Module T1

Solution to problem 1

Two ways to work it.

1. Find P_{pq} and $P^{'}_{pq}$ then $P_{loss} = \left|P_{pq} - P^{'}_{pq}\right|$ where $P^{'}_{pq} = -P_{qp}$
2. Find I_{pq} and then $P_{loss} = 3\left|I_{pq}\right|^2 R$

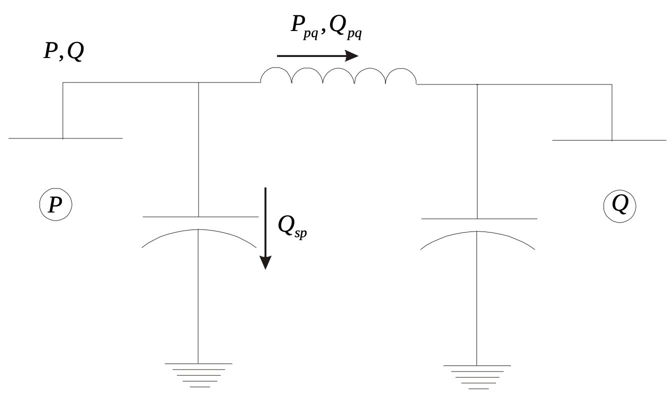
We will do #2.

$$I_{pq} = \frac{V_p\angle\theta_p - V_q\angle\theta_q}{R + jX} = 10^3 \times \frac{63.51\angle 6.0° - 65.06\angle 0°}{30 + j150} = 45\angle 27.26°$$

$$P_{loss} = 3(45)^2(30) = \underline{\underline{0.183MW}} = 0.61 \times 3MW$$

#1:

$$P_{pq} = 2.666$$
$$P_{pq}^{'} = 2.605$$
$$P_{loss/\phi} = 0.061$$



Solution to problem 2

$$P_{pq} = \left|V_p\right|\left|V_q\right| \cdot B \cdot \left(\theta_p - \theta_q\right) = \left(63.51\times10^3\right)\cdot\left(65.06\times10^3\right)\cdot[0.0067]\cdot[0.10472]$$
$$= 2.885MW$$

$$Q_{pq} = \left|V_p\right| \cdot B \cdot \left(\left|V_p\right| - \left|V_q\right|\right) = \left(63.51\times10^3\right)\cdot[0.0067]\cdot[63.51 - 65.06]\times10^3$$
$$= -0.6566MVAR$$

$$Q_{sp} = -\left|V_p\right|^2 \cdot \frac{B_c}{2} = -\left(63.51\times10^3\right)^2 \cdot (0.0001) = -0.4034MVAR$$

So, $Q = Q_{pq} + Q_{sp} = -0.6566 - 0.4034 = -1.06MVAR$

$$\Rightarrow \underline{\underline{P = 3\times 2.885 = 8.655MW}},\quad \underline{\underline{Q = 3\times1.06 = -3.18MVAR}}$$

Solution to problem 3

$$G - jB = \frac{1}{R + jX} = 1.98 - j19.80$$

(a) $P_{pq} = V_p^2 G - V_p V_q G\cos(\phi_p - \phi_q) + V_p V_q B\sin(\phi_p - \phi_q)$

$= 1.15^2(1.98) - (1.15)(0.95)(1.98)\cos(10^\circ) + (1.15(0.95)(19.80)\sin(10^\circ)$

= **4.2445**

So real power into line = 100(4.245) = **424.5 MW**

$P'_{pq} = -P_{pq} = -[V_q^2 G - V_q V_p G\cos(\phi_q - \phi_p) + V_q V_p B\sin(\phi_q - \phi_p)]$

$= -[.95^2(1.98) - (0.95)(1.15)(1.98)\cos(-10^\circ) + (1.15)(0.95)(19.8)\sin(-10^\circ)]$

= **4.0996**

So real power out of the line = 100(4.0996) =**409.96 MW**

(b) $P_{loss} = P_{pq} - P'_{pq}$=424.5-409.96= **14.49 MW**

(c) F = (0.03 / kWhr)(14.49MW)(1000kW/MW) = **$434.70 / hr**

(d) $\gamma = \frac{P_{out}}{P_{in}} = \frac{409.96}{424.5}$ = **96.6 %**

Solution to problem 6

(1) $$P_{pq} = \frac{3(65.06\times10^3)(63.51\times10^3)}{150}\sin(6.0^\circ - 0^\circ) = 8.638\times10^6 W$$

$$Q_{pq} = \frac{3(65.06\times10^3)^2}{150} - \frac{3(65.06\times10^3)(63.51\times10^3)}{150}\cos(6.0^\circ - 0^\circ) = 2.35\times10^6 Var$$

OR

$$P_{pq} = \frac{3\left(65.06\times10^{3}\right)\left(63.51\times10^{3}\right)}{150}\left(\theta_1 - \theta_2\right)$$ Where $\theta_1 \theta_2$ are in radians and Qpq will be the same . This assumption will be made iff G=0 and R=0.

