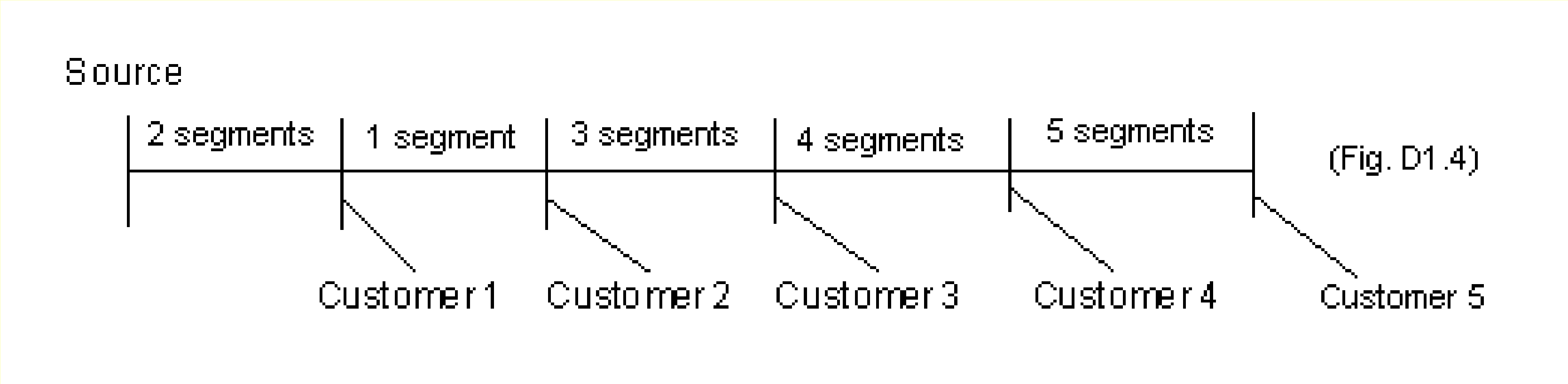
# Module D1-b
# Introduction to Distribution Systems
# (Continued)

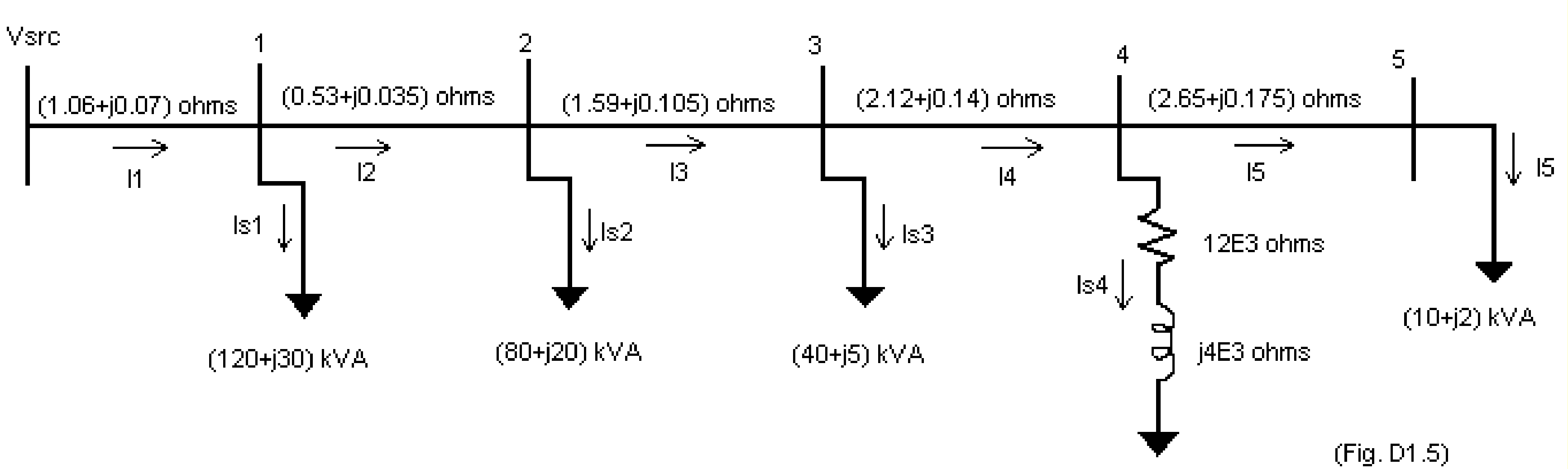
# Example: Determine Vs required to hold V5=11 kV (L-L)




- 1 segment = 1000 ft
- Need to use table to get impedance of each section
- All lines use 4 AWG conductors
- System is 3 phase

Load data is as follows:

bus 1: 120+j30 kVA
bus 2: 80+j20 kVA
bus 3: 40+j5 kVA
bus 4: 12,000+j4000 ohms/phase (Y)
bus 5: 10+j2 kVA


Vsrc
1
2
3
4
5
(1.06+j0.07) ohms
(0.53+j0.035) ohms
(1.59+j0.105) ohms
(2.12+j0.14) ohms
(2.65+j0.175) ohms
I1
I2
I3
I4
I5
Is1
Is2
Is3
Is4
I5
12E3 ohms
j4E3 ohms
(120+j30) kVA
(80+j20) kVA
(40+j5) kVA
(10+j2) kVA

(Fig. D1.5)

# Basic algorithm for problem solution:

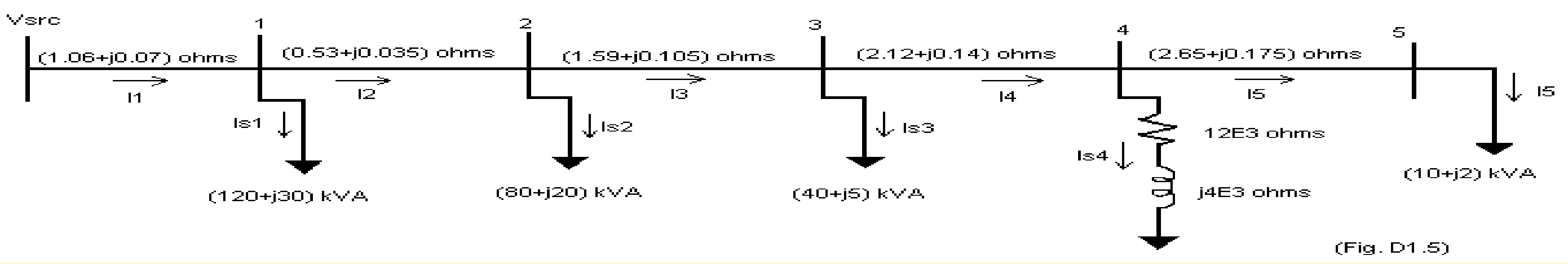
1. $i=5$
2. Find $I_i$ (current flowing into bus i)

   $I_i=I_{i+1}+I_{si}$

   If constant power load, $I_{si}=(S_i/V_i)^*$

   If constant Z load, $I_{si}=V_i/Z_i$
3. Find $V_{i-1}=V_i+I_i(Z_{i-1,i})$
4. $i=i-1$, if not done, go to 2

Note: powers are per phase (or pu) and voltages are L-N (or pu)

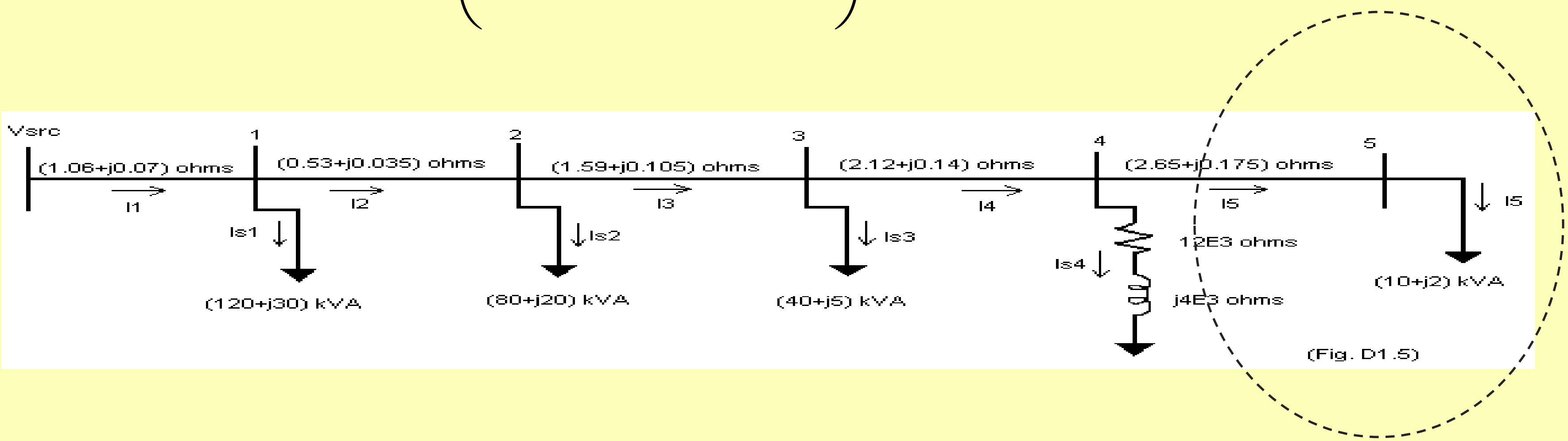


(Fig. D1.5)

We normally use L-N voltage as reference. This time we will use L-L voltage as reference. Our choice will only affect the angles in our calculations.

