the wind direction in the previous hour ϕ_{t-1}. Conventional time series models for defining such relationships include: autoregressive (AR), moving average (MA), and autoregressive moving average (ARMA) [17]. Our wind direction model is a first-order autoregressive Bayesian time series model. Suppose we have observed circular data $\phi_1, \ldots, \phi_{t-1}$ and the corresponding random variables come from the VM distribution, then our model assumes that the random variable Φ_t should also come from the VM distribution $\mathrm{VM}(\mu_t, \kappa)$. In this VM distribution, the variation κ is assumed constant. The location parameter or expectation of Φ_t is related to the previous measurement ϕ_{t-1} through the link function

$$E(\Phi_t) = \mu_t = g(\alpha_1 g^{-1}(\phi_{t-1}) + \alpha_0) \tag{7}$$

where $g(\cdot)$ is a suitable function. Fisher [16] suggested the arctangent function is a feasible choice for the function g in (7). Therefore, our AR(1) model is

$$\begin{aligned} p(\phi_t|\mu_t, \kappa) =& [2\pi * I_0(\kappa)]^{-1} \exp[\kappa \cos(\phi_t - \mu_t)] \\ \mu_t =& 2\tan^{-1}\left(\alpha_1 \tan\left(\frac{\phi_{t-1}}{2}\right) + \alpha_0\right) \\ \alpha_1 \sim& \mathrm{Norm}(0, 10^4) \\ \alpha_0 \sim& \mathrm{Norm}(0, 10^4) \\ \kappa \sim& \mathrm{Gam}(1, 10^{-3}). \end{aligned}$$

where $\sim$ stands for "described by distribution" and ϕ_t and ϕ_{t-1} are wind directions for time t and $t-1$, respectively. The last three rows are prior distributions of the parameters. The normal distributions for α_1 and α_0 and the gamma distribution for κ have large variance so that the data (rather than the prior distribution) determines the relevant parameter values in the posterior distribution. The unnormalized full joint distribution for the model parameters $\alpha_1, \alpha_0, \kappa$ is (with $n = 144$ for 6-day data)

$$\begin{aligned} p(\alpha_1, \alpha_0, \kappa|\phi_1, \ldots, \phi_n) \propto \prod_{t=2}^{n} \{&[2\pi I_0(\kappa)]^{-1} \\ &\times \exp[\kappa \cos(\phi_t - \mu_t)]\} \\ &\times \exp\left(-0.5\alpha_1^2 10^{-4}\right) \\ &\times \exp\left(-0.5\alpha_0^2 10^{-4}\right) \\ &\times \exp(-10^{-3}\kappa). \end{aligned}$$

From the joint distribution, we obtain the unnormalized conditional distribution for every parameter and express them as log-posteriors

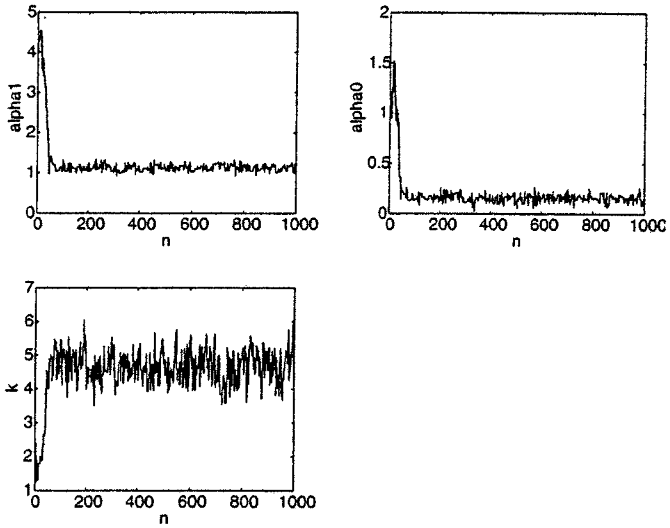

Fig. 1. Convergence of all Markov chains over the sample space $(\alpha_1, \alpha_0, \kappa)$.

