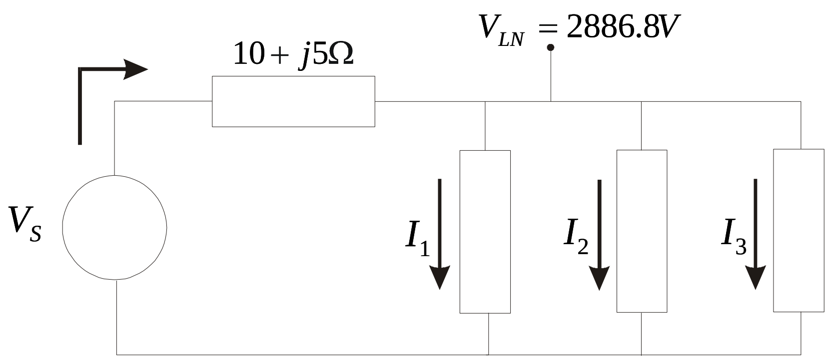
Module B3

Problem 1

The 3-phase loads are connected in parallel. One is a purely resistive load connected in wye. It consumes 300kW. The second is a purely inductive 300kVAR load connected in wye. The third is a purely capacitive 300kVAR load connected in wye. The line-to-line voltage at the load is 5kV. A 3-phase distribution line supplying this load has an impedance of 10+j5 ohms per phase.

(a) Calculate the currents drawn by each load (magnitude and phase).
(b) Indicate the power factor of each load. Remember that non-unity power factors must also include whether they are lagging or leading.
(c) What is the power factor of the entire load? That is, what is the power factor seen by the transmission line at the load end?
(d) Calculate the real and reactive power supplied at the sending end of the distribution line.

Solution to problem 1



(a) $$V_{Load} = \frac{5kV}{\sqrt{3}} = 2886.8\angle 0^\circ$$

Note that:

$$S = V \cdot I^* \Rightarrow I = \left(\frac{S}{V}\right)^*, S = \frac{S_{3\phi}}{3}$$

Therefore:

$S_1 = 100 \times 10^3 W,\ S_2 = j100 \times 10^3 VAR,\ S_3 = -j100 \times 10^3 VAR.$

for purely resistive load: $$I_1 = \left(\frac{S_1}{V_{Load}}\right)^* = \frac{100 \times 10^3}{2886.8} = \underline{\underline{34.64\angle 0^\circ A}}$$

for purely inductive load: $$I_2 = \left(\frac{S_2}{V_{Load}}\right)^* = \frac{-j100 \times 10^3}{2886.8} = \underline{\underline{34.64\angle -90^\circ A}}$$

for purely capacitive load: $$I_3 = \left(\frac{S_3}{V_{Load}}\right)^* = \frac{j100 \times 10^3}{2886.8} = \underline{\underline{34.64\angle 90^\circ A}}$$

Note in the above that for the resistive load, the current and voltage are in phase, for the inductive, the current lags by 90 deg, and for the capacitive, the current leads by 90 deg.

(b) Load 1 : $\underline{\underline{pf_1 = 1.0}}$

Load 2 : $\underline{\underline{pf_2 = 0 \ lagging}}$

Load 3 : $\underline{\underline{pf_3 = 0 \ leading}}$

(c) Need current angle with respect to $V_{Load} = 2886.98\angle 0°$

$$I_{Load} = I_1 + I_2 + I_3 = 34.64\angle 0° - j34.64 + j34.64 = 34.64\angle 0°$$

so V_{Load} and I_{Load} are in phase !

$$\Rightarrow \underline{\underline{pf_{Load} = 1.0}}$$

(d) $V_s = V_{Load} + I_{Load}(Z_t) = 2886.8 + (34.64\angle 0°)\cdot(10 + j5) = 3237.8\angle 3.07°$

$\Rightarrow S_{3\phi} = 3V_s I^*_{Load} = 3\cdot(3237.8\angle 3.07°)\cdot(34.64\angle 0°) = 336{,}472\angle 3.07° VA$

$\underline{\underline{P_{3\phi} = 335.99 kW}}$, $\underline{\underline{Q_{3\phi} = 18.02 kVAR}}$

Alternatively, one could compute losses and add to load :

$$S_{loss(3\phi)} = 3(|I_{Load}|^2 Z_t) = 3(34.64)^2 \cdot (10 + j5) = 35{,}999.9 + j17{,}998.9$$

$\Rightarrow S_{3\phi} = S_{load(3\phi)} + S_{loss(3\phi)} = 300\times 10^3 + 35{,}997.9 + j17{,}998.9 VA$

$\Rightarrow \underline{\underline{P_{3\phi} = 335.99 kW}}$

$\Rightarrow \underline{\underline{Q_{3\phi} = 18.0 kVAR}}$

Problem 2

A three phase load has a per phase impedance, connected in Y, of $100 + j30\Omega$. The line-to-line voltage magnitude at the load is 1500V. The three-phase distribution line supplying this load has an impedance of $10 + j5\Omega/\phi$.