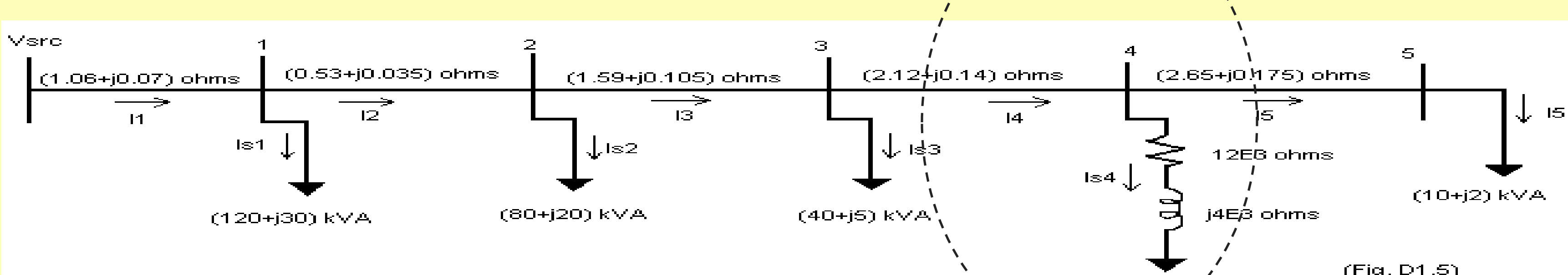
$$V5 = \frac{11000\angle -30^\circ}{\sqrt{3}} = 6350\angle -30^\circ$$

$$I5 = \left(\frac{S}{V}\right)^* = \left(\frac{\left(\frac{10 - j2}{3}\right)\cdot 10^3}{6350\angle 30^\circ}\right) = 0.4021 - j0.3533\ A$$



(Fig. D1.5)

$$V4 = V5 + I5 \cdot Z45$$
$$= (6350\angle -30^\circ) + (0.4021 - j.3533)(2.65 + j0.175)$$
$$= 6352.3\angle -30.002^\circ$$
$$\Rightarrow \quad V4LL = 11002.44\angle -.002^\circ$$
$$IS4 = \frac{V4}{Z4} = \frac{6352.3\angle -30.002^\circ}{12000 + j4000} = 0.33318 - j0.37575$$
$$I4 = I5 + IS4 = (0.4021 - j0.3533) + (0.33318 - j0.37575)$$
$$= (0.7353 - j0.7291)$$



(Fig. D1.5)

$$V3 = V4 + I4 \cdot Z34$$

$$= \left(6352.3\angle -30.002^\circ\right) + \left(0.7353 - j0.7291\right)\left(2.12 + j0.14\right)$$

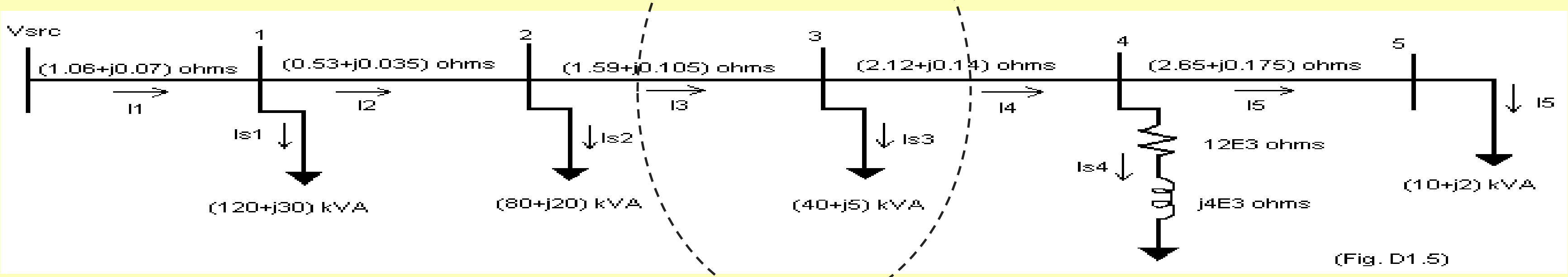
$$= 6354.46\angle -30.006^\circ$$

$$\Rightarrow \quad V3LL = 11006.25\angle -.006^\circ$$

$$IS3 = \frac{\left(\dfrac{40 - j5}{3}\right) \cdot 10^3}{6354.46\angle 30.006^\circ} = 1.6859 - j1.2765$$

$$I3 = I4 + IS3 = \left(0.7353 - j0.7291\right) + \left(1.6859 - j1.2765\right)$$

$$= \left(2.4212 - j2.0055\right)$$




(Fig. D1.5)

$$V2 = V3 + I3 \cdot Z23 = (6354.46\angle -30.006^\circ)$$

$$+ (2.4212 - j2.0055)(1.59 + j0.105)$$

$$= 6359.44\angle -30.0106^\circ$$

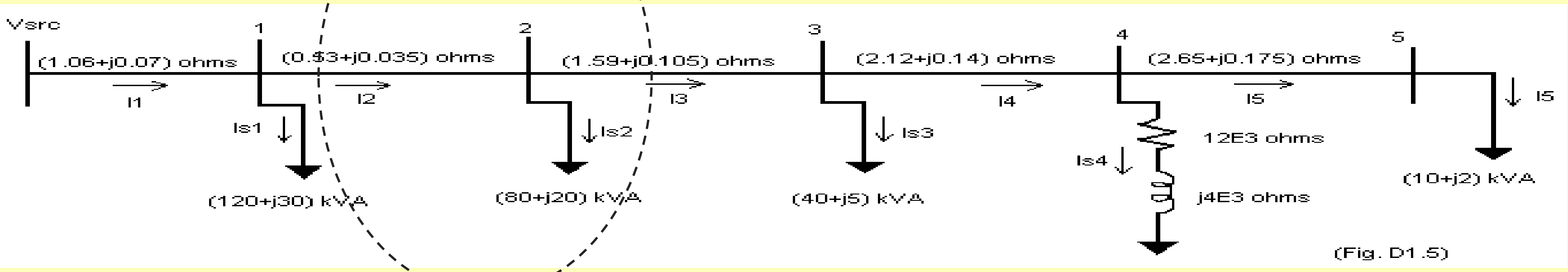
$$\Rightarrow \quad V2LL = 11014.88\angle -.0106^\circ$$

$$IS2 = \frac{\left(\dfrac{80 - j20}{3}\right) \cdot 10^3}{6359.44\angle 30.0106} = 3.10674 - j3.00506$$