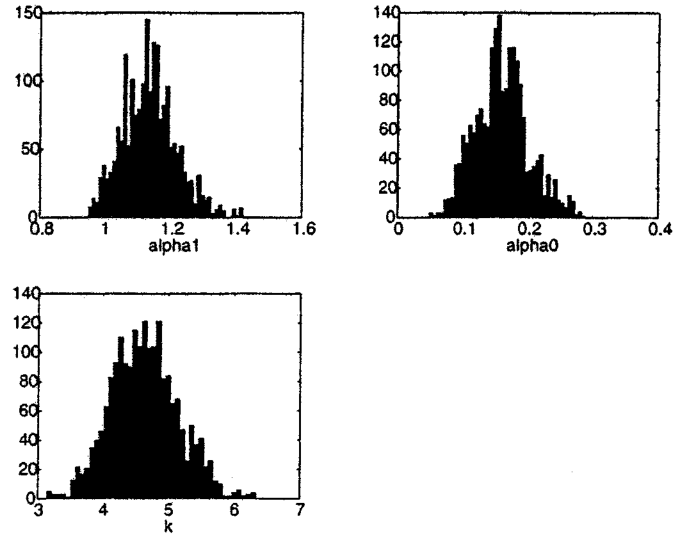

Fig. 2. Posterior distributions of α_1, α_0, and κ. The number of samples $= 2000$.

$$\log(p(\alpha_1|\alpha_0, \kappa, \phi_1, \phi_2, \ldots, \phi_n)) \propto \sum_{t=2}^{n} \left[\kappa \cos\left(\phi_t - 2\tan^{-1}\left(\alpha_1 \tan\frac{\phi_{t-1}}{2} + \alpha_0 \right)\right)\right] - 0.5\alpha_1^2 10^{-4}$$

$$\log(p(\alpha_0|\alpha_1, \kappa, \phi_1, \phi_2, \ldots, \phi_n)) \propto \sum_{t=2}^{n} \left[\kappa \cos\left(\phi_t - 2\tan^{-1}\left(\alpha_1 \tan\frac{\phi_{t-1}}{2} + \alpha_0 \right)\right)\right] - 0.5\alpha_0^2 10^{-4}$$

$$\log(p(\kappa|\alpha_0, \alpha_1, \phi_1, \phi_2, \ldots, \phi_n)) \propto \sum_{t=2}^{n} \left[\kappa \cos\left(\phi_t - 2\tan^{-1}\left(\alpha_1 \tan\frac{\phi_{t-1}}{2} + \alpha_0 \right)\right)\right] - n\log(I_0(\kappa)) - \kappa 10^{-3}$$

The Gibbs sampler with embedded Metropolis steps is applied to obtain the simulations from the posterior distributions. Fig. 1 illustrates how the Markov chains converge to the stationary distribution in our wind direction model, and Fig. 2 shows the posterior distributions of the parameters.

Having obtained samples from the posterior distributions, we need to check the fitness of the model. In [15], a variety of model checking approaches are reviewed. We do not describe them in detail here. The checking results indicate that our wind direction model fits the data.

Once we obtain samples from the posterior distributions, we can use them to obtain the posterior predictive distribution of the wind direction. The process is straightforward. First, load wind direction model parameters $\alpha_0, \alpha_1, \kappa$ and the most recent wind direction ϕ_{t-1} from disk to memory; then, for every simulated set of $(\alpha_0, \alpha_1, \kappa)$, draw a ϕ_t from $\mathrm{VM}(\mu_t, \kappa)$. This gives not just a single estimate for ϕ_t, but a full predictive distribution.

C. Wind Speed Model

Researchers generally model wind speed data with the Weibull distribution. However, histograms of some wind speed data reflect a "spike" located at a low wind speed corresponding to the stall speed of the wind speed measuring instrument, as in Fig. 3. A simple Weibull model will not be able to fit the data in this case. To deal with this truncated data problem, a logistic regression mixture time series model is developed to fit the wind speed data. We use two distributions in this model. A normal distribution with mean equal to the stall speed and a very small variance is used to fit the observed data with values around the stall speed. A Weibull distribution is used to model wind speeds greater than the stall speed. We also introduce z_t as an unobserved indicator of the distribution responsible for the observed wind speed y_t. If $z_t = 1$, the wind speed is from the Weibull distribution; if $z_t = 0$, it is from the Normal distribution. The z_ts are not an integral part of the analysis; they are used to facilitate computation. The joint distribution of the wind speed and the unobserved indicator is

