Global robust adaptive control of power systems

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Abstract: A robust, adaptive control scheme is presented which stabilises a nonlinear model of a power system to disturbances anywhere in the power system. The control is local, in the sense that the control of each machine depends only on information available at the machine. Simulation results are presented which show that the control is very effective against instabilities of current importance such as sustained oscillations following a major system disturbance.

1 Introduction

Over the last decade, the interconnected power system in the United States has become less stable. As a result, the large interconnected subsystems that coordinate their activities have begun to detect sustained oscillations in their simulation analysis. Real oscillations have also been observed. These oscillations usually follow a major disturbance, for instance, removal of a fault or the loss of a major transmission line. One of the more prominent examples of this phenomenon is the 0.7 Hz oscillation that arises in the Western System Coordinating Council (WSCC) following the loss of one of the AC or DC interties between the Pacific Northwest and California [1]. Preventing this sustained oscillation is of great interest since the potential for an oscillation causes lower operational limits to be set on transfer levels on the interties. Oscillations can also arise during 'normal' steady-state operation.

The damping of these oscillations has been studied by a number of investigators. However, the majority of existing literature on power system control is based on linearised generator models, using eigenvalue analysis and pole placement techniques [2–4], model reduction techniques [5], and some modern control theory (adaptive, robust, etc.) [6–10]. Since these control schemes are derived from linearised generator models, they are suitable for small disturbances about a steady-state operating point. Some authors have declared that the controls so designed are also effective for transient stability under certain circumstances [3, 11], but these claims cannot be proved, only substantiated by numerical simulation.

The very nonlinear nature of the generator and system behaviour following a severe disturbance precludes the use of classical linear control techniques. Some authors have tried to bypass the nonlinearity problem by using identification techniques [12, 13] (i.e. observe the data of system inputs and local terminal state variables to establish a linear dynamic equivalent model, and then base the control on the equivalent model). This technique is very hard to apply to the transient problems, because transient periods only last a few cycles, and the identification will further delay the control action.

Many investigators have tried to design controls directly based on nonlinear generator models. Differential geometric theory is the approach used to design these controls [14]. Another technique seen in the literature is exact feedback linearisation [15]. Unfortunately, both techniques need global information of the system, which is still impractical in a real power system. Although the authors of Reference 14 tried to decouple the control, it seems impossible when the system is large.

In a previous paper [16] we developed a global robust control that stabilised a power system for any disturbance, anywhere in the power system. The motivation for this control was the problem of damping the sustained oscillations that now arise in many power systems following severe disturbances. The robust control developed [16] is, so far as we know, the only decentralised robust control, with global stability, that has been formulated for a nonlinear model of a power system. Thus, the control applies to the transient and midterm stability problems as well as the steady-state stability problem. The control has the additional advantage of requiring only local linear feedback.

Although the control [16] guarantees asymptotic stability, it does not provide much insight as to how to determine the local feedback gains. Thus, the tuning of the control becomes a separate problem. This tuning is important, because we would like to shape both the transient response of the system and the control. Shaping the control (through time) is particularly important. Since research on this type of control is quite new, it is too much to expect that the shape of the control can be 'optimised' in any sense. However, we would like the control to be feasible. We will have more to say about this when we discuss the simulation results.

In this paper, we propose a new adaptive control scheme for providing global robust control of a power system. The control is once again from the mechanical

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side and the power system model remains quite simple. However, as shown by the simulation results, the use of adaptive control overcomes the tuning problem discussed above. The adaptive control law frees us from having to specify the local feedback gains. We can set these gains to arbitrary initial values and let the adaptation tune the control through time. Further, as shown by the simulation results, the adaptive control gives a very acceptable transient response for the power system, while yielding a reasonable control.

The simplicity of the machine model used in the proof of robustness is a consequence of using Lyapunov's direct. Even with simple machine models, the Lyapunov function of the overall power system is still algebraically complicated. If more detailed machine models are used, the construction of the Lyapunov function proceeds along exactly the same path as described in the paper, but the algebraic complexity becomes increasingly more difficult. Towards the end of the paper, we will discuss the feasibility of extending this approach to more detailed machine models. Our goal in this paper is to take the next logical steps beyond [16], namely to solve the tuning problem, improve the transient response and make the control feasible.

Despite the simplicity of the machine models used to establish the robustness of the control, we believe the results are worthwhile and give new insight into the problem of global control of a power system. As shown by the simulations, the control typically stabilises the system in less than 10 seconds. Thus, the control does its work primarily in the 'transient period'. The proposed control is global in the sense that it can respond to any disturbance, but, in the simulations, we will concentrate on the phenomenon currently of most interest in the utility industry, namely sustained oscillations. These oscillations are important because they limit the transfer levels on major bulk transmission lines.

2 Problem formulation

In formulating the power system model for constructing and analysing a robust control strategy, one would obviously want as detailed a machine model as possible. In particular, for 20 s response times, we would like to consider the effects of excitation systems and turbine/governor control. The machine models used in the proof of robustness of the proposed adaptive control do not explicitly model either the turbine/governor or exciter dynamics. However, the following points should be noted. The form of the proposed control does allow excitation dynamics to be included in the machine models used in the simulations presented later. Further, the analysis and proof of robustness of the adaptive control does implicitly account for the effects of excitation.

A final observation is the following. As mentioned earlier, both the previous paper [16] and the present paper represent essentially the only extant efforts made at determining a robust, global control scheme for a nonlinear model of a power system. Hence, at this juncture, it is desirable to keep the development of control schemes as clear and constructive as possible. This is impossible with the machine model chosen for this paper. Including the dynamics of the turbine/governor and excitation system leads to extremely complex algebraic expressions that obscure the design of the control. Thus, the machine models chosen are consistent with the present state of this research area. However, an adaptive control based on a more detailed machine model is currently under consideration.

