Module B3

3.1 Sinusoidal steady-state analysis (single-phase), a review

3.2 Three-phase analysis

Kirtley

Chapter 2: AC Voltage, Current and Power

2.1 Sources and Power

2.2 Resistors, Inductors, and Capacitors

Chapter 4: Polyphase systems

4.1 Three-phase systems

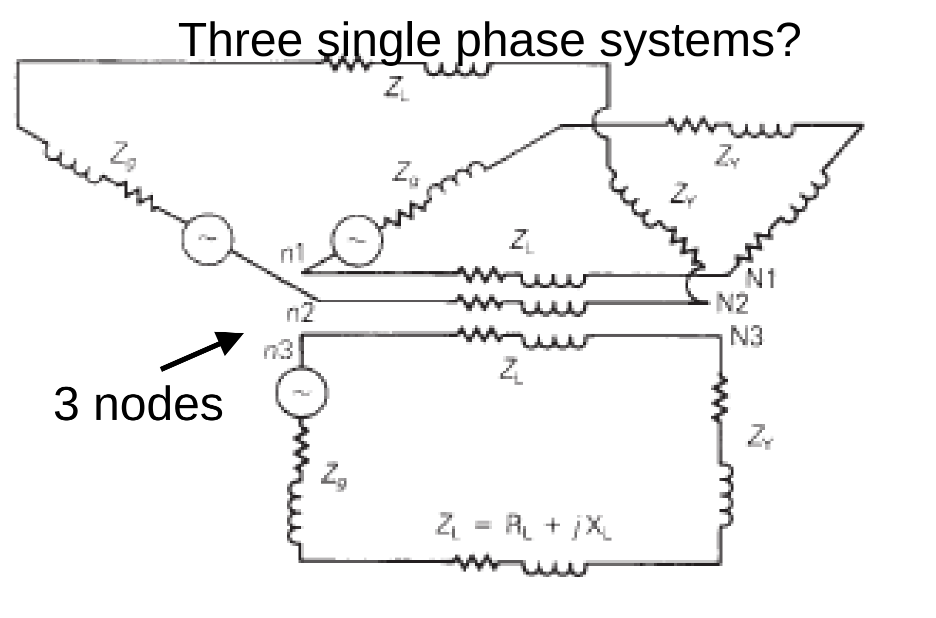
4.2 Line-Line Voltages

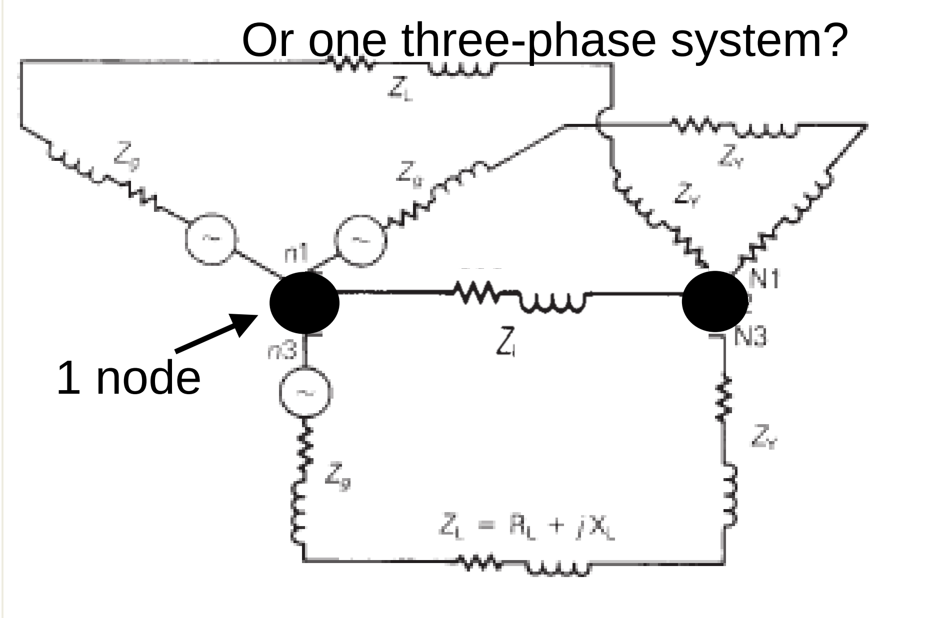
Three-phase power

All of what we have done in the previous slides is for “single phase” circuits. However, almost all transmission systems in the US are 3-phase AC systems (the only exceptions are a few DC transmission lines). Three-phase AC is preferred over single-phase AC because the investment and operating costs per MW of transmission capacity are more attractive, and because a 3-phase system provides constant power (not pulsating as we saw before)

You can see this in the next slide.