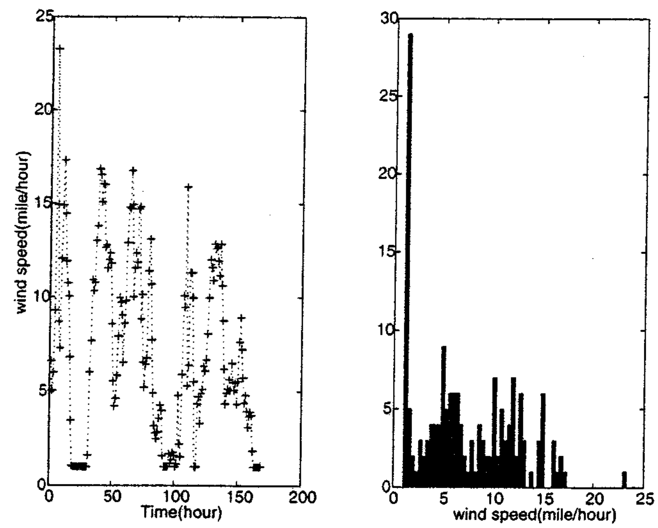

Fig. 3. Six days' wind speed data and its histogram.

$$p(y_t, z_t|\alpha, \beta, \gamma, \delta, \nu) = (\lambda * \mathrm{Weib}(y_t|\mu_t, \nu))^{z_t} *((1-\lambda) * \mathrm{Norm}(y_t|m, c))^{1-z_t}$$

where

$$\lambda = \frac{\exp(\alpha + \beta * y_{t-1})}{1 + \exp(\alpha + \beta * y_{t-1})}$$
$$\log(\mu_t) = \gamma + \delta * \log(y_{t-1})$$
$$m = \min(\text{observed wind speed}), \quad c = 10^{-1}$$

The logistic regression model for λ is used to allow the probability of a stall speed measurement to depend on the most recent wind speed (y_{t-1}). The model for μ_t is the portion of the Weibull model that links the current wind speed to the most recent value (as was done in the VM model for wind direction). As y_t is the only observable random variable, we should integrate (or sum) over the unobserved z_t to get the cumulative distribution function (cdf) function for the marginal model

$$\begin{aligned} F(y_t|\alpha,\beta,\gamma,\delta,\nu) =& Pr(Y \le y_t|z_t = 1)Pr(z_t = 1) \\ &+ Pr(Y \le y_t|z_t = 0)Pr(z_t = 0) \\ =& \lambda \int_{-\infty}^{y_t} \text{Weib}(x|\mu_t,\nu)dx \\ &+ (1-\lambda)\int_{-\infty}^{y_t} \text{Norm}(x|m,c)dx \end{aligned}$$

Therefore, the marginal pdf for y_t is

$$\begin{aligned} p(y_t|\alpha,\beta,\gamma,\delta,\nu) =& \frac{F(y_t|\alpha,\beta,\gamma,\delta,\nu)}{dy_t} \\ =& \lambda \text{Weib}(y_t|\mu_t,\nu) \\ &+ (1-\lambda)\text{Norm}(y_t|m,c). \end{aligned}$$

The joint likelihood for the n observations is then

$$\begin{aligned} &p(\alpha,\beta,\gamma,\delta,\nu|y_1,\ldots,y_n) \\ &= \prod_{t=2}^{n} \left\{ \left(\frac{\exp(\alpha + \beta * y_{t-1})}{1 + \exp(\alpha + \beta * y_{t-1})} * \text{Weib}(y_t|\mu_t,\nu) \right) \right. \\ &\quad \left. + \left(\frac{1}{1 + \exp(\alpha + \beta * y_{t-1})} * \text{Norm}(y_t|m,c) \right) \right\}. \end{aligned} \tag{8}$$

For six days and one sample/h, there are $n = 144$ observations.

To follow the Bayesian approach, one would combine the likelihood (8) with a prior distribution for the parameters as was done for wind direction. It is difficult to perform a Bayesian analysis of the mixture model however so we instead estimate the parameters by maximum-likelihood. For mixture models it is especially convenient to estimate the parametes for this mixture model using the EM algorithm with the z_ts included as missing variables.

Let $\theta = (\alpha,\beta,\gamma,\delta,\nu)$ be the parameter vector and y be the observed variable. We need to find θ that maximizes $p(y|\theta)$, averaging over the unobserved indicators, the z_ts. We do this by referring back to the joint likelihood for y_t and z_t, $p(y,z|\theta) = \prod_{i=2}^{n} p(y_t,z_t|\theta)$. The EM algorithm works as follows:

- Estimate a starting value for the parameters θ^0
- For step t:
  – E-step: get $E(\log p(y,z|\theta)|y,\theta^{t-1})$ where the expectation is with respect to the conditional distribution of the latent indicators given the current estimate of the parameters and the observed data $p(z|y,\theta^{t-1})$. This is the expected value of the complete data (y and z) likelihood (hence the name "expecation step").
  – M-step: set the value of θ^t as the value of θ that maximizes the $E(\log p(y,z|\theta)|y,\theta^{t-1})$ computed in the E-step. Note that the result of the E-step includes y and θ with z replaced by its expected value (see below). It is this result that we now maximize of θ (hence the name "maximization step").