In this paper, we consider a power system of n generators, where the model of machine i, $i = 1, \ldots, n$ is given by

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i + P_{g_i} = P_{m_i} \tag{1}$$

where δ_i is the angle of machine i relative to the synchronous angle of the system, and M_i, D_i, P_{g_i}, and P_{m_i}, are the inertia constant, damping coefficient, electrical output power, and mechanical input power, respectively, of machine i.

The coupling of the machine dynamics arises through power conservation in the related network equations. More specifically, the electrical power outputs P_{g_i}, $i = 1, \ldots, n$, of the generators, satisfy the power flow equations:

$$\begin{aligned} P_{g_i} &= P_{e_i} + P_{l_i} \\ P_{e_i} &= E_i^2 G_{ii} + E_i \sum_{j \in m(i)} E_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \\ \delta_{ij} &= \delta_i - \delta_j \end{aligned} \tag{2}$$

with the variables in the powerflow equations defined as follows. E_i is the magnitude of the voltage behind transient reactance of the ith machine, P_{e_i} is the electrical bus output power at machine i; P_{l_i} is the local electrical load power at machine i; G_{ii} is the driving point conductance at machine i; G_{ij} is the mutual conductance between the machines i and j; B_{ij} is the mutual susceptance between the machines i and j; $m(i)$ is the set of busses which are directly connected to bus i.

It is worth observing at this point that when exciter dynamics are included in the machine model, E_i is time-varying, and $E_{i_{min}} \leqslant E_i \leqslant E_{i_{max}}$. In the proof of robustness we will not require E_i to be fixed, only that E_i be bounded below and above. Thus, the stability proof does account implicitly for the effect of excitation.

Letting $\dot{\delta}_i = \omega_i$, eqn. 1 can be rewritten in state form as

$$\begin{aligned} \dot{\delta}_i &= \omega_i \\ \dot{\omega}_i &= \frac{1}{M_i}(P_{m_i} - P_{g_i} - D_i \omega_i) \end{aligned} \tag{3}$$

Let the equilibrium point of eqn. 3 be characterised by δ_i^r, ω_i^r, $P_{m_i}^r$, and $P_{g_i}^r$. That is, δ_i^r, ω_i^r, $P_{m_i}^r$, and $P_{g_i}^r$ are the solutions of the following equations.

$$\begin{aligned} \dot{\delta}_i^r &= \omega_i^r = 0 \\ \dot{\omega}_i^r &= 0 = \frac{1}{M_i}(P_{m_i}^r - P_{g_i}^r - D_i \omega_i^r) \\ P_{g_i}^r &= P_{e_i}^r + P_{l_i} \\ \delta_{ij}^r &= \delta_i^r - \delta_j^r \\ P_{e_i}^r &= E_i^2 G_{ii} + E_i \sum_{j \in m(i)} E_j (G_{ij} \cos \delta_{ij}^r + B_{ij} \sin \delta_{ij}^r) \end{aligned}$$

Because the system parameters G_{ii}, G_{ij} and B_{ij} are assumed to be unknown, in the sense that they are not known exactly, the equilibrium point cannot be found explicitly. In the next Section, we design a controller which controls the power system smoothly to a desired steady state operating point. The desired steady-state operating point is not necessarily equal to the actual steady-state operating point. In general, the desired operating point is a 'best guess' of the actual operating point, based on imperfect knowledge of the system parameters and topology. In most actual cases the operating points will be 'close' together. We emphasise that the actual steady-state operating point does not need to be known to implement the proposed control. It is introduced as a mathematical convenience to facilitate the proof of robustness.

It is worth noting that not knowing the precise steady-state operating point is not a practical limitation. In practice, once the disturbance has been controlled, the system operators can move the system to the exact operating point they desire, just as they routinely move this operating point many times per day.

3 A robust adaptive controller

The control developed in this paper will stabilise the power system to an arbitrary major disturbance anywhere in the power system. It is in this sense that we mean the control is global. One of the system disturbances that we correct in simulation is a sustained oscillation that arises after the disturbance is removed. However, we emphasise that this is not the only potential postdisturbance system instability that the control can operate against. To that end we also consider a case where the system is unstable without control.

The overall control consists of n local controllers, one at each machine, with each local controller using only information available locally, that is at the machine. Before we introduce the controller, we define the error states:

$$x_{1i} = \delta_i^d - \delta_i \quad x_{2i} = \omega_i^d - \omega_i \tag{4}$$

The variables x_{1i} and x_{2i} represent the errors between the states of the power system and the desired trajectory. Since we wish to drive the system to steady state, we must have $\omega_i^d = 0$, so that eqn. 4 becomes

$$x_{1i} = \delta_i^d - \delta_i \quad x_{2i} = -\omega_i \tag{5}$$

The proposed robust, adaptive, local, controller for the ith machine is given by

$$\begin{aligned} u_i &= P_{m_i}^d - P_{m_i} \\ &= -k_{1i}(\delta_i^d - \delta_i) - k_{2i}(\omega_i^d - \omega_i) - \hat{C}_i(t)g_i - \Delta\hat{P}_{m_i}(t) \\ &= -k_{1i}x_{1i} - k_{2i}x_{2i} - \hat{C}_i(t)g_i(x_{1i}, x_{2i}) - \Delta\hat{P}_{m_i}(t) \end{aligned} \tag{6}$$

or

$$\begin{aligned} P_{m_i} &= P_{m_i}^d + k_{1i}x_{1i} \\ &\quad + k_{2i}x_{2i} + \hat{C}_i(t)g_i(x_{1i}, x_{2i}) + \Delta\hat{P}_{m_i}(t) \end{aligned} \tag{7}$$

where

$$g_i(x_{1i}, x_{2i}) = \text{sign}\left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right) \tag{8}$$

Several comments need to be made at this point. We will ultimately replace the sign function in eqn. 8 with a continuous approximation. Our motivation is to first present the proof of robustness based on the sign function, because the proof is simple and easy to follow. We will then replace the sign function by its continuous approximation. The proof in this case proceeds in the same way, but the algebraic complexity is much greater. It is hoped that the first proof will make the second easier to follow. At this juncture the role of the term $\hat{C}_i(t)$ is not at all clear. However, it will become clear as we proceed. $\Delta\hat{P}_{m_i}(t)$ is the estimate of the so-called mechanical power mismatch defined shortly.

