Problem 7

A balanced, three-phase load having a power factor of 0.8 lagging is supplied by a transmission line carrying 300 amps at 115 kV line-to-line. Compute the three-phase real and reactive power *delivered* to the load.

Solution to problem 7

$$S_{3\phi} = \sqrt{3}\cdot|V_{LL}|\cdot|I| = \left(\sqrt{3}\right)\cdot\left(115\cdot 10^{3}\right)\cdot\left(300\right) = 59.756\ MVA$$

$$pf = 0.8 \Rightarrow \theta = \cos^{-1}(0.8) = 36.9^{\circ} \Rightarrow \sin(\theta) = 0.6$$

$$P = (59.756)\cdot(0.8) = 47.8 MW$$

$$Q = (59.756)\cdot(0.6) = 35.879\ MVAR$$

Problem 8

A balanced, three-phase, delta-connected load consumes 50-j20 kVA at a line-to-line voltage of 13.8 kV. Compute the per-phase impedance of this load assuming a series connection between R and X.

Solution to problem 8

$$S = \frac{|V_{LL}|^2}{Z^*} \Rightarrow Z = \frac{|V_{LL}|^2}{S^*} = \frac{(13.8\cdot 10^3)^2}{(50+j20)\cdot 10^3} = 3283 - j1313\Omega$$

$$Z_\Delta = 3\cdot Z_Y \Rightarrow Z_\Delta = 3\cdot(3283 - j1313) = 9849 - j3939\Omega$$

$$R = 9849\Omega$$

$$X = 3939\Omega(Capacitive)$$

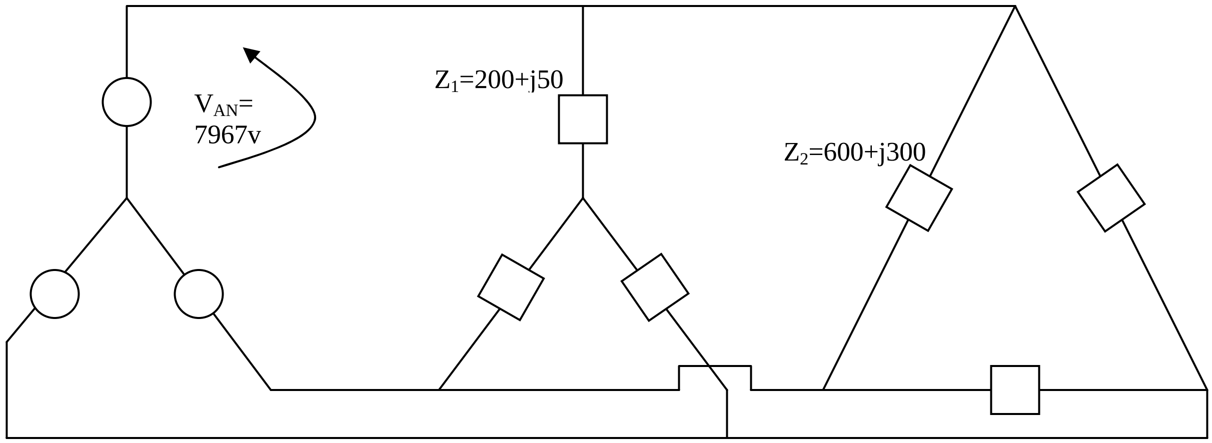
Problem 9

A three-phase wye-connected load having impedance of Z_1=200+j50 ohms per phase is connected in parallel with a three phase delta-connected load having impedance of Z_2=600+j300 ohms per phase. The load is supplied by a three-phase wye-connected generator that is directly interconnected with the loads (i.e., there is no transmission line between the generator and the loads). The voltage magnitude of the generator is 13.8 kV line-to-line. Assume that the phase to neutral voltage at the generator is the angle reference.