$$I2 = I3 + IS2 = (2.4212 - j2.0055)$$

$$+ (3.10674 - j3.00506)$$

$$= (5.5299 - j5.0106)$$



$$V1 = V2 + I2 \cdot Z12$$

$$= (6359.44\angle -30.0106°) + (5.5299 - j5.0106)(0.53 + j0.035)$$

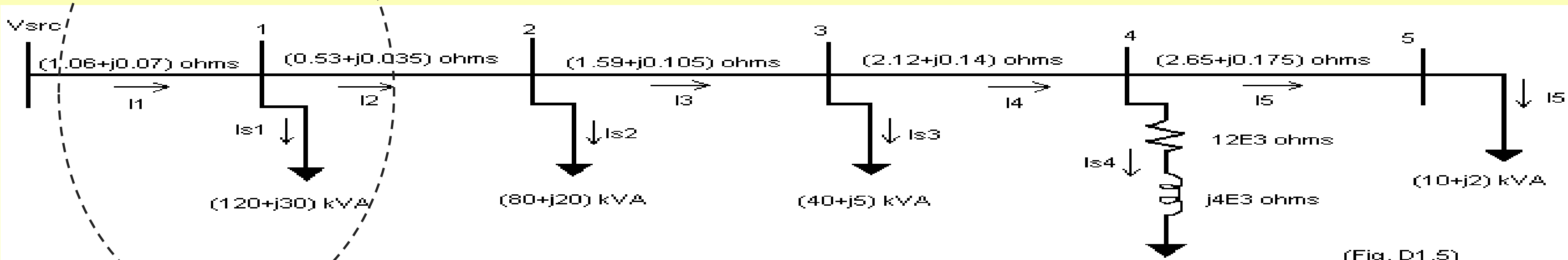
$$= 6363.36\angle -30.0158°$$

$$\Rightarrow \quad V1LL = 11021.66\angle -.0158°$$

$$IS1 = \frac{\left(\dfrac{120 - j30}{3}\right)\cdot 10^3}{6363.36\angle 30.0158} = 4.6568 - j4.50523$$

$$I1 = I2 + IS1 = (5.5279 - j5.0106) + (4.6568 - j4.50523)$$

$$= (10.1847 - j9.5158)$$




(Fig. D1.5)

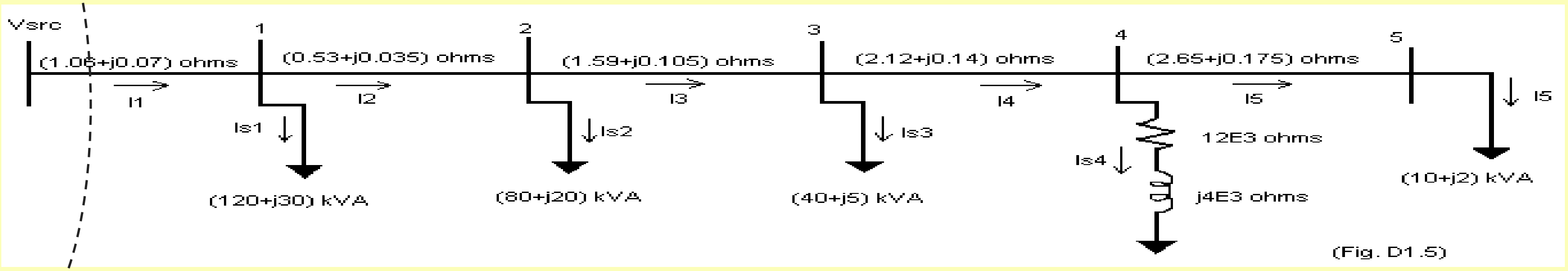
$$
\begin{aligned}
VSRC &= V1 + I1 \cdot ZS1 \\
&= \left(6363.36\angle -30.0158^\circ\right) + \left(10.1847 - j9.5158\right)\left(1.06 + j0.07\right) \\
&= 6377.97\angle -30.0372^\circ
\end{aligned}
$$

$$
\Rightarrow \quad VSRC, LL = 11046.98\angle -.0372^\circ
$$

$$
I_{SRC} = I1
$$

$$
P_{SRC} = P_{IN} = 3\,\mathrm{Re}\left\{\left(V_{SRC}\right)\left(I_{SRC}\right)^{*}\right\} = \mathrm{Re}\left\{\left(\left(6377.97\angle -30.0372^\circ\right)\left(10.1847 - j9.5158\right)^{*}\right)\right\}
$$

$$
= 3\,\mathrm{Re}\left\{\left(86{,}613.79 + j20025.28\right)\right\} = 3(86{,}613.79) = 259{,}841.4 \text{ watts}
$$



(Fig. D1.5)

$$Losses:$$

$$I5^2 \cdot R45 = .7592$$

$$I4^2 \cdot R34 = 2.273$$

$$I3^2 \cdot R23 = 15.716$$

$$I2^2 \cdot R12 = 29.502$$

$$I1^2 \cdot RS1 = 205.935$$

$$TOTAL = 3(254.19) = 762.57 = P_{LOSS}$$

$$Efficiency\left(\eta\right):$$

$$\eta = \frac{P_{IN} - P_{LOSS}}{P_{IN}} \times 100\%$$

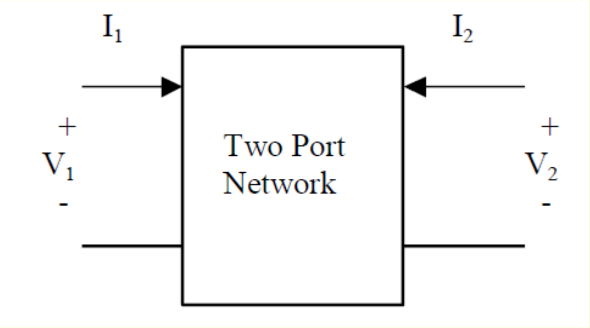
$$\eta = \frac{259814.4 - 762.57}{259814.4} \times 100\% = 99.71\%$$

$$regulation\ \%reg:$$

$$\%reg = \frac{\left|V_{LL,SRC}\right| - \left|V_{LL,5}\right|}{\left|V_{LL,SRC}\right|} \times 100\%$$

$$\%reg = \frac{11046.98 - 11000}{11046.98} \times 100\% = 0.425\%$$

# Two Port Networks



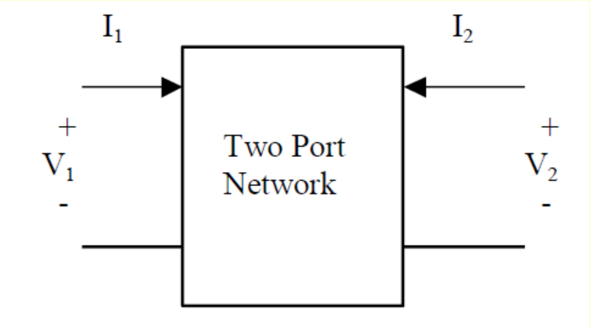

A two-port network is a linear network having two pairs of terminals: input and output.

It can contain any of the following components:

- R, L, C
- Transformers
- Op-amps
- Dependent sources

But it cannot contain *independent* sources.

# Two Port Networks

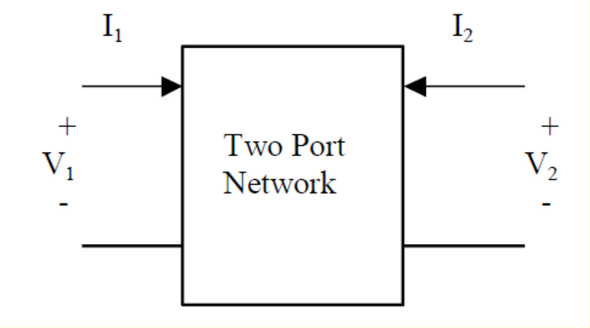



We may use two-ports to describe a number of different types of circuits including:

- Transistor circuits
- Transformers
- Transmission lines
- Distribution lines

Two ports are attractive because, no matter how complex what is inside the box, we may describe it with only four parameters! There are six "4-parameter sets" as given on the next slide.

# Two Port Networks




| | |
|---|---|
| Z (impedance) parameters | $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ |
| Y (admittance) parameters | $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Y \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ |
| H (hybrid) parameters | $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = H \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$ |
| G (inverse hybrid) parameters | $\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = G \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$ |
| a (transmission) parameters | $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = A \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$ |
| b (inverse transmission) parameters | $\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = B \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$ |

a-parameters and ABCD parameters are the same except in ABCD, we reverse the direction of the current $I_2$.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = T \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

# Two Port Networks

Given two-port equations, each different set of parameters may be determined, one parameter at a time, according to the following procedure:

1. Pull out the equation containing the parameter of interest;
2. Solve the equation for the parameter of interest.
3. Set to zero the current or voltage necessary to eliminate the remaining parameter from the equation.

# Example

Consider the Y-parameters.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Let's determine $y_{21}$.