The closed-loop dynamics of the ith machine in terms of x_{1i} and x_{2i} are

$$\begin{aligned} \dot{x}_{1i} &= x_{2i} \\ \dot{x}_{2i} &= -\dot{\omega}_i \\ &= -\frac{1}{M_i}(P_{m_i} - P_{g_i} - D_i\omega_i) \\ &= -\frac{1}{M_i}(P_{m_i} - P_{g_i} - D_i\omega_i) \\ &\quad + \frac{1}{M_i}(P_{m_i}^r - P_{g_i}^r - D_i\omega_i^r) \\ &= \frac{1}{M_i}[(P_{m_i}^r - P_{m_i}) - (P_{g_i}^r - P_{g_i}) - D_i(\omega_i^r - \omega_i)] \\ &= \frac{1}{M_i}[(P_{m_i}^d - P_{m_i}) + (P_{m_i}^r - P_{m_i}^d) \\ &\quad - (P_{g_i}^r - P_{g_i}) - D_i(\omega_i^r - \omega_i)] \\ &= \frac{1}{M_i}[-k_{1i}x_{1i} - k_{2i}x_{2i} - \hat{C}_i(t)g_i(x_{1i}, x_{2i}) \\ &\quad - \Delta\hat{P}_{m_i}(t) + \Delta P_{m_i} - (P_{g_i}^r - P_{g_i}) - D_i x_{2i}] \\ &= \frac{1}{M_i}[-k_{1i}x_{1i} - (k_{2i} + D_i)x_{2i} + f_i(x) \\ &\quad - \hat{C}_i(t)g_i(x_{1i}, x_{2i}) + \Delta P_{m_i} - \Delta\hat{P}_{m_i}(t)] \end{aligned}$$

where

$$\begin{aligned} \Delta P_{m_i} &= P_{m_i}^r - P_{m_i}^d \\ f_i(x) &= P_{g_i}^r - P_{g_i} \\ &= P_{e_i}^r + P_{l_i} - P_{e_i} - P_{l_i} \\ &= P_{e_i}^r - P_{e_i} \end{aligned}$$

Thus, the error equation of the ith machine can be described by the following dynamic equations:

$$\dot{x}_i = A_i x_i + B_i f_i(x) - B_i\hat{C}_i(t)g_i(x_i) - B_i\psi_i \tag{9}$$

where

$$x_i = \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix} \quad A_i = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_{1i}}{M_i} & -\dfrac{k_{2i} + D_i}{M_i} \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ \dfrac{1}{M_i} \end{bmatrix}$$

and

$$\psi_i(t) = \Delta\hat{P}_{m_i}(t) - \Delta P_{m_i} \tag{10}$$

The mechanical power mismatch is defined as $\Delta P_{m_i} = P_{m_i}^r - P_{m_i}^d$. Since $P_{m_i}^r$ is an unknown constant, ΔP_{m_i} is an unknown constant as well. Fig. 1 is the block diagram of

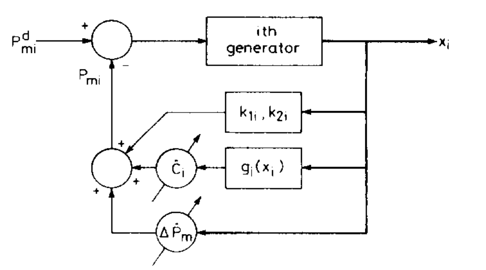

Fig. 1 *Block diagram of individual machine control*

ith generator under the proposed local control. The error dynamics of the whole system can be written in the following form:

$$\dot{x} = Ax + BF(x) - BG(x) - B\Psi \tag{11}$$

where

$$x = [x_1^T, x_2^T, \ldots, x_n^T]^T$$
$$A = \mathrm{diag}\,[A_1, A_2, \ldots, A_n]$$
$$B = \mathrm{diag}\,[B_1, B_2, \ldots, B_n]$$
$$F(x) = [f_1(x), f_2(x), \ldots, f_n(x)]^T$$
$$G(x) = [\hat{C}_1(t)g_1(x_1), \hat{C}_2(t)g_2(x_2), \ldots, \hat{C}_n(t)g_n(x_n)]^T$$
$$\Psi = [\psi_1(t), \psi_2(t), \ldots, \psi_n(t)]^T$$

The functions f_i contain the parameters and steady-state information about the power system and therefore cannot be determined. Consequently, they are treated as uncertainties in the following analysis. To bound the f_i we write

$$\begin{aligned} f_i(x) &= P_{e_i}^r - P_{e_i} \\ &= (E_i^r)^2 G_{ii} + E_i^r \sum_{j \in m(i)}^{n} E_j^r(G_{ij}\cos\delta_{ij}^r + B_{ij}\sin\delta_{ij}^r) \\ &\quad - E_i^2 G_{ii} - E_i \sum_{j \in m(i)}^{n} E_j(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) \\ &= ((E_i^r)^2 - E_i^2)G_{ii} \\ &\quad + \sum_{j \in m(i)}^{n} G_{ij}(E_i^r E_j^r \cos\delta_{ij}^r - E_i E_j \cos\delta_{ij}) \\ &\quad + \sum_{j \in m(i)}^{n} B_{ij}(E_i^r E_j^r \sin\delta_{ij}^r - E_i E_j \sin\delta_{ij}) \end{aligned}$$

Although the f_i are unknown, it is easy to see that

$$|f_i(x)| \leqslant C_i \tag{12}$$

where $C_i = (E_{i_{max}}^2 - E_{i_{min}}^2)G_{ii} + \max_{j \in m(i)}[2E_{i_{max}}E_{j_{max}}(|G_{ij}| + |B_{ij}|)]s_i$ are unknown constants, $i = 1, 2, \ldots, n$, and s_i is the number of buses directly connected to bus i.