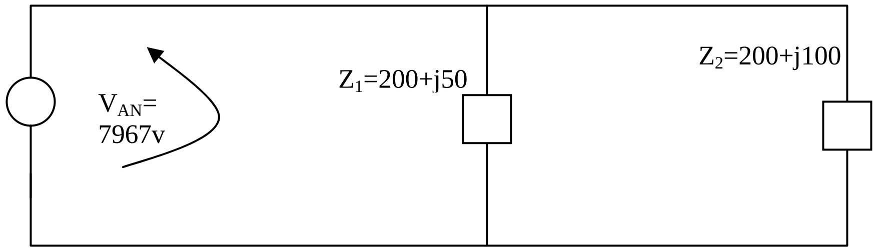
1. Draw the three-phase circuit. Clearly identify the numerical values of one line to neutral source voltage phasor and one-phase impedance for each of loads 1 and 2.
2. Draw the per-phase circuit. Clearly identify the numerical values of the source voltage phasor and the per-phase impedances of loads 1 and 2.
3. Compute the three-phase complex power consumed by each load and the total, complex three-phase power consumed by the two loads.
4. Show that the total, complex three-phase power consumed by the two loads can be computed using the line current and the line-to-line value of the source voltage.

Solution to problem 9

1.



2.



3. We could use $S_1 = 3|V_{AN}|^2/Z_1^*$, $S_2 = 3|V_{AN}|^2/Z_2^*$, or we could get the current instead. Let's do it by getting the current.

   $I_1=V_{AN}/Z_1$=7967/(200+j50)= 37.4918 - j9.3729,
   ➔ $S_1=3V_{AN}(I_1)^*$=3(7967)(37.4918 + j9.3729)= (896.09 +j224.02)kva
   ➔ P_1=896.1 kW, Q_1=224.0 kVAR
   $I_2=V_{AN}/Z_2$=7967/(200+j100)= 31.8680 -j15.9340
   ➔ $S_2=3V_{AN}(I_2)^*$=3(7967)(31.8680 +j15.9340)= (761.68e+j380.84)kva
   ➔ P_2=761.7 kW, Q_2=380.8 kVAR
   $S_{Total}=S_1+S_2$=1657.8+j604.86
   ➔ P_{Total}=1657.8 kW, Q_{Total}=604.9 kVAR

4. $I_T=I_1+I_2$= 69.3598 -j25.3069= 73.83 $\angle$-20.05, $|V_{Line}|$=13,800
   ➔ S_{Total}=(√3)(13,800)(73.83){cos(20.05)+jsin(20.05)}= 1657.8+j604.9

Problem 10

Consider a balanced three-phase source supplying a balanced Y- or Δ- connected load with the following instantaneous voltages and currents.

$$v_{an} = \sqrt{2}|V_p|\cos(\omega t + \theta_v) \qquad i_a = \sqrt{2}|I_p|\cos(\omega t + \theta_i)$$

$$v_{bn} = \sqrt{2}|V_p|\cos(\omega t + \theta_v - 120°) \qquad i_b = \sqrt{2}|I_p|\cos(\omega t + \theta_i - 120°)$$

$$v_{cn} = \sqrt{2}|V_p|\cos(\omega t + \theta_v - 240°) \qquad i_c = \sqrt{2}|I_p|\cos(\omega t + \theta_i - 240°)$$

where $|V_p|$ and $|I_p|$ are the magnitudes of the rms phase voltage and current, respectively. Show that the total instantaneous power provided to the load, as the sum of the instantaneous powers of each phase, is a constant.

Solution for Problem 10

Consider a balanced three-phase source supplying a balanced Y- or Δ- connected load with the following instantaneous voltages

$$v_{an} = \sqrt{2}\left|V_p\right|\cos(\omega t + \theta_v)$$

$$v_{bn} = \sqrt{2}\left|V_p\right|\cos(\omega t + \theta_v - 120\circ)$$

$$v_{cn} = \sqrt{2}\left|V_p\right|\cos(\omega t + \theta_v - 240^\circ)$$

For a balanced load the phase currents are

$$i_a = \sqrt{2}\left|I_p\right|\cos(\omega t + \theta_i)$$

$$i_b = \sqrt{2}\left|I_p\right|\cos(\omega t + \theta_i - 120^\circ)$$

(2.41)

$$i_c = \sqrt{2}\left|I_p\right|\cos(\omega t + \theta_i - 240^\circ)$$

where $\left|V_p\right|$ and $\left|I_p\right|$ are the magnitudes of the rms phase voltage and current, respectively. The total instantaneous power is the sum of the instantaneous power of each phase, given by