(2) $$I = \frac{V_p - V_q}{Z} = \frac{\left(65.06\angle 6.0^\circ - 63.51\angle 0^\circ\right)10^{3}}{30 + j150} = 45.1366\angle 1.36^\circ$$

$$P_{loss} = 3\left|I\right|^{2} R = 3\left|45.14\right|^{2} 30 = 183386\,W$$

(3) $$P_{pq}{}' = P_{pq} - P_{loss}$$

Solution to problem 7

(a) Transmission Line :

$$X_{p.u.1} = 1 + j2, \; S_{base} = 20\,MVA, \; V_{base} = 15.0\,kV$$

$$X_{p.u.2} = X_{p.u.1}\left[\frac{S_{base_2}}{S_{base_1}}\right]\cdot\left[\frac{V_{base_1}}{V_{base_2}}\right]^{2}, = \left[\frac{10}{20}\right]\cdot\left[\frac{15}{13.8}\right]^{2}\cdot\left(1 + j2\right)$$

$$= \underline{\underline{0.591 + j1.18}}$$

Load :

$$Z_{\Omega} = 300 + j50\Omega \Rightarrow Z_{p.u.} = \frac{Z_{\Omega}}{Z_{base}}; \; Z_{base} = \frac{V_{base}^{2}}{S_{base}} = \frac{\left(13.8\times10^{3}\right)^{2}}{10\times10^{6}}$$

$$\Rightarrow Z_{base} = 19.044\Omega \Rightarrow Z_{p.u.} = \frac{300 + j50}{19.044} = \underline{\underline{15.753 + j2.625}}$$

Load voltage :

$$V_L = \frac{13.0}{13.8} = \underline{\underline{0.942}}$$

(b) $$S_{load} = \frac{V_{load}^{2}}{Z_{load}^{*}} = \frac{\left(0.942\right)^{2}}{15.753 - j2.625} = 0.0548 + j0.0091 \Rightarrow \underline{\underline{Q_c = 0.0091}}$$

(c) $$I = \frac{V_{load}}{Z_{load}} = \frac{0.942\angle 0^\circ}{15.753 + j2.625} = 0.0582 - j0.0097 = 0.059\angle -9.46^\circ$$

$$V_t = V_{load} + I\cdot\left(Z_{line}\right) = 0.942 + \left(0.0582 - j0.0097\right)\cdot\left(0.591 + j1.18\right)$$

$$= 0.9897 + j0.0629 = \underline{\underline{0.9898\angle 3.64^\circ}}$$

$$E_f = V_t + I\cdot\left(jX_s\right) = 0.9878 + j0.0629 + \left(0.0582 - j0.0097\right)\cdot\left(j0.2\right)$$

$$= 0.9897 + j0.0745 = \underline{\underline{0.9925\angle 4.30^\circ}}$$

(d) $$G - jB = \frac{1}{Z} = \frac{1}{0.591 + j1.18} = 0.3393 - j0.6775$$

$$
\begin{aligned}
P_{pq} &= V_p^2 G - V_p V_q G \cdot \cos(\Delta\theta) + V_p V_q B \cdot \sin(\Delta\theta) \\
&= (0.9898)^2 \cdot (0.3393) - (0.9898)\cdot(0.942)\cdot(0.3393)\cdot\cos(3.64°) \\
&\quad + (0.9898)\cdot(0.942)\cdot(0.6775)\sin(3.64°) \\
&= 0.3324 - 0.3157 + 0.0401 \\
&= \underline{\underline{0.0568}}
\end{aligned}
$$

$$
\begin{aligned}
Q_{pq} &= V_p^2 B - V_p V_q B \cdot \cos(\Delta\theta) - V_p V_q G \cdot \sin(\Delta\theta) \\
&= (0.9898)^2 \cdot (0.6775) - (0.9898)\cdot(0.942)\cdot(0.6775)\cdot\cos(3.64°) \\
&\quad - (0.9898)\cdot(0.942)\cdot(0.3393)\sin(3.64°) \\
&= 0.66375 - 0.63042 + 0.0201 \\
&= \underline{\underline{0.0132}}
\end{aligned}
$$

(e) Lower, because power flowing into line also include line losses
check : $I^2 R = (0.059)^2 (15.753) = \underline{\underline{0.0548}}$

(f)

$$P_{out} = \frac{V_t E_f}{X_s}\sin\delta = \frac{(0.9898)\cdot(0.9925)}{0.2}\sin(4.3-3.64) = \underline{\underline{0.0566}}$$

$$Q_{out} = \frac{V_t E_f}{X_s}\cos\delta - \frac{V_t^2}{X_s} = \frac{(0.9898)\cdot(0.9925)}{0.2}\cos(4.3-3.64) - \frac{(0.9898)^2}{0.2} = \underline{\underline{0.0130}}$$

(g) They should be the same since the power coming out of generator is same as power going into transmission line.

(h) This model does not include charging capacitance.

Solution to problem 8

Converting angles to radians, we have:

(a)

$P_{13} = |V_1|\,|V_3|\,B(\theta_1-\theta_3) = (1.0)\,(0.98)\,(10)\,(0-(-0.3489)) = 3.419$ pu

$Q_{13} = |V_1|\,B(|V_1|-|V_3|) = (1.0)\,(10)\,(1.0-0.98) = 0.2$ pu

(b)

$P_{12} = |V_1|\,|V_2|\,B(\theta_1-\theta_2) = (1.0)\,(0.96)\,(5)\,(0-(-0.1744)) = 0.84$ pu

$Q_{12} = |V_1|\,B(|V_1|-|V_2|) = (1.0)\,(5)\,(1.0-0.96) = 0.2$pu

(c) $P_1 = P_{13} + P_{12} = 3.419 + 0.84 = 4.27$

Solution to problem 9

(a) False
(b) False
(c) False
(d) False
(e) False
(f) True