Step1: Pull out the equation containing the parameter of interest:

$$I_2 = y_{21} V_1 + y_{22} V_2$$

Step2: Solve the equation for parameter of interest:

$$y_{21} = \frac{I_2 - y_{22} V_2}{V_1}$$

Step3: Set to zero the current or voltage necessary to eliminate the remaining parameter from the equation:

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2 = 0}$$

# Your turn

Consider the Y-parameters.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Let's determine $y_{11}$.

Step1: Pull out the equation containing the parameter of interest:

Step2: Solve the equation for parameter of interest:

Step3: Set to zero the current or voltage necessary to eliminate the remaining parameter from the equation:

# Your turn

Consider the Y-parameters.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Let's determine $y_{11}$.

Step1: Pull out the equation containing the parameter of interest:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

Step2: Solve the equation for parameter of interest:

$$y_{11} = \frac{I_1 - y_{12}V_2}{V_1}$$

Step3: Set to zero the current or voltage necessary to eliminate the remaining parameter from the equation:

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

# Another example

Consider the Z-parameters.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Let's determine $z_{11}$.

Step1: Pull out the equation containing the parameter of interest:

$$V_1 = z_{11} I_1 + z_{12} I_2$$

Step2: Solve the equation for parameter of interest:

$$z_{11} = \frac{V_1 - z_{12} I_2}{I_1}$$

Step3: Set to zero the current or voltage necessary to eliminate the remaining parameter from the equation:

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2 = 0}$$

# Conclusion

Repeated application of this procedure results in following relations:

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \tag{D1.24}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \tag{D1.25}$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \tag{D1.26}$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \quad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \quad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \quad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \tag{D1.27}$$

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad a_{12} = \left.\frac{V_1}{I_2}\right|_{V_2=0} \quad a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} \quad a_{22} = \left.\frac{I_1}{I_2}\right|_{V_2=0} \tag{D1.28}$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0} \quad b_{12} = \left.\frac{V_2}{I_1}\right|_{V_1=0} \quad b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0} \quad b_{22} = \left.\frac{I_2}{I_1}\right|_{V_1=0} \tag{D1.29}$$

Question1: What does it mean to set a voltage to 0?

➔Short the two terminals across which that voltage appears!

Question2: What does it mean to set a current to 0?

➔Open the branch through which that current flows!

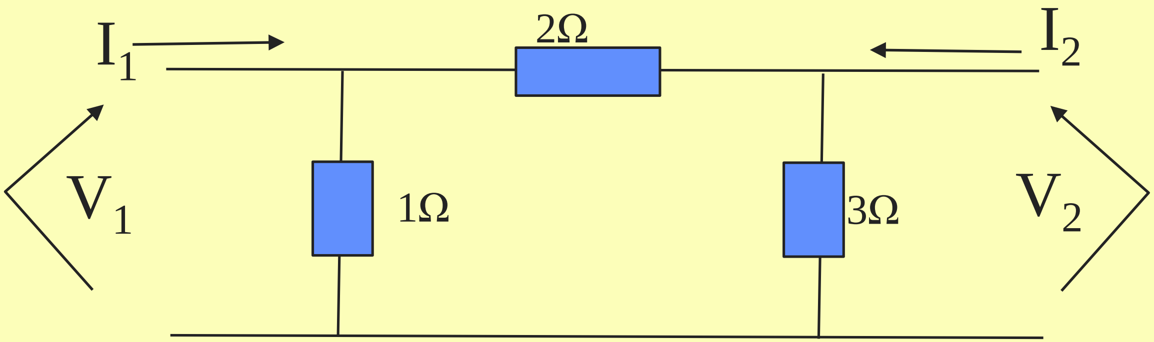
# Question1: What does it mean to set a voltage to 0?

➔Short the two terminals across which that voltage appears!

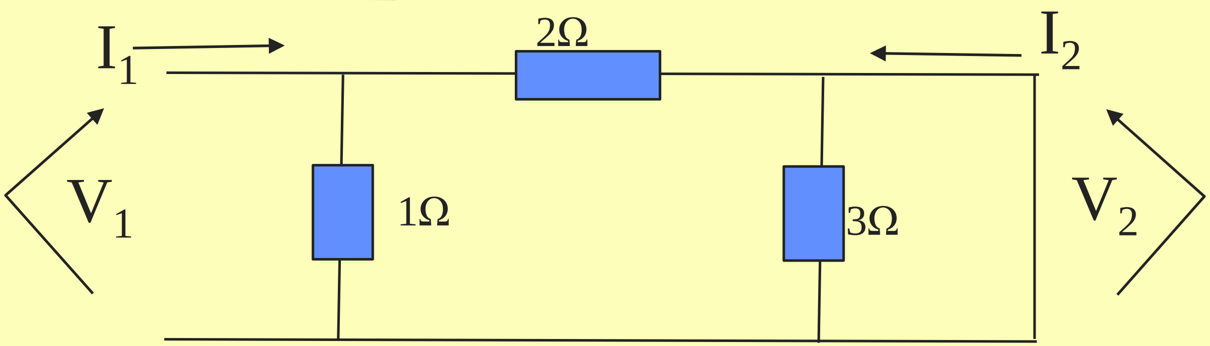
# How to obtain parameters: example 1.

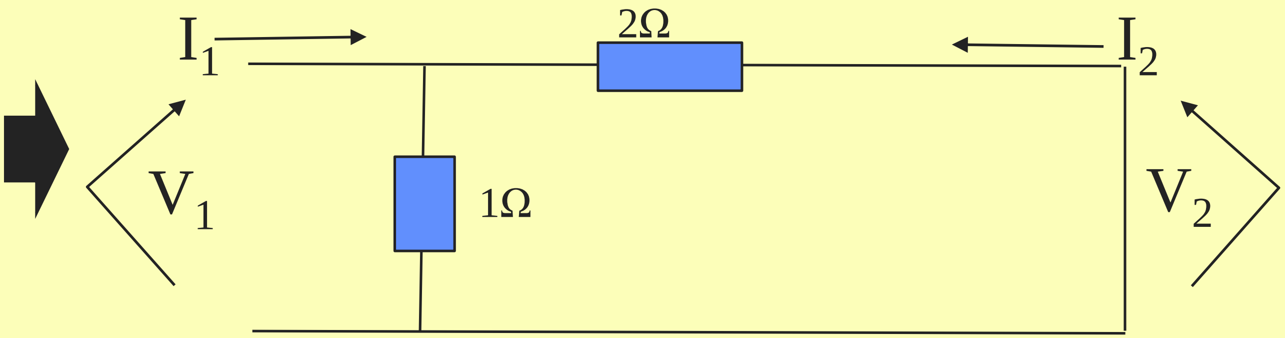
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$




$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$



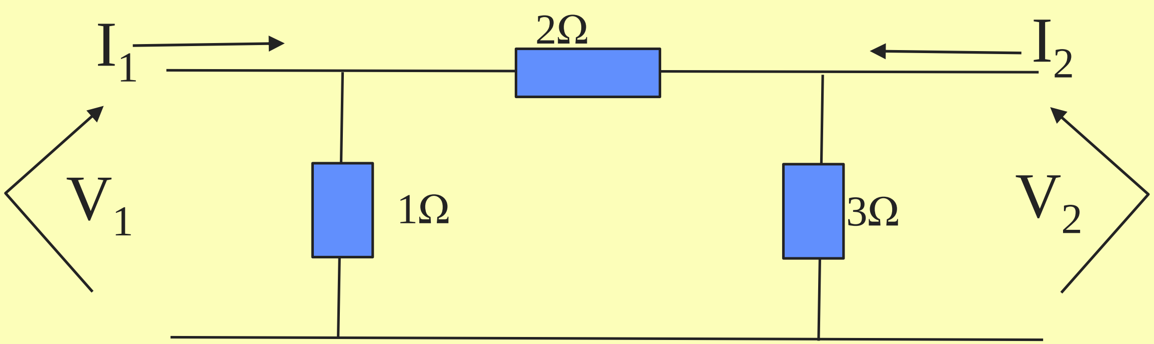

$$\left.\frac{V_1}{I_2}\right|_{V_2=0} = -2$$

$$\Rightarrow \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-1}{2} = y_{21}$$

# Question1: What does it mean to set a voltage to 0?

➔ Short the two terminals across which that voltage appears!

