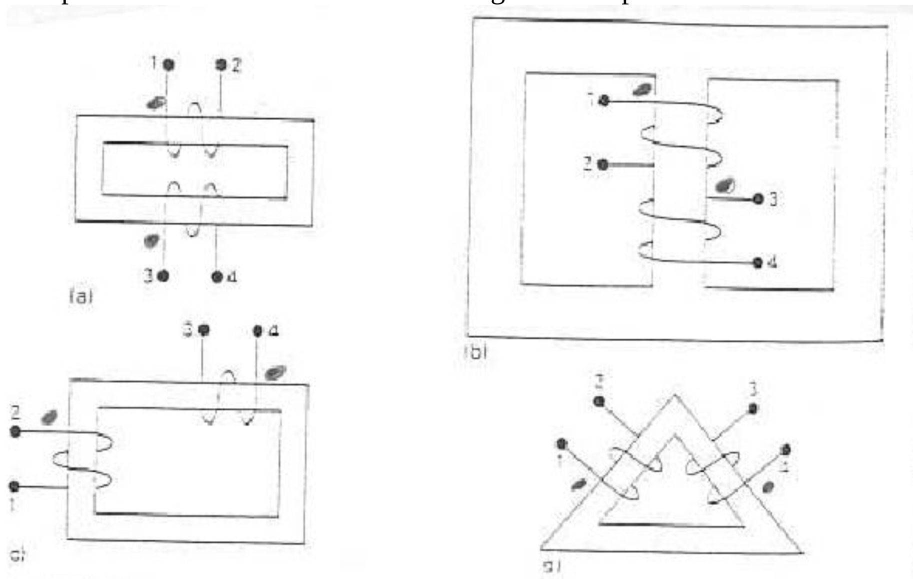
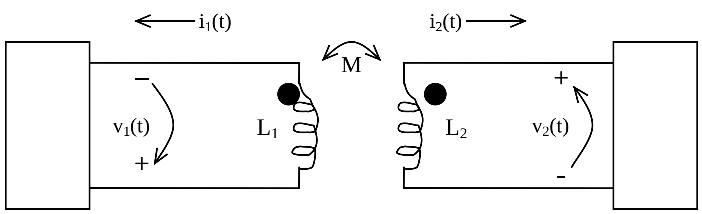
EE 303 Homework on Transformers, Dr. McCalley.

1. The physical construction of four pairs of magnetically coupled coils is shown below. Assume that the magnetic flux is confined to the core material in each structure (no leakage). Show two possible locations for the dot markings on each pair of coils.



In each case, another correct answer is to dot the other (undotted) pair of terminals.

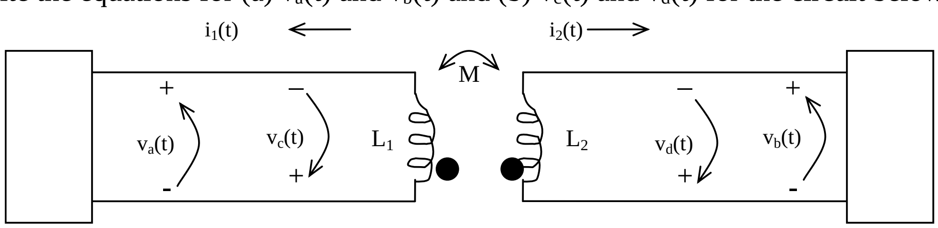
2. Write the equations for $v_1(t)$ and $v_2(t)$ for the circuit below.



$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = -L_2 \frac{di_2(t)}{dt} - M \frac{di_1(t)}{dt}$$

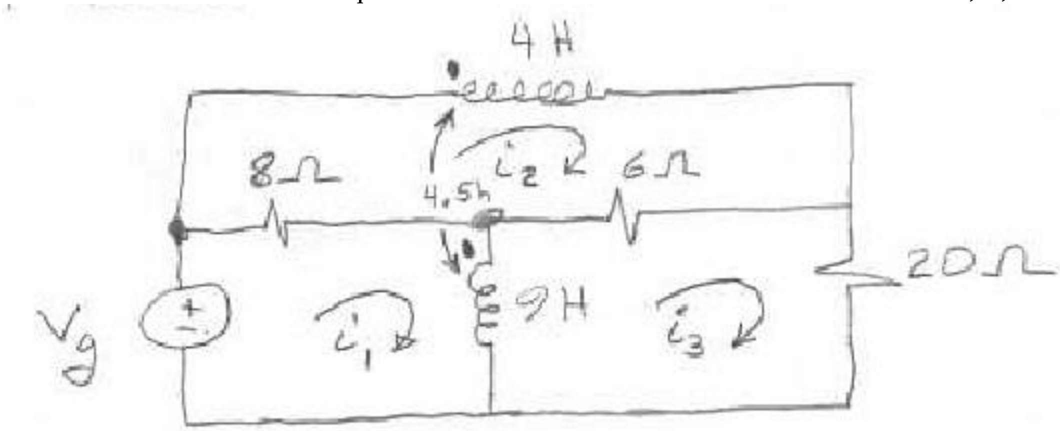
3. Write the equations for (a) $v_a(t)$ and $v_b(t)$ and (b) $v_c(t)$ and $v_d(t)$ for the circuit below.