We briefly describe a few details associated with the E and M steps in the current case. In E-step, $E(\log p(y,z|\theta)|y,\theta^{t-1})$ depends only on $E(z_t|y,\theta^{t-1})$ for $t = 2,\ldots,n$. This is easily obtained for the mixture model as

$$\begin{aligned} &E(z_t|y,\theta^{t-1}) \\ &= Pr(z_t = 1|y,\theta^{t-1}) \\ &= \frac{\lambda^{t-1}\text{Weib}(y_t|\mu_t^{t-1},\nu^{t-1})}{\lambda^{t-1}\text{Weib}(y_t|\mu_t^{t-1},\nu^{t-1}) + (1-\lambda^{t-1})\text{Norm}(y_t|m,c)} \end{aligned} \tag{9}$$

where

$$\lambda^{t-1} = \frac{\exp(\alpha^{t-1} + \beta^{t-1} * y_{t-1})}{1 + \exp(\alpha^{t-1} + \beta^{t-1} * y_{t-1})}$$

and

$$\log \mu_t^{t-1} = \gamma^{t-1} + \delta^{t-1} * \log(y_{t-1}).$$

In the M-step, we use the Newton–Raphson algorithm to maximize the Weibull and normal portions of the complete data (y,z) likelihood. Once again, diagnostics indicate that the model fits the original wind speed data.

D. Ambient Temperature Model

The ambient temperature model is a first-order autoregressive Bayesian time series model. We difference the original data before analysis to remove the 24-h periodic component. If x_t is the temperature at time t, we then define $y_t = x_t - x_{t-24}$. The 24-h

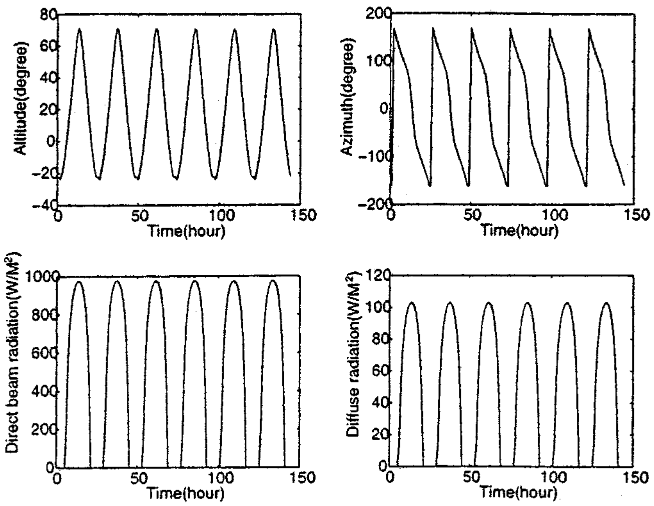

Fig. 4. Solar altitude, azimuth, direct beam radiation, and diffuse radiation for July 1, 1998, in Ames, IA.

temperature change y_t is used for statistical analysis. The model for temperature is very similar to the wind direction model. As a result, we only list the assumed likelihood and prior distributions for it

$$\begin{aligned} y_i|V_i, \mu_i, \tau \sim & \text{Norm}\left(\mu_i, \frac{\tau}{V_i}\right) \\ \mu_i = & \alpha_1 y_{i-1} + \alpha_0 \\ V_i \sim & \text{Inverse} - \chi^2(\nu, 1) \\ \alpha_1 \sim & \text{Norm}(0, 10^4) \\ \alpha_0 \sim & \text{Norm}(0, 10^4) \\ \tau \sim & \text{Gam}(1, 10^{-3}). \end{aligned}$$

One noteworthy point is that in this case all of the Gibbs sampling conditional distributions are known distributions that can be sampled from directly.

E. Solar Radiation Model

The solar radiation model is a deterministic model. When we compute solar heating gain for a conductor line, we need to know the hourly direct beam radiation I_B, diffuse radiation I_d, solar altitude H_s, and solar azimuth γ_s. These variables can be computed directly with a set of equations [14]. The results of calculations using these equations are shown in Fig. 4.