$$p_{3\phi} = v_{an}i_a + v_{bn}i_b + v_{cn}i_c$$

Substituting for the instantaneous voltages and currents

$$\begin{aligned} p_{3\phi} &= 2\left|V_p\right|\left|I_p\right|\cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \\ &+ 2\left|V_p\right|\left|I_p\right|\cos(\omega t + \theta_v - 120^\circ)\cos(\omega t + \theta_i - 120^\circ) \\ &+ 2\left|V_p\right|\left|I_p\right|\cos(\omega t + \theta_v - 240^\circ)\cos(\omega t + \theta_i - 240^\circ) \end{aligned}$$

Using the trigonometric identity cosx cosy =cos(x-y) + cos(x+y)

$$\begin{aligned} p_{3\phi} &= \left|V_p\right|\left|I_p\right|\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right] \\ &+ \left|V_p\right|\left|I_p\right|\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 240^\circ)\right] \\ &+ \left|V_p\right|\left|I_p\right|\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 480^\circ)\right] \end{aligned}$$

The three double frequency cosine terms are out of phase with each other by 120° and add up to zero, and the three-phase instantaneous power is

$$p_{3\phi} = 3\left|V_p\right|\left|I_p\right|\cos\theta$$

$\theta = \theta_v - \theta_i$ is the angle between phase voltage and phase current or the impedance angle.

Problem 11

A three-phase line has an impedance of 2+j4 ohms/phase, and the line feeds two balanced three-phase loads that are connected in parallel. The first load is Y-connected and has an impedance of 30+j40 ohms/phase. The second load is delta-connected and has an impedance of 60-j45 ohms/phase. The line is energized at the sending end from a three-phase balanced supply of line voltage 207.85 volts. Taking the phase voltage V_a as reference, determine:

a. The current, real power, and reactive power drawn from the supply.
b. The line voltage at the combined loads.
c. The current per phase in each load.
d. The total real and reactive powers in each load and the line.

Solution for Problem 11

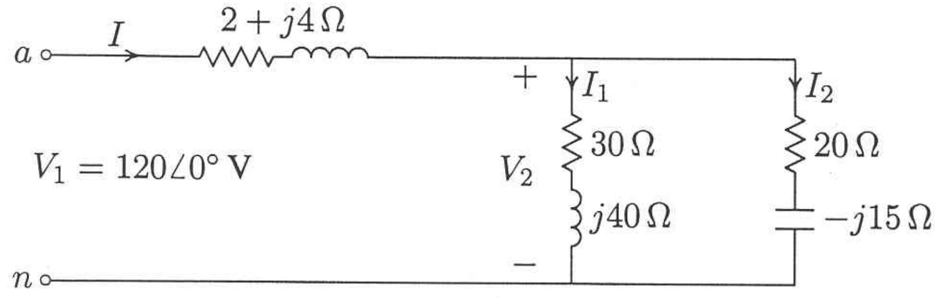
(a) The Δ-connected load is transformed into an equivalent Y. The impedance per phase of the equivalent Y is

$$Z_2 = \frac{60 - j45}{3} = 20 - j15\ \Omega$$

The phase voltage is

$$V_1 = \frac{207.85}{\sqrt{3}} = 120\ \text{V}$$

The single-phase equivalent circuit is shown in the following figure.



The total impedance is

$$Z = 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)}$$
$$= 2 + j4 + 22 - j4 = 24\ \Omega$$

with the phase voltage V_{an} as reference, the current in phase *a* is

$$I = \frac{V_1}{Z} = \frac{120\angle 0^\circ}{24} = 5\ \text{A}$$

The three-phase power supplied is

$$S = 3V_1 I^* = 3(120\angle 0^\circ)(5\angle 0^\circ) = 1800\ \text{W}$$

(b) The phase voltage at the load terminal is

$$V_2 = 120\angle 0^\circ - (2 + j4)(5\angle 0^\circ) = 110 - j20$$
$$= 111.8\angle -10.3^\circ\ \text{V}$$