We now come to the main result of this paper.

Theorem 1: The error system (eqn. 11) is guaranteed to be asymptotically stable, if, for every i, $i = 1, 2, \ldots, n$,

(i) $k_{1i} > 0$, $k_{2i} > 0$

$$\text{(ii) } \dot{\hat{C}}_i(t) = \begin{cases} \left| x_{1i} + \dfrac{M_i}{D_i} x_{2i} \right| & \hat{C}_i(t) < C_{i_{max}} \\ 0 & \hat{C}_i(t) \geqslant C_{i_{max}} \end{cases} \tag{13}$$

with $\hat{C}_i(t_0) < C_{i_{max}}$, where $C_{i_{max}}$ is any constant greater than the upper bound of C_i

$$\text{(iii) } \Delta\dot{\hat{P}}_{m_i}(t) = x_{1i} + \frac{M_i}{D_i} x_{2i} \tag{14}$$

More specially, the states x_{1i} and x_{2i}, $i = 1, 2, \ldots, n$, approach zero asymptotically, and consequently any possible oscillation in the power system decays to zero asymptotically.

Proof: The proof uses Lyapunov's direct method. Under the assumptions of the theorem, choose the Lyapunov function candidate to be

$$V = \tfrac{1}{2}x^T P x + \tfrac{1}{2}\Phi^T\Phi + \tfrac{1}{2}\Psi^T\Psi \tag{15}$$

with

$$\Phi = [\phi_1, \ldots, \phi_n]^T \quad \Psi = [\psi_1, \ldots, \psi_n]^T$$

where

$$\phi_i = \hat{C}_i(t) - C_i \quad \psi_i = \Delta\hat{P}_{m_i}(t) - \Delta P_{m_i}$$

If we choose

$$P = \mathrm{diag}\,\{P_1 \quad P_2 \quad \cdots \quad P_n\}$$

$$P_i = \begin{bmatrix} D_i + k_{2i} + \dfrac{M_i}{D_i}k_{1i} & M_i \\ M_i & \dfrac{M_i^2}{D_i} \end{bmatrix}$$

it is easy to see that P is positive definite, and

$$V = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} [\tfrac{1}{2}x_i^T P_i x_i + \tfrac{1}{2}\phi_i^2 + \tfrac{1}{2}\psi_i^2] \tag{16}$$

We must establish that $\dot{V}$ is at least negative semidefinite, or sufficiently, that the $\dot{V}_i$, $i = 1, \ldots, n$, are all negative semidefinite. The derivative of V_i with respect to time can be written as

$$\dot{V}_i = \tfrac{1}{2}\dot{x}_i^T P_i x_i + \tfrac{1}{2}x_i^T P_i \dot{x}_i + \phi_i\dot{\phi}_i + \psi_i\dot{\psi}_i \tag{17}$$

Substituting the right-hand side of eqn. 3 for $\dot{x}_i$, and noting that

$$x_i^T P_i B_i = x_{1i} + \frac{M_i}{D_i} x_{2i}$$

yields

$$\begin{aligned} \dot{V}_i &= \tfrac{1}{2}x_i^T(A_i^T P_i + P_i A_i)x_i + f_i(x)x_i^T P_i B_i \\ &\quad - \hat{C}_i(t)g_i(x_i)x_i^T P_i B_i - \psi_i x_i^T P_i B_i + \phi_i\dot{\phi}_i + \psi_i\dot{\psi}_i \\ &= -k_{1i}x_{1i}^2 - k_{2i}\frac{M_i}{D_i}x_{2i}^2 + f_i(x)\left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right) \\ &\quad - \hat{C}_i(t)g_i(x_i)\left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right) \\ &\quad - \psi_i\left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right) + \phi_i\dot{\phi}_i + \psi_i\dot{\psi}_i \end{aligned}$$

Given the definition of g_i, we note that

$$\hat{C}_i(t)g_i(x_i)\left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right) = \hat{C}_i(t)\left|x_{1i} + \frac{M_i}{D_i}x_{2i}\right|$$

This allows us to write

$$\begin{aligned} \dot{V}_i &\leqslant -k_{1i}x_{1i}^2 - k_{2i}\frac{M_i}{D_i}x_{2i}^2 + C_i\left|x_{1i} + \frac{M_i}{D_i}x_{2i}\right| \\ &\quad - \hat{C}_i(t)\left|x_{1i} + \frac{M_i}{D_i}x_{2i}\right| - \psi_i\left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right) \\ &\quad + \phi_i\dot{\phi}_i + \psi_i\dot{\psi}_i \\ &= -k_{1i}x_{1i}^2 - k_{2i}\frac{M_i}{D_i}x_{2i}^2 + \phi_i\left[\dot{\phi}_i - \left|x_{1i} + \frac{M_i}{D_i}x_{2i}\right|\right] \\ &\quad + \psi_i\left[\dot{\psi}_i - \left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right)\right] \\ &= \begin{cases} -k_{1i}x_{1i}^2 - k_{2i}\dfrac{M_i}{D_i}x_{2i}^2 & \hat{C}_i(t) < C_{i_{max}} \\ -k_{1i}x_{1i}^2 - k_{2i}\dfrac{M_i}{D_i}x_{2i}^2 - (C_{i_{max}} - C_i)\left|x_{1i} + \dfrac{M_i}{D_i}x_{2i}\right| & \\ & \hat{C}_i(t) = C_{i_{max}} \end{cases} \end{aligned}$$

where the last two steps are based on the definition of $\hat{C}_i(t)$ given in the statement of the theorem.