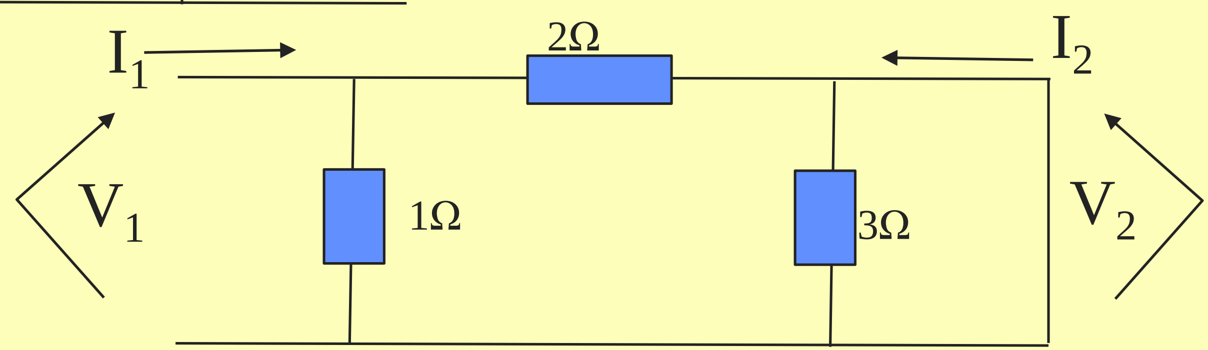
## How to obtain parameters: example 1.

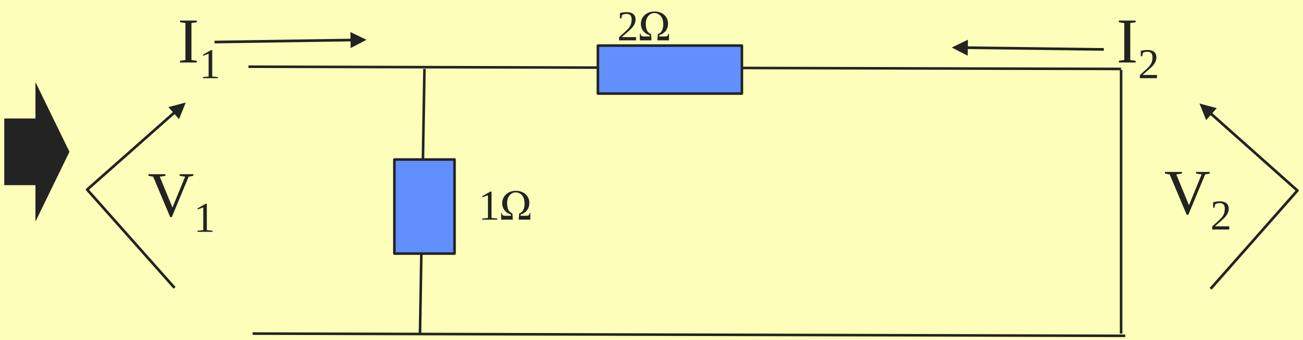



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$




$$\left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{2(1)}{3}$$

$$\Rightarrow \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{3}{2} = y_{11}$$

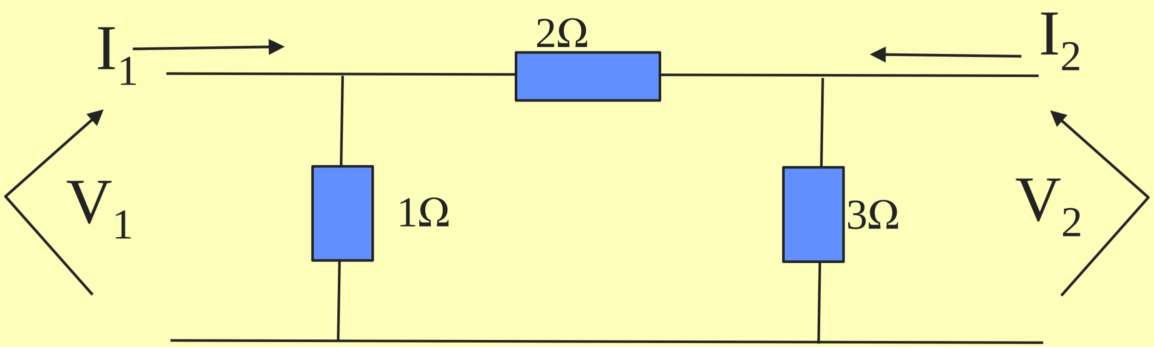
# Question1: What does it mean to set a voltage to 0?

➔Short the two terminals across which that voltage appears!

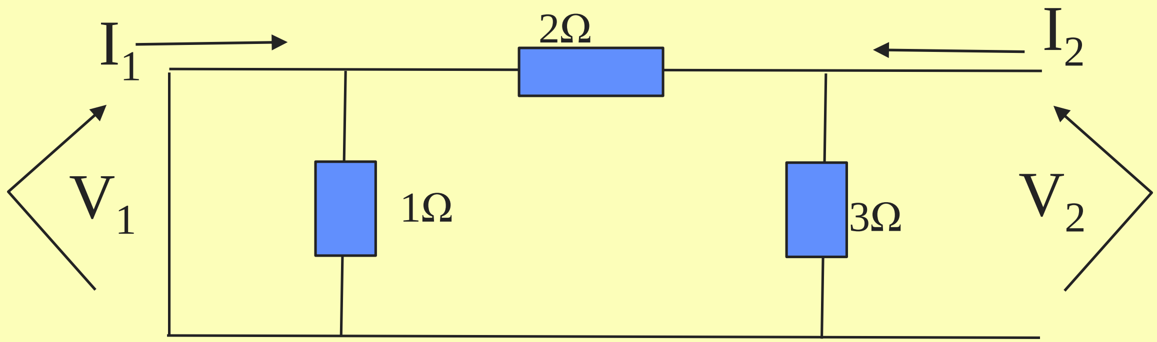
## How to obtain parameters: example 1.

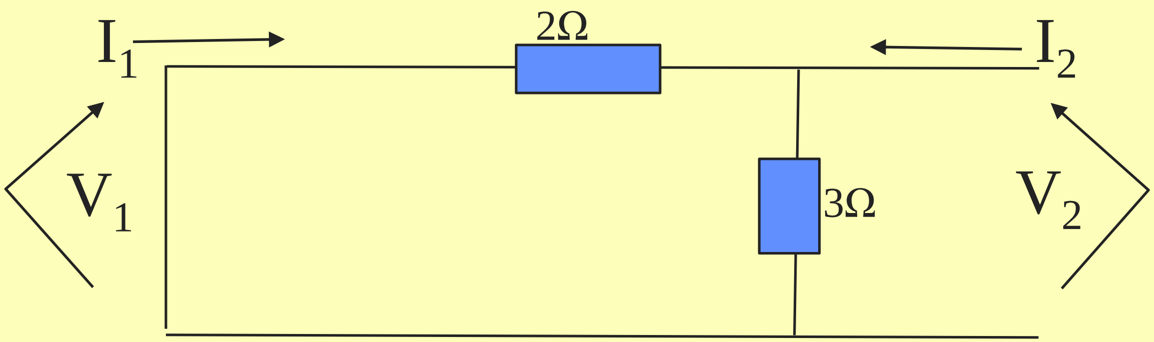
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$




$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$




$$\left.\frac{V_2}{I_1}\right|_{V_1=0} = -2$$

$$\Rightarrow \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-1}{2} = y_{12}$$

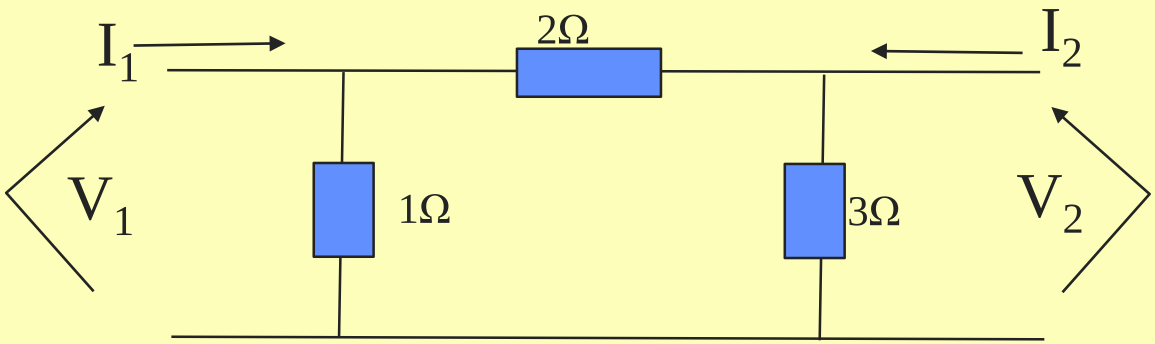
# Question1: What does it mean to set a voltage to 0?