(a) $V_a(t) = -L_1 \frac{di_1(t)}{dt} \; M \frac{di_2(t)}{dt}$, $V_b(t) = -L_2 \frac{di_2(t)}{dt} - M \frac{di_1(t)}{dt}$

(b) $V_c(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$, $V_d(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$

4. Write a set of mesh current equations that describe the circuit below in terms of i_1, i_2, and i_3.

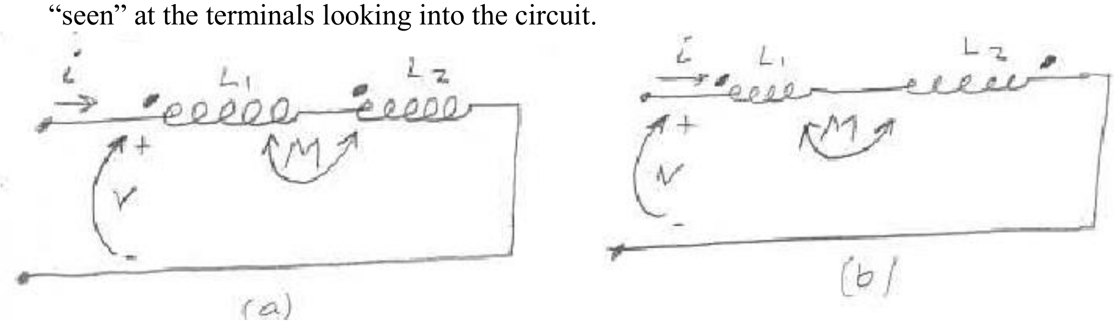


Top Loop: $-4\frac{di_2}{dt}-4.5\left[\frac{di_1}{dt}-\frac{di_3}{dt}\right]+6\left[i_3-i_2\right]+8\left[i_1-i_2\right]=0$

Left Loop: $v_g-8\left[i_1-i_2\right]-9\left[\frac{di_1}{dt}-\frac{di_3}{dt}\right]-4.5\frac{di_2}{dt}=0$

Right Loop: $9\left[\frac{di_1}{dt}-\frac{di_3}{dt}\right]+4.5\frac{di_2}{dt}-6\left[i_3-i_2\right]-20i_3=0$

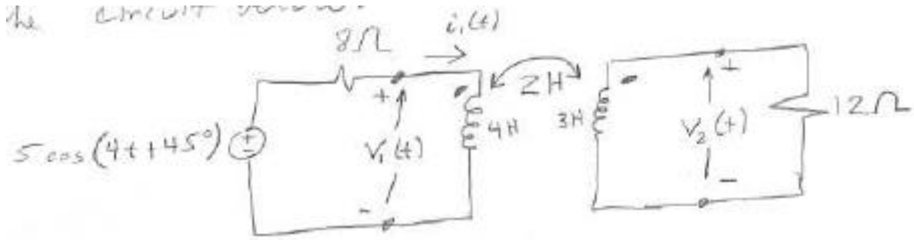
5. A pair of coupled inductors is connected in two different ways as shown below. In each case, find the differential equation relating v(t) and i(t), and then find the equivalent inductance "seen" at the terminals looking into the circuit.



a) $v(t)=L_1di/dt+Mdi/dt+L_2di/dt+Mdi/dt$
   ➔ $v(t)=[L_1+L_2+2M]di/dt$
   ➔ $L_{eq}=L_1+L_2+2M$

b) $v(t)=L_1di/dt-Mdi/dt+L_2di/dt-Mdi/dt$
   ➔ $v(t)=[L_1+L_2-2M]dt/dt$
   ➔ $L_{eq}=L_1+L_2-2M$

6. Use phasors to find the voltage $v_2(t)$ in the circuit below.



Since we need not do any power calculations in this problem, we may express all phasors using the maximum value as the magnitude. One important concept you must use here is that the differential equations, which characterize the relationships for *any time* under *any type of excitation,* may be converted to phasor relations for performing analysis under *steady-state conditions* when the excitation is *sinusoidal*. The transformation requires that you replace differentiation by "jω" in the phasor domain. One can see why if you just differentiate $i(t)=|I|\sin\omega t$. Then you get $di/dt=|I|\omega\cos\omega t$, which is just the original function, scaled by ω and rotated forward by 90 degrees, i.e., $di/dt=|I|\omega\sin(\omega t+90)$. So you see that the phasor for i(t) is just $I=|I|\angle 0$, and the phasor for di/dt is just $\omega|I|\angle 90=\omega|I|\angle 0\angle 90=j\omega I$.

$$\text{Left Loop: } 5\cos(4t+45°)-8i_1-\left[4\frac{di_1}{dt}+2\frac{di_2}{dt}\right]=0$$

$$\text{Right Loop: } 3\frac{di_2}{dt}+2\frac{di_1}{dt}-12i_2=0$$

Now transform to phasor notation:

$$\text{Left Loop: } 5\angle 45°-8\bar{I}_1-j\omega 4\bar{I}_1-j\omega 2\bar{I}_2=0$$

$$\text{Right Loop: } j\omega 3\bar{I}_2+j\omega 2\bar{I}_2-12\bar{I}_2=0$$

The above two equations may be solved – this effort results in I_2=.138 /_ -141.35°. Then, $V_2=-I_2R$=1.656/_38.65°.