Since $(C_{i_{max}} - C_i)|x_{1i} + (M_i/D_i)x_{2i}| \geqslant 0$, $i = 1, 2, \ldots, n$, we finally have

$$\dot{V}_i(t) \leqslant -x_i^T Q_i x_i \tag{18}$$

for all $\hat{C}_i(t)$, where

$$Q_i = \begin{bmatrix} k_{1i} & 0 \\ 0 & k_{2i}\dfrac{M_i}{D_i} \end{bmatrix} \quad i = 1, 2, \ldots, n \tag{19}$$

The Q_i are positive definite, thus we have $\lambda_{min}(Q_i) > 0$. Now we separate V_i into two parts, V_{1i} and V_{2i}, that is $V_i = V_{1i} + V_{2i}$, where $V_{1i} = \frac{1}{2}x_i^T P_i x_i$ and $V_{2i} = \frac{1}{2}\phi_i^2 + \frac{1}{2}\psi_i^2$ respectively. It is easy to see that V_{1i} and V_{2i} are both positive definite, and

$$V_{1i} \leqslant \lambda_{max}(P_i)\|x_i\|^2$$

Using eqn. 18 and this last expression, we can write

$$\begin{aligned} \dot{V}_i(t) &\leqslant -x_i^T Q_i x_i \\ &\leqslant -\lambda_{min}(Q_i)\|x_i\|^2 \\ &\leqslant -\frac{\lambda_{min}(Q_i)}{\lambda_{max}(P_i)} V_{1i} \\ &\stackrel{\text{def}}{=} -\lambda_1 V_{1i} \end{aligned} \tag{20}$$

Integrating eqn. 20 on both sides and rearranging yields

$$V_{1i}(t) + \lambda_1 \int_{t_0}^{t} V_{1i}(\tau)\, d\tau \leqslant V_i(t_0) \tag{21}$$

Taking the limit as t approaches infinity on both sides of eqn. 21, we have

$$\lim_{t\to\infty}\left[V_{1i}(t) + \lambda_1 \int_{t_0}^{t} V_{1i}(\tau)\, d\tau \right] \leqslant V_i(t_0) < +\infty \tag{22}$$

Thus, it follows from the continuity of the solution and from the continuity and positive definiteness of V_{1i} that $\lim_{t\to\infty} V_{1i}(\|x_i\|) = 0$. Since V_{1i} is positive definite, we have $\lim_{t\to\infty}\|x_i\| = 0$.

Several remarks are worth making at this point.

(i) The control consists of linear feedback terms and two adaptive terms. The first adaptive term $\hat{C}_i(t)g_i(x_{1i}, x_{2i})$, is the adaptive tuning of the linear feedback gains, which, associated with the linear feedback gains, guarantees robustness of the system stability. The second adaptive term $\Delta\hat{P}_{mi}(t)$, changes continuously according to the local error states, it smoothes the transient trajectorise of the system.

(ii) The control guarantees that the system approaches the postdisturbance steady state, chosen arbitrarily by the system operator, asymptotically.

(iii) The only constraint placed on the robust gains, k_{1i} and k_{2i}, is that they must be positive. Thus no tuning is required.

(iv) Since the proposed control uses adaptation, it is not necessary to know the system parameters. This is extremely important in a large power system model where, under the best of circumstances, we have only good estimates of these parameters.

(v) The proof of robustness, using Lyapunov's direct method only guarantees system stability; it does not provide any information about the transient behaviour of the system or the shape of the control. That is the next issue to be addressed.

As we will subsequently see from the simulations, the definition of g_i used in the proof results in a very unsatisfying control. To correct that problem we now repeat the proof using a continuous approximation to the sign function. We provide an outline of the proof, since it follows almost identically the proof we have just completed.

To smooth the mechanical power, we choose g_i to be:

$$g_i(x_{1i}, x_{2i}) = -\frac{x_{1i} + \dfrac{M_i}{D_i}x_{2i}}{\left|x_{1i} + \dfrac{M_i}{D_i}x_{2i}\right| + \varepsilon_i e^{-\beta_i t}} \tag{23}$$

where the exponential term should be reset at the time the fault is cleared.

We use the same Lyapunov function and the same adaptive laws. The proof is as follows.