➔Short the two terminals across which that voltage appears!

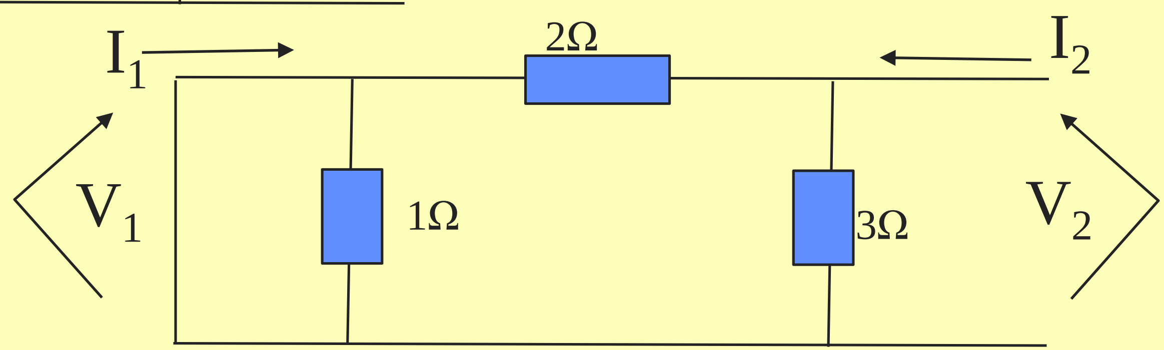
## How to obtain parameters: example 1.

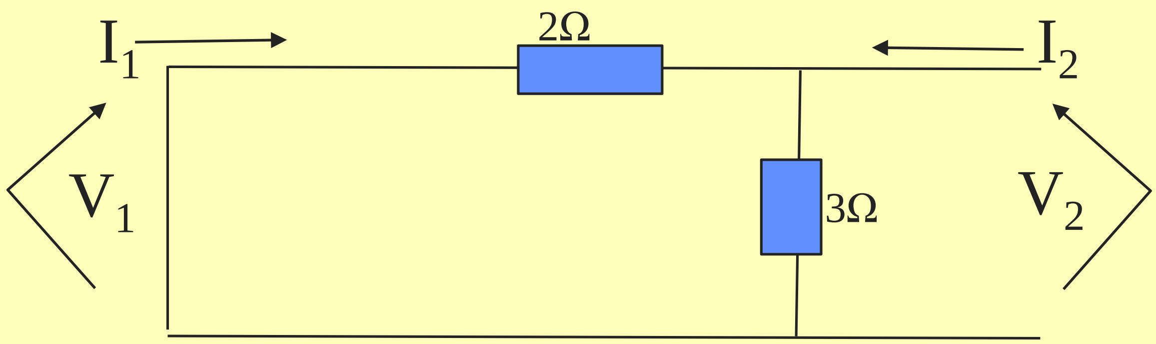
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$




$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$




$$\left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{2(3)}{5} = \frac{6}{5}$$

$$\Rightarrow \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{5}{6} = y_{22}$$

# Summary

## How to obtain parameters: example 1.

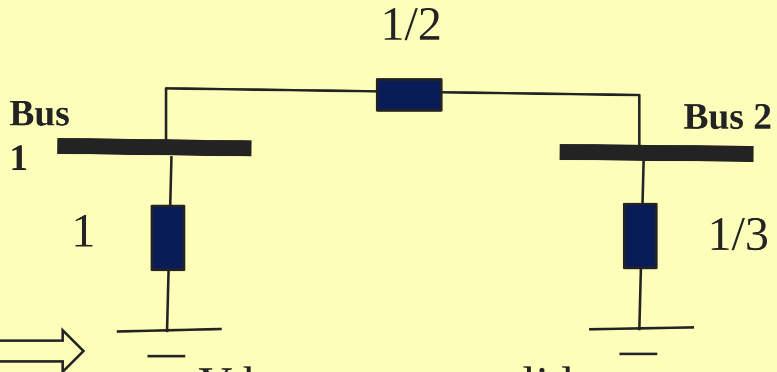
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

<u>Aside (to connect to previous work): Recall Y-bus construction</u>:
(1) off-diagonal Y-bus elements are negated admittances of lines connecting buses corresponding to Y-bus row/column nums.

(2) diagonal Y-bus elements are sum of all admittances connected to bus with corresponding bus num.

➔ $3/2 = y_{1s} + 1/2$ ➔ $y_{1s} = 1$

➔ $5/6 = y_{2s} + 1/2$ ➔ $y_{2s} = 2/6 = 1/3$

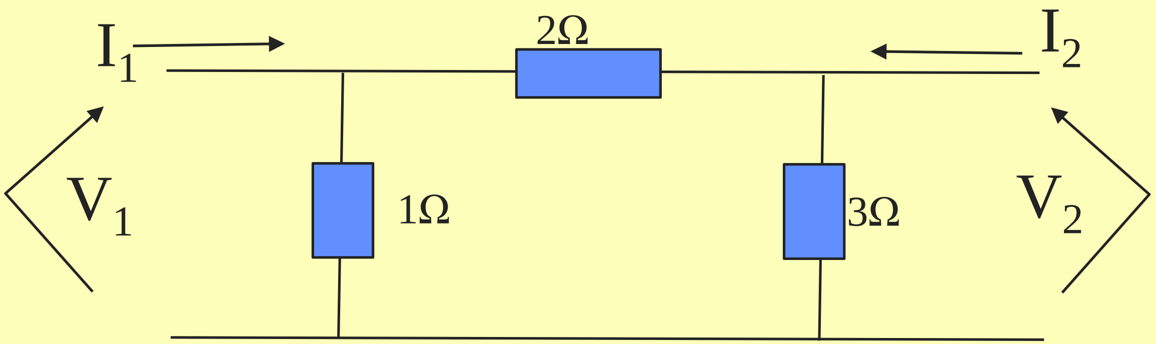



Y-bus: see next slide to see if network is same!!!

# Summary

## How to obtain parameters: example 1.

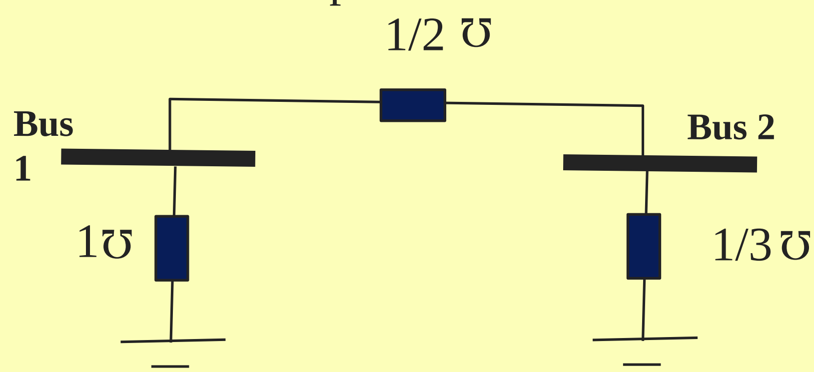
The original network




# Are they the same?

➔Note the numbers in the top network are impedances whereas the numbers in the bottom network are admittances.

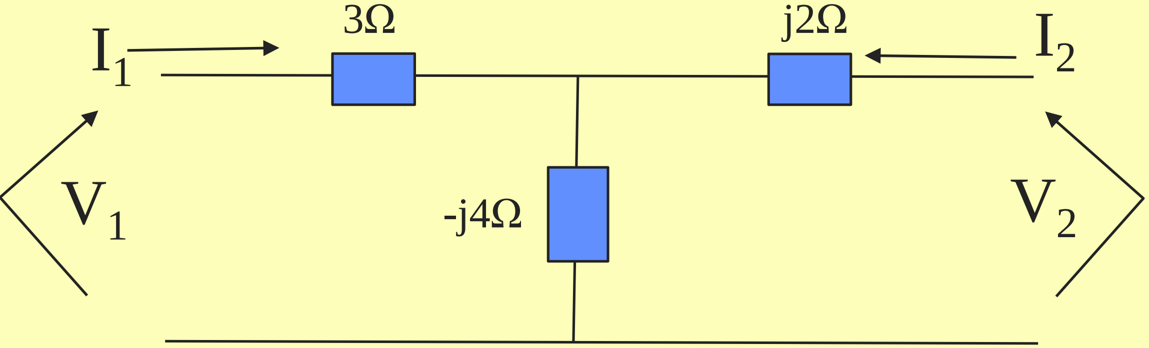
The Y-bus resulting from the two-port calculations



# Question2: What does it mean to set a current to 0?

➔Open the branch through which that current flows!