(a) Calculate the line-to-line voltage magnitude at the sending end of the distribution line.

(b) Calculate the real and reactive power supplied at the sending end of the distribution line.

Solution to problem 2

(a) $V_{AN} = \dfrac{1500}{\sqrt{3}} = 866.025V \Rightarrow I_A = \dfrac{V_{AN}}{Z_{LOAD}} = \dfrac{866.025}{100 + j30} = 7.945 - j2.384A = 8.295\angle -16.7° A$

$$\begin{aligned} V_{sending,AN} &= V_{AN} + I_A \cdot Z_{Line} = 866.025\angle 0° + (8.295\angle -16.7°)\cdot(10 + j5) \\ &= 957.39 + j15.89 = 957.53\angle 0.95° V \\ &\Rightarrow \underline{\underline{|V_{Sending,AB}| = 957.53 \cdot \sqrt{3} = 1658.5V}} \end{aligned}$$

(b) $S = 3\cdot V_{sending,AN} \cdot I^*_a = 3\cdot(957.53\angle 0.95°)\cdot(8.295\angle 16.7°) = 22{,}706.5 + j7224.7 VA$

$\Rightarrow \underline{\underline{P = 22.706 kW, \ \ Q = 7.225 kVAR}}$

Problem 3

A three-phase load consumes 100kVA at 0.7 pf lagging. The line-to-line voltage magnitude at the load is 1500V. The three-phase distribution line supplying this load has an impedance of $10 + j5\Omega / \phi$

(a) Calculate the line-to-line voltage magnitude at the sending end of the distribution line.
(b) Calculate the real and reactive power supplied at the sending end of the distribution line.

Solution to problem 3

(a) Note that $\theta=\cos^{-1}(0.7)=45.57$ deg (the angle is positive because the pf is lagging), and sin(45.7)=0.714.

$$V_{an} = \frac{1500}{\sqrt{3}} = 866.025\angle 0° V,\quad S_{1\phi} = \frac{100\times 10^3}{3}(0.7 + j0.714) VA$$

$$I_L = \left(\frac{S_{1\phi}}{V_{an}}\right)^* = \frac{33.3\times 10^3 \times (0.7 - j0.714)}{866.025} = 38.45\angle -45.57° A$$

$$V_{sending,an} = V_{an} + I_L \cdot Z_L = 866.025\angle 0° + 38.45\angle -45.57° \cdot (10 + j5) = 1279.2\angle -6.211°$$

$$\underline{\underline{|V_{sending,AB}| = \sqrt{3}\times 1279.2 = 2215.3V}}$$

(b)

$$S_{sending} = 3\cdot (V_{an,sending})\cdot (I_L)^* = 3(1279.2\angle -6.211°)\cdot (38.45\angle -45.57^{\circ}) = 147.556\angle 39.36° VA$$

$$\Rightarrow \underline{\underline{P_{sending} = 114.09kW,\quad Q_{sending} = 93.58kVAR}}$$

Problem 4

The complex power absorbed by a three-phase load is 1500kVA at 0.8 pf lag

$P_{/\phi} =$ ______________ $Q_{/\phi} =$ ______________

If the Line voltage at the load in problem 1 is 8660.2540 V, what is the voltage magnitude across each phase of the load, if the load is connected as follows,

$|V_d| =$ ______________ $|V_y| =$ ______________

What is the magnitude of line current drawn by this load?

$|I_L| =$ ______________

Solution to problem 4

The complex power absorbed by a three-phase load is 1500kVA at 0.8 pf lag

Note that $\theta=\cos^{-1}(0.8)=36.87$ deg (the angle is positive because the pf is lagging), and $\sin(36.87)=0.6$. Then $P_{1\phi}=1500(0.8)/3=400$ kW, $Q_{1\phi}=1500(0.6)/3=300$ kW.

$P_{1Ø}$ = 400 kW Q/Ø = 300 kVAR

If the Line voltage at the load in problem 1 is 8660.2540 V, what is the voltage magnitude across each phase of the load, if the load is connected as follows,

$|V_\Delta|$ = 8660.254 V $|V_y|$ = 5000 V= $\dfrac{8660.254}{\sqrt{3}}$

What is the magnitude of line current drawn by this load?