$$\begin{aligned} \dot{V}_i &= \tfrac{1}{2}\dot{x}_i^T P_i x_i + \tfrac{1}{2}x_i^T P_i \dot{x}_i + \phi_i\dot{\phi} + \psi_i\dot{\psi}_i \\ &\leqslant -k_{1i}x_{1i}^2 - k_{2i}\frac{M_i}{D_i}x_{2i}^2 + C_i\left|x_{1i} + \frac{M_i}{D_i}x_{2i}\right| \\ &\quad - \hat{C}_i(t)\frac{\left(x_{1i} + \dfrac{M_i}{D_i}x_{2i}\right)^2}{\left|x_{1i} + \dfrac{M_i}{D_i}x_{2i}\right| + \varepsilon_i e^{-\beta_i t}} \\ &\quad - \psi_i\left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right) + \phi_i\dot{\phi}_i + \psi_i\dot{\psi}_i \\ &= -k_{1i}x_{1i}^2 - k_{2i}\frac{M_i}{D_i}x_{2i}^2 + C_i\left|x_{1i} + \frac{M_i}{D_i}x_{2i}\right| \\ &\quad - \hat{C}_i(t)\frac{\left(x_{1i} + \dfrac{M_i}{D_i}x_{2i}\right)^2 - \varepsilon_i^2 e^{-2\beta_i t} + \varepsilon_i^2 e^{-2\beta_i t}}{\left|x_{1i} + \dfrac{M_i}{D_i}x_{2i}\right| + \varepsilon_i e^{-\beta_i t}} \\ &\quad - \psi_i\left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right) + \phi_i\dot{\phi}_i + \psi_i\dot{\psi}_i \\ &\leqslant -k_{1i}x_{1i}^2 - k_{2i}\frac{M_i}{D_i}x_{2i}^2 - \phi_i\left|x_{1i} + \frac{M_i}{D_i}x_{2i}\right| \\ &\quad + \varepsilon_i C_{i_{max}} e^{-\beta_i t} - \hat{C}_i(t)\frac{\varepsilon_i^2 e^{-2\beta_i t}}{\left|x_{1i} + \dfrac{M_i}{D_i}x_{2i}\right| + \varepsilon_i e^{-\beta_i t}} \\ &\quad - \psi_i\left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right) + \phi_i\dot{\phi}_i + \psi_i\dot{\psi}_i \\ &\leqslant -k_{1i}x_{1i}^2 - k_{2i}\frac{M_i}{D_i}x_{2i}^2 + \phi_i\left[\dot{\phi}_i - \left|x_{1i} + \frac{M_i}{D_i}x_{2i}\right|\right] \\ &\quad + \psi_i\left[\dot{\psi}_i - \left(x_{1i} + \frac{M_i}{D_i}x_{2i}\right)\right] + \varepsilon_i C_{i_{max}} e^{-\beta_i t} \\ &= \begin{cases} -k_{1i}x_{1i}^2 - k_{2i}\dfrac{M_i}{D_i}x_{2i}^2 + \varepsilon_i C_{i_{max}} e^{-\beta_i t} & \hat{C}_i(t) < C_{i_{max}} \\ -k_{1i}x_{1i}^2 - k_{2i}\dfrac{M_i}{D_i}x_{2i}^2 + \varepsilon_i C_{i_{max}} e^{-\beta_i t} - (C_{i_{max}} - C_i)\left|x_{1i} + \dfrac{M_i}{D_i}x_{2i}\right| & \hat{C}_i(t) = C_{i_{max}} \end{cases} \\ &\leqslant -x_i^T Q_i x_i + \varepsilon_i C_{i_{max}} e^{-\beta_i t} \end{aligned}$$

Using the same 'separation' approach we used earlier, we can write

$$\dot{V}_i(t) \leq -\lambda_1 V_{1i} + \varepsilon_i C_{i_{max}} e^{-\beta_i t} \tag{24}$$

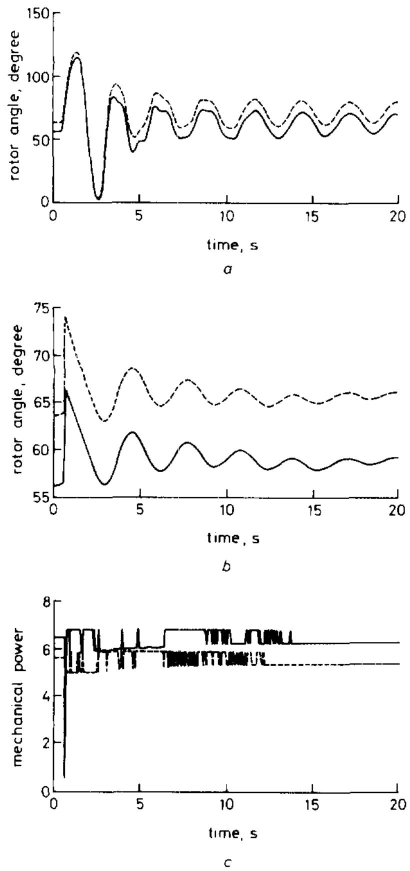

Fig. 2 *System response to disturbance I*
a rotor angles of machines 6 and 7 with no control
b rotor angles of machines 6 and 7 with primitive control
c mechanical power of machines 6 and 7 with primitive control

Integrating inequality eqn. 24 on both sides and rearranging, we have

$$V_{1i}(t) + \lambda_1 \int_{t_0}^{t} V_{1i}(\tau)\, d\tau \leq V_i(t_0) + \frac{\varepsilon_i C_{i_{max}}}{\beta} e^{-\beta_i t_0} \tag{25}$$

Taking the limit as t approaches infinity on both sides of inequality eqn. 25, yields

$$\lim_{t \to \infty} \left[V_{1i}(t) + \lambda_1 \int_{t_0}^{t} V_{1i}(\tau)\, d\tau \right] \leq V_i(t_0) + \frac{\varepsilon_i C_{i_{max}}}{\beta} e^{-\beta_i t_0} < +\infty \tag{26}$$

Thus, it follows from the continuity of the solution and from the continuity and positive definiteness of V_{1i} that $\lim_{t \to \infty} V_{1i}(\|x_i\|) = 0$. Since V_{1i} is positive definite, we have $\lim_{t \to \infty} \|x_i\| = 0$.

In the above discussion, we assumed that the ratio of machine inertia and damping constant is known. It is well known that this ratio is approximately the same for all machines, and a common simplifyng assumption in power system analysis is to assign the same ratio to all machines. However, this assumption is not necessary in the present analysis, that is the proposed control can still be proved to be valid with a minor change of the condition of the theorem.