# How to obtain parameters: example 2.




$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 3 - j4$$

The series combination of the 3Ω and the –j4Ω impedances.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = -j4$$

The voltage across and current through the –j4Ω impedance.

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -j4$$

The voltage across and current through the –j4Ω impedance.

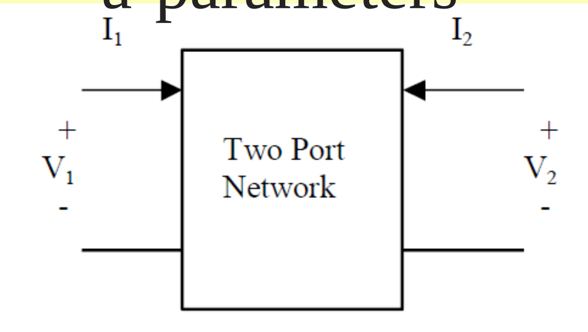
$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = j2 - j4 = -j2$$

The series combination of the j2Ω and the –j4Ω impedances.

# a-parameters and ABCD parameters

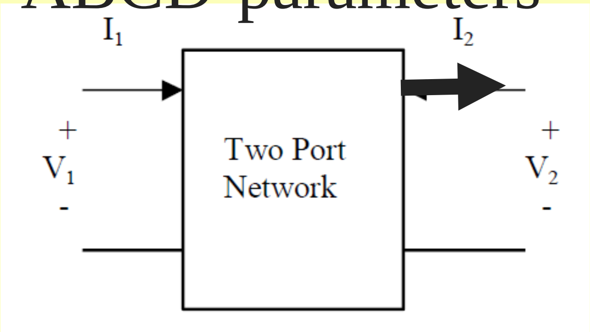
The “a-parameters” (also called “transmission parameters” and “ABCD parameters”) are useful for analysis of dist ccts because they provide the ability to compute input voltage and current as a function of output voltage and current.

a-parameters




$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

ABCD-parameters



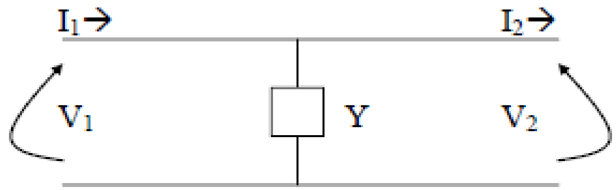

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

The ABCD parameters are the same except for the direction of $I_2$.

The a-parameters are more common in circuit theory, including electronic cct design; the ABCD parameters are common in power.

# ABCD parameters

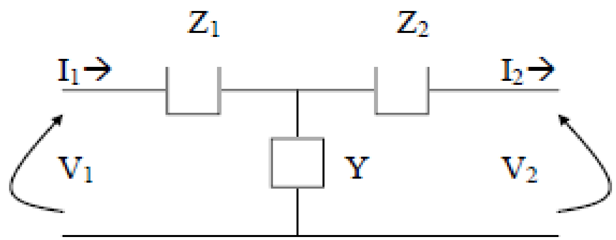
(a)




$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

**Fig. D1.14: ABCD parameters for "I" circuit**

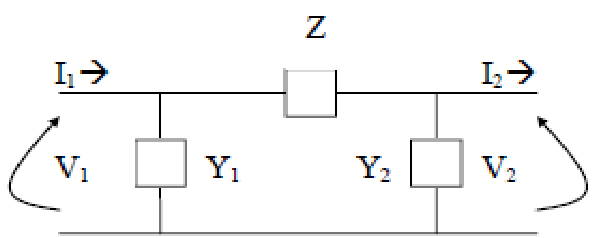
(b)




$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1+YZ_1 & Z_1+Z_2+YZ_1Z_2 \\ Y & 1+YZ_2 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

**Fig. D1.15: ABCD parameters for "T" circuit**

(c)




$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1+Y_2Z & Z \\ Y_1+Y_2+ZY_1Y_2 & 1+Y_1Z \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

**Fig. D1.16: ABCD parameters for "π" circuit**

**Strong suggestion before final exam: Prove that above 3 matrices indeed give ABCD parameters for the given configuration.**

# ABCD parameters

Below, I have done (b) for you. You should first do (a) (it is easiest) and then do (c). One of these is likely to be on the exam.

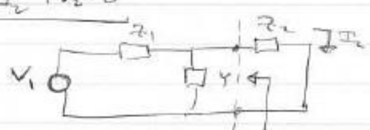
(b) $A = \frac{V_1}{V_2}\Big|_{I_2=0}$

Use voltage division:

$$V_2 = V_1\left[\frac{1/Y}{Z_1 + \frac{1}{Y}}\right].$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{Z_1 + \frac{1}{Y}}{1/Y} = Z_1Y + 1 = A$$

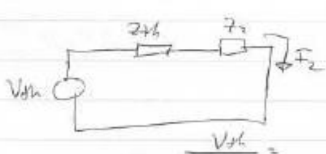
$B = \frac{V_1}{I_2}\Big|_{V_2=0}$




Use Thevenin:

$$Z_{th} = \frac{Z_1 \cdot \frac{1}{Y}}{Z_1 + \frac{1}{Y}} = \frac{Z_1}{1 + Z_1Y}$$

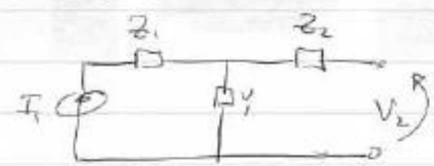
$$V_{th} = V_1\left[\frac{1/Y}{\frac{1}{Y} + Z_1}\right] = V_1\left(\frac{1}{1+Z_1Y}\right)$$




$$I_2 = \frac{V_{th}}{Z_{th} + Z_2} = \frac{V_1\left(\frac{1}{1+Z_1Y}\right)}{\frac{Z_1}{1+Z_1Y} + Z_2}$$

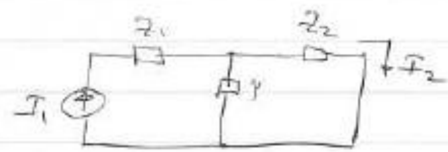
$$\Rightarrow \frac{V_1}{I_2} = \frac{\frac{Z_1}{1+Z_1Y} + Z_2}{\frac{1}{1+Z_1Y}} = \frac{Z_1 + Z_2(1+Z_1Y)}{1} = Z_1 + Z_2 + Z_1Z_2Y = B$$

$C = \frac{I_1}{V_2}\Big|_{I_2=0}$




$$\Rightarrow V_2 = \frac{I_1}{Y} \Rightarrow \frac{I_1}{V_2} = Y = C$$

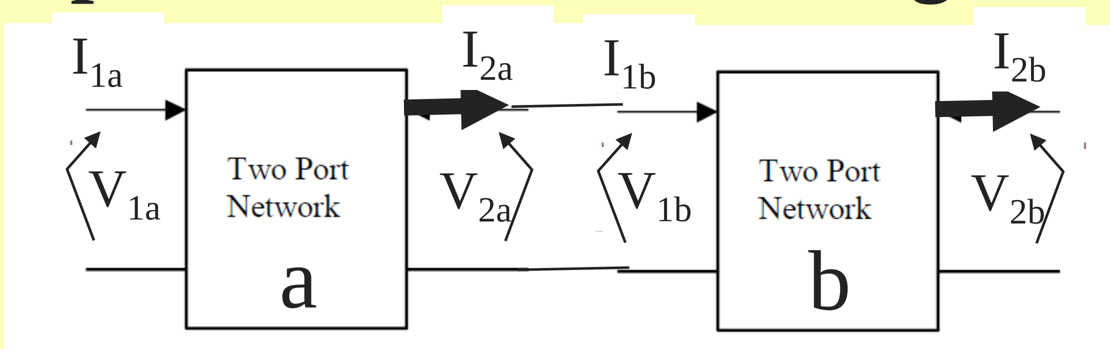
$D = \frac{I_1}{I_2}\Big|_{V_2=0}$




Use current division: $I_2 = I_1\left[\frac{1/Y}{\frac{1}{Y} + Z_2}\right]$

$$\Rightarrow I_2 = I_1\left(\frac{1}{1+YZ_2}\right) \Rightarrow \frac{I_1}{I_2} = 1 + YZ_2 = D$$