III. Current–Temperature (CT) Model

The current-temperature (CT) model is used to compute the conductor temperature given a certain current value and a set of predicted weather conditions. The CT model is a dynamic current–temperature model based on the IEEE Standard [4]. Joule heating (P_j), solar heating (P_s), convective cooling (P_c), and radiative cooling (P_r) are considered in the CT model. Therefore, we have

$$mC_p \frac{d\theta_c}{dt} = P_j + P_s - P_c - P_r \tag{10}$$

where θ_c is conductor temperature; m is mass; and C_p is specific heat.

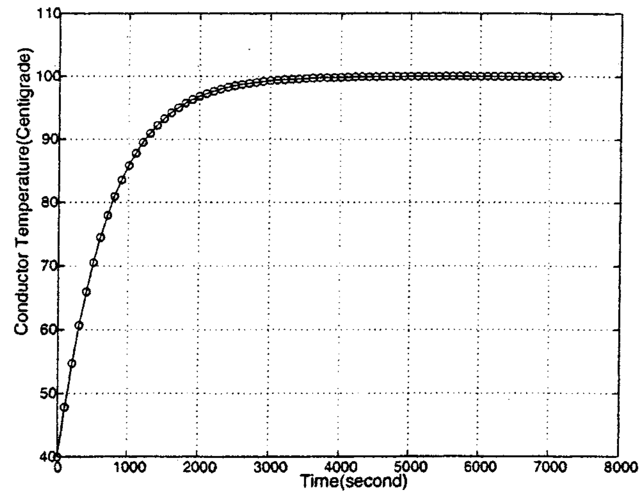

Fig. 5. Simulation of conductor temperature.

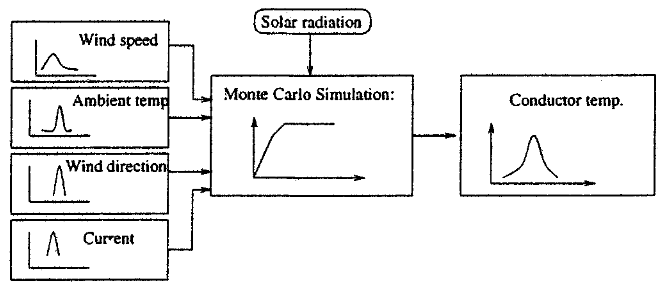

Fig. 6. MC simulation for conductor temperature.

Equation (10) is used to simulate the thermal behavior of the conductor by numerical integration. Normally, the thermal time constant of most conductors is about 10–30 min. Therefore, we only need to simulate for 1 h. The conductor temperature after 1 h (3600 s) is very near to steady state and will not change very much, as observed in Fig. 5. From Section II, we obtain predictive distributions for wind speed (U), wind direction (ϕ), and ambient temperature (θ_a)—although it should be noted that our notation for some of these quantities is different than the notation in Section II. For solar radiation (S), we use a single deterministic value in the computation for the next hour. For current (I) we estimate its distribution from state estimation programs and sensitivity calculations assuming multivariate normal distribution of operating conditions[2]. With these predictive weather and current distributions for the next hour, we use MC simulation to obtain the predictive distribution of conductor temperature. For a given set of values $(U, \phi, \theta_a, I, S)$, we assume conditions remain unchanged for the next hour. With this assumption, we perform a numerical integration over a 1-h period to obtain the steady-state conductor temperature. We use the temperature at 1 h $(\theta_c\ (1\ \text{h}))$ as the conductor temperature for this set of weather and current conditions, assuming, conservatively, that sag follows temperature decreases instantaneously. This process is repeated for each set of values $(U, \phi, \theta_a, I, S)$, yielding a collection of different values of $\theta_c\ (1\ \text{h})$. Repeating the procedure with a large number of samples of $(U, \phi, \theta_a, I, S)$, we obtain the predictive conductor temperature distribution, as shown in Fig. 6.

We assume that we may select a conductor critical temperature (θ_{cf}) within which the operation of the line is safe, in that the line will not violate its minimum clearance requirement as defined by National Electrical Safety Code (NESC). This choice could be made conservatively as $K\theta_D$, where θ_D is the maximum steady-state design temperature of the conductor, and K is chosen accounting for the condition of the line, with $0 < K \leq 1$. The probability $Pr(\theta_c \geq \theta_{cf})$ is an indicator of the risk associated with this situation. When we use MC simulation, we estimate $Pr(\theta_c \geq \theta_{cf})$ by the fraction of simulations with θ_c (1 h) greater than θ_{cf}. Therefore

$$Pr(\theta_c \geq \theta_{cf}) = \frac{N_f}{N} \tag{11}$$

where N_f is the number of simulations which obtain a θ_c (1 h) greater than θ_{cf}, and N is the total number of simulations.