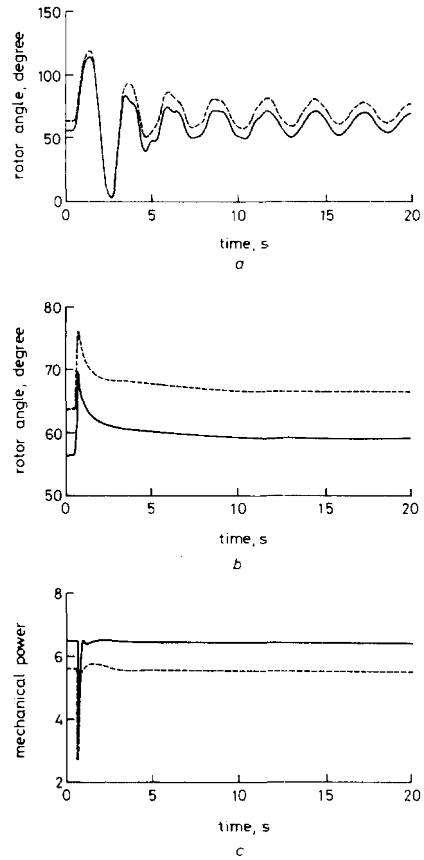

Fig. 3 *System response to disturbance I*
a rotor angles of machines 6 and 7 without control
b rotor angles of machines 6 and 7 using adaptive control
c mechanical power of machines 6 and 7 under adaptive control

We write the damping coefficient D_i' as a nominal, known part D_i, plus an unknown part d_i. That is

$$D_i' = D_i + d_i \tag{27}$$

Replacing D_i in eqn. 27 by D_i' and using the same control eqn. 6, the ith subsystem becomes

$$\dot{x}_i = A_i x_i + B_i f_i(x) - B_i \hat{C}_i(t) g_i(x_i) - B_i \psi_i + B_i d_i x_{2i} \tag{28}$$

where all the definitions are the same as in eqn. 9. Choosing the same Lyapunov function, we have

$$\dot{V}_i \leq -x_i^T Q_i' x_i + \varepsilon_i C_{i_{max}} e^{-\beta_i t}$$

where

$$Q_i' = \begin{bmatrix} k_{1i} & d_i \\ 0 & (k_{2i} + d_i) \dfrac{M_i}{D_i} \end{bmatrix} \quad i = 1, 2, \ldots, n \tag{29}$$

If we change the condition (i) of the theorem to $k_{1i} > 0$ and $k_{2i} > |d_i|$, we know that the matrix Q_i' is positive definite. Using the same argument in the above proof, we can get the same result.

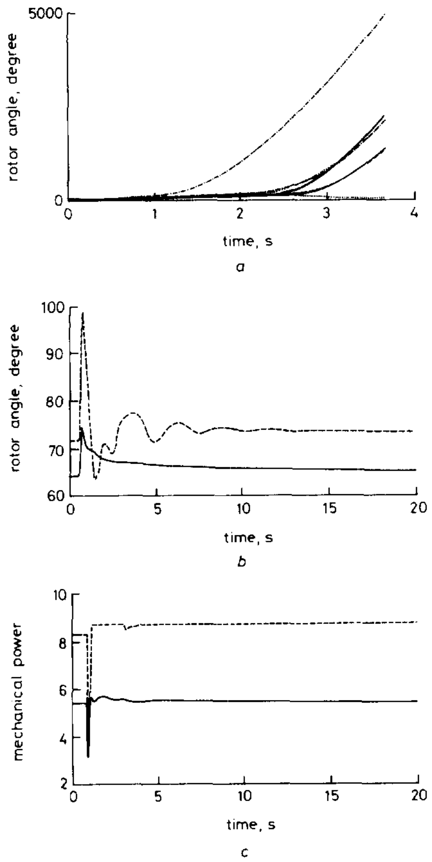

Fig. 4 *System response to disturbance 2*
a rotor angles of machines 8 and 9 without control
b rotor angles of machines 8 and 9 using adaptive control
c mechanical power of machines 8 and 9 under adaptive control

4 Validation of the control algorithms

The proposed control algorithms were tested using the loading of the 39 Bus New England System given [17]. The topology of the system and the load data are readily available. A two-axis machine model is used for all generators, and every generator has excitation control with the exception of machine 10 which is used as the reference. This machine has an inertia constant ten times greater than that of any other machine in the system and thus is a good candidate for the reference. The mechanical power of all machines is per unit on a 100 MVA base.

To test the control, we choose two scenarios. Disturbance 1 is a fault at bus 2 at time $t = 0.5$ s, which is cleared at $t = 0.6$ s by dropping the line from bus 2 to bus 1. This is a six-cycle fault, quite long by most standards, but our goal was to produce a sustained oscillation using the loading given [17]. Disturbance 2 is a three phase fault at bus 26 initiated at $t = 0.5$ s and cleared after six cycles by dropping the line from bus 26 to 25.

Fig. 2*a* shows the angle trajectories of machine 6 and 7, the others are similar, with the simulation carried out with no control. The sustained oscillation in this case is clearly evident. The trajectories of the other machines are similar. Fig. 2*b* is the angle trajectories of machine 6 and 7 with the 'primitive' controller, that is the controller where g_i is a sign function. The oscillations are damped, but take about 20 seconds to die out. The mechanical input power is shown in Fig. 2*c*, and as can be seen, the changes in mechanical power are frequency and abrupt, making the control unacceptable.

Fig. 3*b* again shows disturbance 1 but with g_i as given by eqn. 23. As can be seen in Fig. 3*b*, there is no oscillation and the system is near steady state after about three seconds. Again the rotor angles are for machine 6 and 7. Further, the mechanical power changes are acceptable. More specifically, the power dips once and then returns to the original steady state values. This kind of control resembles fast valving. However, the size and speed of the power drop is probably beyond the capability of existing fast valving. The results shown in Fig. 3 indicate that the robust adaptive controller is very effective.