$|I_L|$ = 100 A

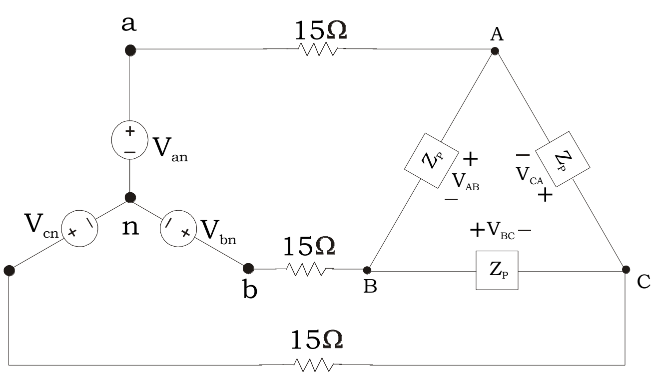
$$|I_L| = \frac{1500x10^3}{\sqrt{3}\times 8660.254}$$

Problem 5

In the circuit shown below, V_{an} = 12,000 + j 0 V (rms). Assume positive phase sequence. The balanced source supplies 1.5 MW and 0.3 MVAR from its terminals to the three phase balanced line and load. Find:

a) The rms line current.

b) Z_p



Solution to problem 5

(a) $S_{3\phi} = (1.5 + j0.3)*10^6 VA$

$$S_{1\phi} = \frac{1}{3}(1.5 + j0.3)*10^6 VA$$

Assume a per-phase circuit. We don't know what the load's phase impedance is, but we don't care, since we know it consumes $s_{1\phi}$. The line current will be the same as the phase current, i.e., $\mathbf{I}_{ab}=\mathbf{I}_{an}$.

$$V_{an} I_{an}^* = S_{1\phi} = 12{,}000 I_{an}^*$$

$$I_{an}^* = \frac{(1.5 + j0.3)x10^6}{3 \times 12{,}000} = 42.492\angle 11.31^\circ A$$

$$I_{an} = \underline{\underline{42.492\angle -11.31^\circ A}}$$

(b) $Z_p = 3Z_{py}$

$$Z_{py} = \frac{V_{an}}{I_{aA}}$$

$$V_{AN} = V_{aN} - I_{aA} * 15 = 12{,}000 - 42.492\angle -11.31 * 15$$

$$= 11{,}375.68\angle 0.6296^\circ V$$

$$Z_{Py} = \frac{11{,}375.68\angle 0.6296^\circ}{42.492\angle -11.31^\circ} = 267.7135\angle 11.939^\circ$$

$$Z_P = 3x267.7135\angle 11.939 = \underline{\underline{803.1405\angle 11.939^\circ \Omega}}$$

Problem 6

A three phase source is supplying a balanced three phase load over a transmission line having impedance of Z_L=2+j20 ohms per phase. The voltage at the source end of the transmission line is 2887 ∠0 volts line to neutral. The current supplied through the transmission line is I_L=100 ∠-30° amperes.

1. Determine the power factor seen by the source, and specify whether it is leading or lagging.
2. Determine the voltage (line to neutral) at the load.
3. Determine the power factor of the load, and specify whether the load is
   a. leading or lagging
   b. inductive or capacitive
4. Determine the real and reactive power consumed by the load.

Solution to problem 6

1. pf=cos(30)=0.8660, and it is lagging.

2. V_{LOAD}=2887∠0-100∠-30(2+j20)=713.8-j1632=2366.5∠-43.6

3. pf angle=angle at which voltage leads the current=-43.6-(-30)=-13.6, so pf=cos(-13.6)=0.972, …and the current is leading the voltage! This means the power factor is leading (part a) and the load must be capacitive (part b).

4. S=3V_{LOAD} $(I)^*$=3(2366.5∠-43.6)(100∠+30)=690044-j166939=709.950∠-13.6 kVA

Problem 7

A balanced, three-phase load having a power factor of 0.8 lagging is supplied by a transmission line carrying 300 amps at 115 kV line-to-line. Compute the three-phase real and reactive power *delivered* to the load.

Solution to problem 7

$$S_{3\phi} = \sqrt{3} \cdot |V_{LL}| \cdot |I| = \left(\sqrt{3}\right) \cdot \left(115 \cdot 10^{3}\right) \cdot \left(300\right) = 59.756 MVA$$

$$pf = 0.8 \Rightarrow \theta = \cos^{-1}\left(0.8\right) = 36.9^{\circ} \Rightarrow \sin\left(\theta\right) = 0.6$$

$$P = (59.756) \cdot (0.8) = 47.8 MW$$

$$Q = (59.756) \cdot (0.6) = 35.879 MVAR$$