IV. Spatial Correlation for Ambient Temperature

A. Temperature Regions

Transmission lines may extend several hundred miles over several states. This can be problematic because the approach described in Section II and Section III requires data on wind speed, wind direction, and temperature all along the line. A simple way to implement the approach described in this paper is to assume a constant (and very conservative) value for the wind speed component and just predict temperature. The simulation approach would then always use the same wind speed and only temperature and current would vary. If this is done, then it is possible to utilize temperature data from the internet rather than from on-site weather monitors, as it is usually possible to obtain data from observing stations relatively close to the line. We should set up a grid over the area we are studying. For each cell in the grid, there should be an observing station in the cell. The more observing stations we have, the more cells will be in the grid, and the more accurate will be our results. We assume the temperature is uniform in a cell. Once the grid is set up, we can check how many cells a transmission line will traverse.

B. Correlations

Weather conditions at different locations are correlated. When we generate values for ambient temperature, wind speed, or wind direction, we obtain independent samples from the marginal distributions at every location. In fact, these random variables have some dependencies among them. The rank order correlation method is adopted to pair the random variables. This method is not limited by the type of distribution, and we only need use the one-dimensional marginal distributions. In this method, we first obtain marginal distributions for each random variable. Next, each random variable is generated independently from the corresponding marginal distribution. In the last step, we pair the generated random variables. Our goal is to make the generated random variables have a correlation matrix similar to that computed from the raw weather data.

We obtain the correlation matrix from raw data by evaluating Spearman's r [18] for each pair of variables

$$r = 1 - \left(\frac{6\sum(\Delta R^2)}{N(N^2-1)}\right) \tag{12}$$

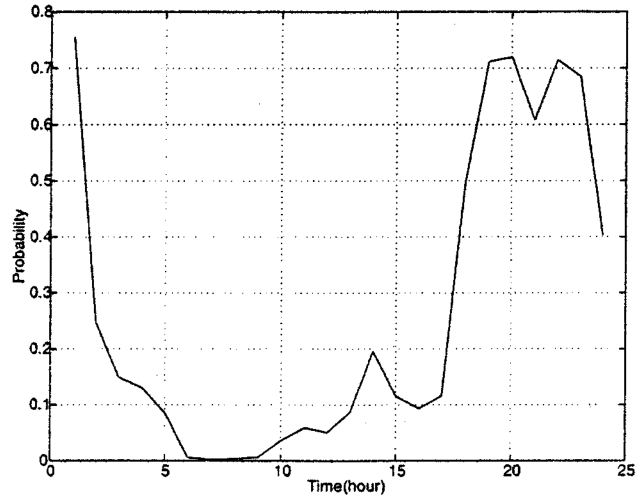

Fig. 7. Line risk variation with time.

where N is the number of data pairs, and ΔR is the difference of ranks of a pair.

After we obtain the k by k (k is the number of the variables) correlation matrix R, we want to generate random variables which have the same correlation matrix R. To do this, we produce an N by k matrix W in which each column contains randomly mixed Van Waerdon scores [18]. Van Waerdon scores are normal scores, $\Phi^{-1}(i/(N+1))$, $i = 1, 2, \ldots, N$, where Φ^{-1} is the inverse function of the standard normal distribution cdf function. We use the Cholesky factorization to decompose the matrix $R = U^T \times U$, where U is an upper triangular matrix. Then, we multiply W and U to obtain $O = W \times U$. The O matrix is an N by k order matrix which will be used to pair the samples independently generated from the marginal distributions. It is the the order in each column of O that contains the correlation information. Now, we have an N by k matrix of samples S, where the columns of S are random variables generated independently from the marginal distribution of each random variable. We sort each column in S with the order corresponding to the same column in matrix O. Having been sorted, matrix S will have a correlation matrix which is nearly equal to R. Each row of S will be used as a sample for k random variables; the random variables are no longer independent.

V. Example

A transmission line runs from Castana to Ames in Iowa. We utilize only two weather monitors on the line; one at Castana and one at Ames, in order to illustrate the method in a simple fashion, realizing that actual implementation would require more than this. The conductor type is Drake (26/7). The critical conductor temperature is selected as $\theta_{cf} = 100\,^\circ$C.