# ABCD parameters – Cascading connections




$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \underbrace{\begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix}}_{T_a} \begin{bmatrix} V_{2a} \\ I_{2a} \end{bmatrix} \qquad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \underbrace{\begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}}_{T_b} \begin{bmatrix} V_{2b} \\ I_{2b} \end{bmatrix}$$

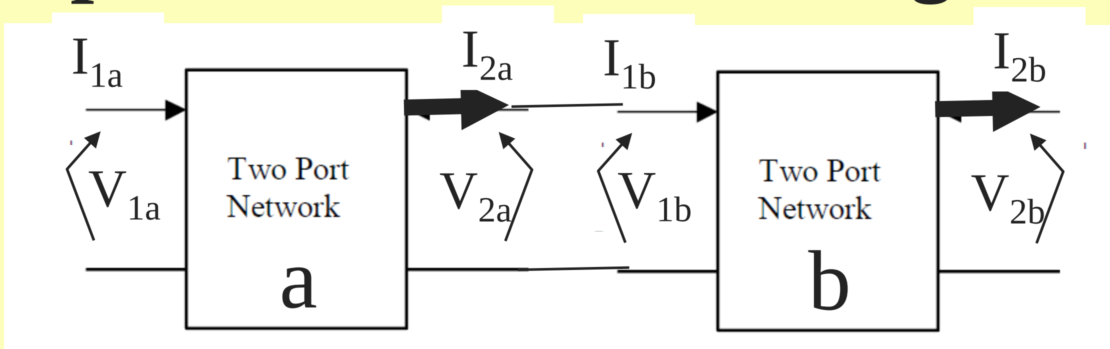
These are the same, i.e.,

$$\begin{bmatrix} V_{2a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

…so let's substitute RHS of right expression in for RHS vector of left expression…

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \underbrace{\begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix}}_{T_a} \underbrace{\begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix}}_{T_b} \begin{bmatrix} V_{2b} \\ I_{2b} \end{bmatrix} = T_a T_b \begin{bmatrix} V_{2b} \\ I_{2b} \end{bmatrix}$$

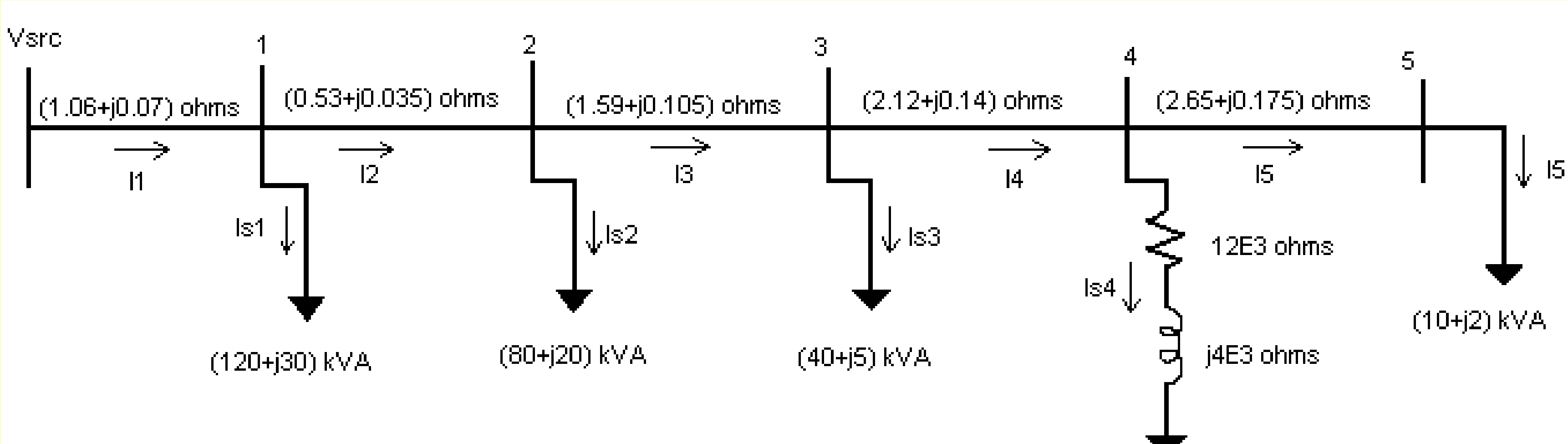
# ABCD parameters – Cascading connections



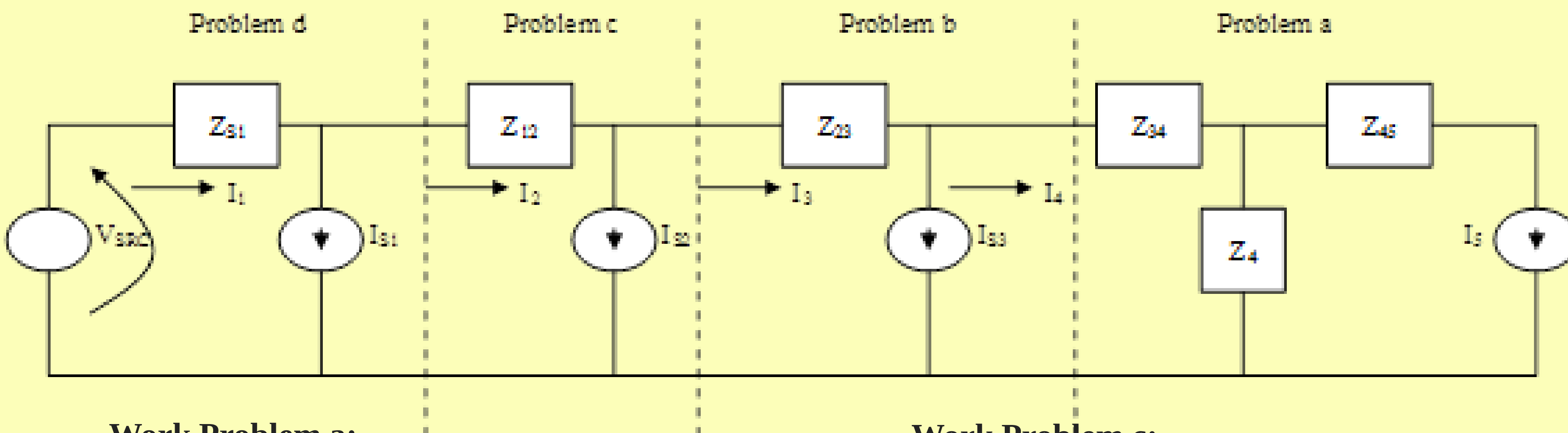

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \underbrace{\begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix}}_{T_a} \underbrace{\begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix}}_{T_a} \begin{bmatrix} V_{2a} \\ I_{2a} \end{bmatrix} = T_a T_b \begin{bmatrix} V_{2a} \\ I_{2a} \end{bmatrix}$$

So cascaded two-ports may be assessed from output (on the right) to input (on the left) by using the product of their individual ABCD parameters. Nice ☺.

We are now in position to re-work our voltage regulation problem on the 5-bus distribution feeder…see next slide. Do this in preparation for exam as well.




(Fig. D1.5)




**Work Problem a:**

- **Compute $I_5$**
- **Get ABCD parameters**
- **Compute $[\ V_3 \quad I_4]^T$**

**Work Problem b:**

- **Compute $I_{S3}$ and then $I_3$**
- **Get ABCD parameters**
- **Compute $[V_2 \quad I_3]^T$**

**Work Problem c:**

- **Compute $I_{S2}$ and then $I_2$**
- **Get ABCD parameters**
- **Compute $[V_1 \quad I_2]^T$**

**Work Problem d:**

- **Compute $I_{S1}$ and then $I_1$**
- **Get ABCD parameters**
- **Compute $[V_{SRC} \quad I_1]^T$**