The line voltage at the load terminal is

$$V_{2ab} = \sqrt{3}\angle 30^\circ V_2 = \sqrt{3}(111.8)\angle 19.7^\circ = 193.64\angle 19.7^\circ\ \text{V}$$

(c) The current per phase in the Y-connected load and in the equivalent Y of the Δ load is

$$I_1 = \frac{V_2}{Z_1} = \frac{110 - j20}{30 + j40} = 1 - j2 = 2.236\angle -63.4^\circ\ \text{A}$$

$$I_2 = \frac{V_2}{Z_2} = \frac{110 - j20}{20 - j15} = 4 + j2 = 4.472\angle 26.56^\circ\ \text{A}$$

The phase current in the original Δ-connected load, i.e., I_{ab} is given by

$$I_{ab} = \frac{I_2}{\sqrt{3}\angle -30^\circ} = \frac{4.472\angle 26.56^\circ}{\sqrt{3}\angle -30^\circ} = 2.582\angle 56.56^\circ\ \text{A}$$

(d) The three-phase power absorbed by each load is

$$S_1 = 3V_2 I_1^* = 3(111.8\angle -10.3^\circ)(2.236\angle 63.4^\circ) = 450\ \text{W} + j600\ \text{var}$$
$$S_2 = 3V_2 I_2^* = 3(111.8\angle -10.3^\circ)(4.472\angle -26.56^\circ) = 1200\ \text{W - } j900\ \text{var}$$

The three-phase power absorbed by the line is

$$S_L = 3(R_L + jX_L)|I|^2 = 3(2 + j4)(5)^2 = 150 \text{ W} + j300 \text{ var}$$

It is clear that the sum of load powers and line losses is equal to the power delivered from the supply, i.e.,

$$S_1 + S_2 + S_L = (450 + j600) + (1200 - j900) + (150 + j300)$$
$$= 1800 \text{ W} + j0 \text{ var}$$

Problem 12

A three-phase line has an impedance of 0.4+j2.7 ohms per phase. The line feeds two balanced three-phase loads that are connected in parallel. The first load is absorbing 560.1kVA at 0.707 power factor lagging. The second load absorbs 132 kW at unity power factor. The line-to-line voltage at the load end of the line is 3810.5 volts. Determine:

a. The magnitude of the line voltage at the source end of the line.
b. Total real and reactive power loss in the line.
c. Real power and reactive power supplied at the sending end of the line.

Solution for Problem 12

(a) The phase voltage at the load terminals is

$$V_2 = \frac{3810.5}{\sqrt{3}} = 2200 \text{ V}$$

The total complex power is

$$S_{R(3\phi)} = 560.1(0.707 + j0.707) + 132 = 528 + j396$$
$$= 660\angle 36.87^\circ \text{ kVA}$$

With the phase voltage V_2 as reference, the current in the line is

$$I = \frac{S^*_{R(3\phi)}}{3V_2^*} = \frac{660{,}000\angle -36.87^\circ}{3(2200\angle 0^\circ)} = 100\angle -36.87^\circ \text{ A}$$

The phase voltage at the sending end is

$$V_1 = 2200\angle 0^\circ + (0.4 + j2.7)100\angle -36.87^\circ = 2401.7\angle 4.58^\circ \text{ V}$$

The magnitude of the line voltage at the sending end of the line is

$$|V_{1L}| = \sqrt{3}|V_1| = \sqrt{3}(2401.7) = 4160 \text{ V}$$

(b) The three-phase power loss in the line is

$$S_{L(3\phi)} = 3R|I|^2 + j3X|I|^2 = 3(0.4)(100)^2 + j3(2.7)(100)^2$$
$$= 12 \text{ kW} + j81 \text{ kvar}$$

(c) The three-phase sending power is

$$S_{S(3\phi)} = 3V_1 I^* = 3(2401.7\angle 4.58^\circ)(100\angle 36.87^\circ) = 540 \text{ kW} + j477 \text{ kvar}$$

It is clear that the sum of load powers and the line losses is equal to the power delivered from the supply, i.e.,

$$S_{S(3\phi)} = S_{R(3\phi)} + S_{L(3\phi)} = (528 + j396) + (12 + j81) = 540 \text{ kW} + j477 \text{ kvar}$$