A. Risk Variation With Time

We fix a distribution for the line current as a normal distribution with mean of 992 A and standard deviation of 5 A, where the uncertainty in line flow is caused by uncertainty in loading condition [2]. Simulations to determine risk variation with time provide the probability that the conductor temperature exceeds the critical value for Castana, as shown in Fig. 7. Results for

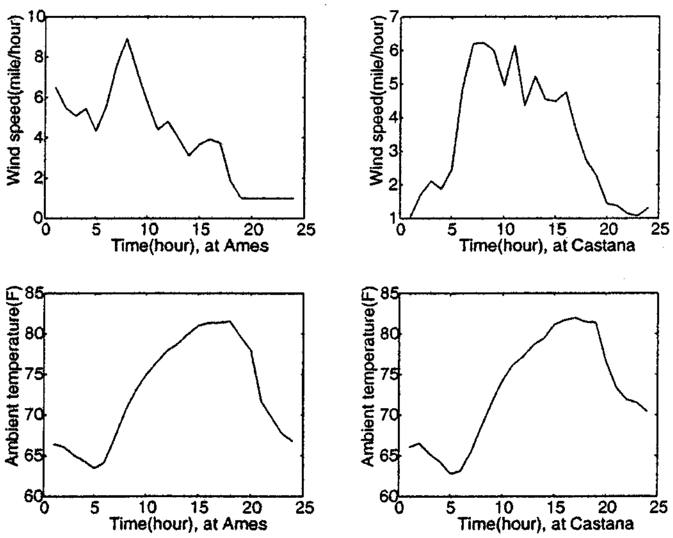

Fig. 8. Observed wind speeds and ambient temperature at Ames and Castana on July 1, 1998.

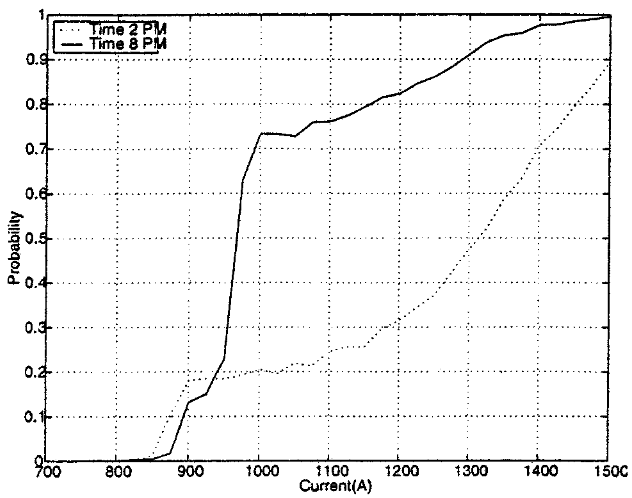

Fig. 9. Line risk variation with current.

Ames are similar. From 5:00 AM to 5:00 PM, the thermal overload risk is relatively low. During evening and night, the risk is high. This is because, although the ambient temperatures are low at night, the wind speeds are also very low at night. We verify this from the observed wind speed and ambient temperature data at Ames and Castana in Fig. 8. This confirms that transmission line risk is highly sensitive to wind speed, as is well known.

B. Line Risk Variation With Current

To assess the effect of current we fix two times of interest, 2:00 PM and 8:00 PM, to see how risk varies with current mean value (assuming standard deviation is 5 A). Fig. 9 shows how current values affect the risk. Note at 8:00 PM there is a substantial increase of risk for this line when current value rises from 900 A to 1000 A. Overload risk can be very sensitive to loading when the wind speed is low.

C. Line Risk With and Without Correlation

We consider the effect of critical temperature and the importance of modeling correlation. We fix the current distribution as

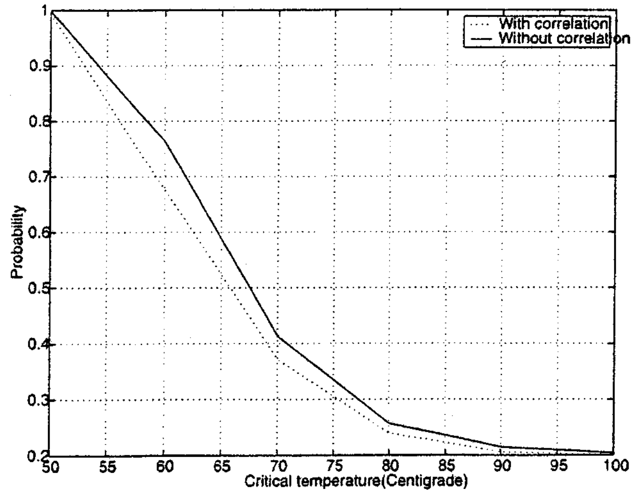

Fig. 10. Line risk with or without considering correlation.

normal with mean 992 A and standard deviation of 5 A, and we consider the time 2:00 PM. Critical conductor temperatures are varied from 50 °C to 100 °C. Naturally, if the critical temperature is low, the risk is greater. If we do not consider the correlation between ambient temperature and wind speed, we find that the risk is overestimated (see Fig. 10). This is because the MC simulations include many unrealistic scenarios when no correlation is considered. Therefore, the correlations should be included in risk computation for better estimation.