To further test the adaptive controller, we used disturbance 2. Fig. 4*a* shows that the system without control is unstable. Fig. 4*b* shows the same case with control (g_i given by eqn. 23). The system is stable and reaches steady state after about 10 seconds with only a few swings. The rotor angles shown are for machines 8 and 9, the two machines closest to the fault. The rotor angle deviations of the other machines are smaller and are omitted. Fig. 4*c* shows the changes in mechanical power. Again, the changes, aside from the first dip, are relatively small. The flat area following the first dip is a result of limiting the maximum mechanical power to 1.05 of the initial mechanical power.

5 Summary and conclusions

In this paper we have introduced a robust adaptive controller based on a nonlinear model of the power system. We have shown that the control is robust against arbitrary disturbances anywhere in the power system. The simulation results presented show that the control is very effective and feasible, given that one is willing to put control on every machine in the system. Further, the transient response is quite acceptable.

The great asset of the adaptive controller presented here is that it completely obviates the need to tune the robust controllers on each of the machines. This is a crucial advantage, since this tuning process could be quite laborious. Further, the tuning would have to be done offline using a particular power system topology. Since the configuration of any power system changes on a minute-by-minute basis the best that could be done without adaptation would be to try to tune using a 'worst' case topology and loading. This approach would be inferior to using adaptation which is impervious to changes in system topology. Another advantage of the adaptive controller is that it will bring the system to any desired steady state if there is sufficient mechanical power available. Robust control without adaptation will always stabilise the system, but not to an arbitrary steady state.

We have noted earlier that the modelling process accounts for the presence of excitation control in the sense that we do not require that the internal voltages of the machines remain constant, only that the voltages are bounded. We have not accounted, either implicitly or explicitly for turbine governor control. That is the next refinement which we will address in a future paper.

We make a final observation on the feasibility of the control scheme. It is unlikely that any utility will be willing to put the proposed control on every machine in a power system to guarantee that the system response is stable for every contingency, at least not at present. The interconnected grid is still relatively stable, and utilities can obtain satisfactory security by designing control only for a few specific disturbances that are viewed as 'worst case' scenarios. However, if current trends continue, the stability of the interconnected grid may degrade to the point where the system is vulnerable to many different disturbances. In this case, a successful control strategy will have to be global in scope, and this paper provides some guidelines for developing such a strategy.

6 References

1 MANSOUR, Y.: 'Application of eigenanalysis to the Western North American Power System', *in* 'Eigenanalysis and frequency domain methods for system dynamic performance' (Power Engineering Society publication 90TH0292-3-PWR)

2 DEMELLO, F.P., and CONCORDIA, C.: 'Concepts of synchronous machine stability as affected by excitation control', *IEEE Trans.*, 1969, **PAS-88**, (4), pp. 316–329

3 KUNDUR, P., KLEIN, M., ROGERS, G.J., and ZYWNO, M.S.: 'Application of power system stablizers for enhancement of overall system stability', *IEEE Trans.*, 1989, **PS-4**, (2), pp. 614–626

4 CHEN, C., and HSU, Y.: 'Power system stability improvement using dynamic output feedback compensation', *Elect. Power Syst. Res.*, 1987, **12**, pp. 37–39

5 FELIACHI, A., ZHANG, X., and SIMS, C.S.: 'Power system stabilizers design using optimal reduced order models, Part I and II', *IEEE Trans.*, 1988, **PS-3**, (4), pp. 1670–1684

6 YOUSEF, H., and SIMAAN, M.A.: 'Model reference adaptive control for large scale systems with application to power systems', *IEE Proc. D*, 1991, **138**, (4), pp. 321–326

7 SONG, Y.H.: 'Novel adaptive control scheme for improving power system stability', *IEE Proc. C*, 1992, **139**, (5), pp. 423–426

8 WANG, Y., ZHOU, R., and WEN, C.: 'Robust load-frequency controller design for power systems', *IEE Proc. C*, 1993, **140**, (1), pp. 11–16

9 LEE, K.A., YEE, H., and TEO, C.Y.: 'Self-tuning algorithm for automatic generation control in an interconnected power system', *Elect. Power Syst. Res.*, 1991, **20**, pp. 157–165

10 LIM, C.M., and HIYAMA, T.: 'Application of a self-tuning control scheme to a power system with multi-model oscillation', *Elect. Power Syst. Res.*, 1992, **24**, pp. 91–98

11 WALKER, P.A., SERAG, A.M., and ABDALLA, O.H.: 'Integrated excitation and turbine control in a multimachine power system', *IEE Proc. C*, 1989, **136**, (6), pp. 331–340

12 WU, Q.H., and HOGG, B.W.: 'Self-tuning control for turbogenerators in multimachine power systems', *IEE Proc. C*, 1990, **137**, (2), pp. 146–158

13 MUSAAZI, M.K., JOHNSON, R.B.I., and CORY, B.J.: 'Multimachine system transient stability improvement using transient power system stabilizers (TPSS)', *IEEE Trans.*, 1986, **EC-1**, (4), pp. 34–38

14 LU, Q., SUN, Y., and LEE, G.K.F.: 'Nonlinear optimal excitation control for multimachine systems', *in* 'IFAC power systems modeling and control applications' (Brussels, Belgium, 1988), pp. 27–32

15 GAO, L., CHEN, L., EAN, Y., and MA, H.: 'A nonlinear control design for power systems', *Automatica*, 1992, **28**, (5), pp. 975–979

16 QU, Z., DORSEY, J.F., BOND, J., and McCALLEY, J.: 'Robust transient control of power systems', *IEEE Trans.*, 1992, **CS-39**, CAS-I, (6), pp. 470–476

17 WESTERN ELECTRIC CORPORATION: 'Phase II: frequency domain analysis of low frequency oscillations in large electric power system', Final report EPRI EL-2348 Vol. 1, Electric Power Reserch Institute, Palo Alto, California, 1982