VI. CONCLUSION

A transmission line risk assessment algorithm has been developed for the purpose of making operating decisions. Statistical models are applied to provide effective predictions for ambient temperature, wind speed, and wind direction of the next hour. A mixture model is used to solve a truncated data problem in the wind speed model. The risk assessment tool offers an improved decision-making tool for operators than the traditional method in that risk associated with near-future operating conditions may be assessed.

REFERENCES

[1] J. McCalley, V. Vittal, and N. Abi-Samra, "An overview of risk based security assessment," in *Proc. IEEE PES Summer Meeting*, Edmonton, AB, Canada, July 18–22, 1999, pp. 173–178.
[2] J. McCalley, V. Vittal, N. Abi-Samra, H. Wan, and Y. Dai, "Voltage risk assessment," in *Proc. IEEE PES Summer Meeting*, Edmonton, AB, Canada, July 18–22, 1999, pp. 179–184.
[3] H. Wan, J. McCalley, and V. Vittal, "Increasing thermal rating by risk analysis," *IEEE Trans. Power Syst.*, vol. 14, pp. 815–821, Aug. 1999.
[4] *IEEE Standard for Calculating the Current–Temperature Relationship of Bare Overhead Conductors*, IEEE Std. 738-1993, Nov. 1993.
[5] H. E. House and P. D. Tuttle, "Current-carrying capacity of ACSR," *AIEE Trans. Power App. Syst.*, pp. 1169–1178, Feb. 1958.
[6] D. A. Douglass, D. Lawry, A. Edris, and E. Bascom, "Dynamic thermal ratings realize circuit load limits," *IEEE Comput. Appl. Power*, vol. 13, no. 1, pp. 38–44, Jan. 2000.
[7] D. A. Douglass and A. Edris, "Real-time monitoring and dynamic thermal rating of power transmission circuits," *IEEE Trans. Power Delivery*, vol. 11, pp. 1407–1415, July 1996.
[8] *www.demicon.com and www.davisnet.com/weater/* [Online]
[9] *www.apptech.com* [Online]
[10] "Complex interactive networks/systems initiative: Overview and progress report for joint EPRI/US Department of Defense University research," EPRI, Palo Alto, CA, Rep. TP-114 660, 2000.

[11] D. F. Stephen, H. L. Shenh, and A. F. Roosevelt, "Dynamic thermal line ratings—Part I: Dynamic ampacity rating algorithm," *IEEE Trans. Power App. Syst.*, vol. PAS-102, pp. 1858–1864, June 1983.

[12] Y. Mizuno, Nakamura, H. K. Adomah, and K. Naito, "Assessment of thermal deterioration of transmission line conductor by probabilistic method," *IEEE Trans. Power Delivery*, vol. 13, pp. 267–271, Jan. 1998.

[13] D. Koval and R. Billinton, "Determination of transmission line ampacities by probability and numerical methods," *IEEE Trans. Power App. Syst.*, vol. PAS-89, pp. 1485–1492, Sept./Oct. 1970.

[14] V. Morgan, *Thermal Behavior of Electrical Conductors-Steady, Dynamic & Fault-Current Ratings*, Somerset, U.K.: Research Studies Press, 1991.

[15] A. Gelman, J. Carlin, H. Stern, and D. Rubin, *Bayesian Data Analysis*. London, U.K.: Chapman & Hall, 1995.

[16] N. Fisher, *Statistical Analysis of Circular Data*. Cambridge, U.K.: Cambridge Univ. Press, 1993.

[17] G. Box, *Time Series Analysis, Forecasting, and Control*. San Francisco, CA: Holden-Day, 1994.

[18] R. Iman and W. Conover, "A distribution-free approach to inducing rank order correlation among input variables," *Communications in Statistics: Simulation and Computation*, vol. 11, no. 3, pp. 311–334, 1982.

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