Three-phase power



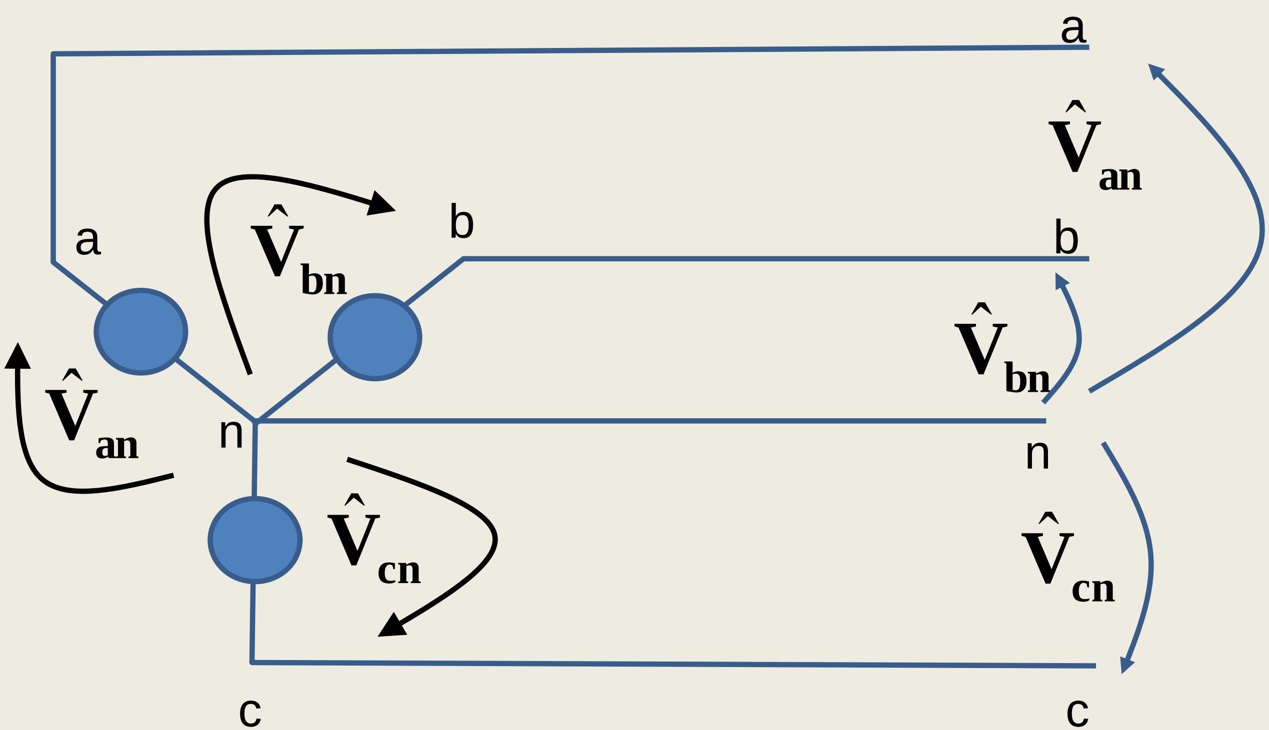


Three single phase systems	One three-phase system
6 wires	4 wires; capital savings!
Each neutral carries full load current	Neutral carries little or no current and can therefore be much smaller; capital savings!
Each neutral carries full load current	Neutral carries little or no current, therefore has little losses; operational savings!
Each single phase circuit delivers instantaneous power that varies at 2ω. Large generators & motor loads vibrate.	We will show that three phase circuits deliver constant instantaneous power; large generators and motors run smoothly.

Three-phase power

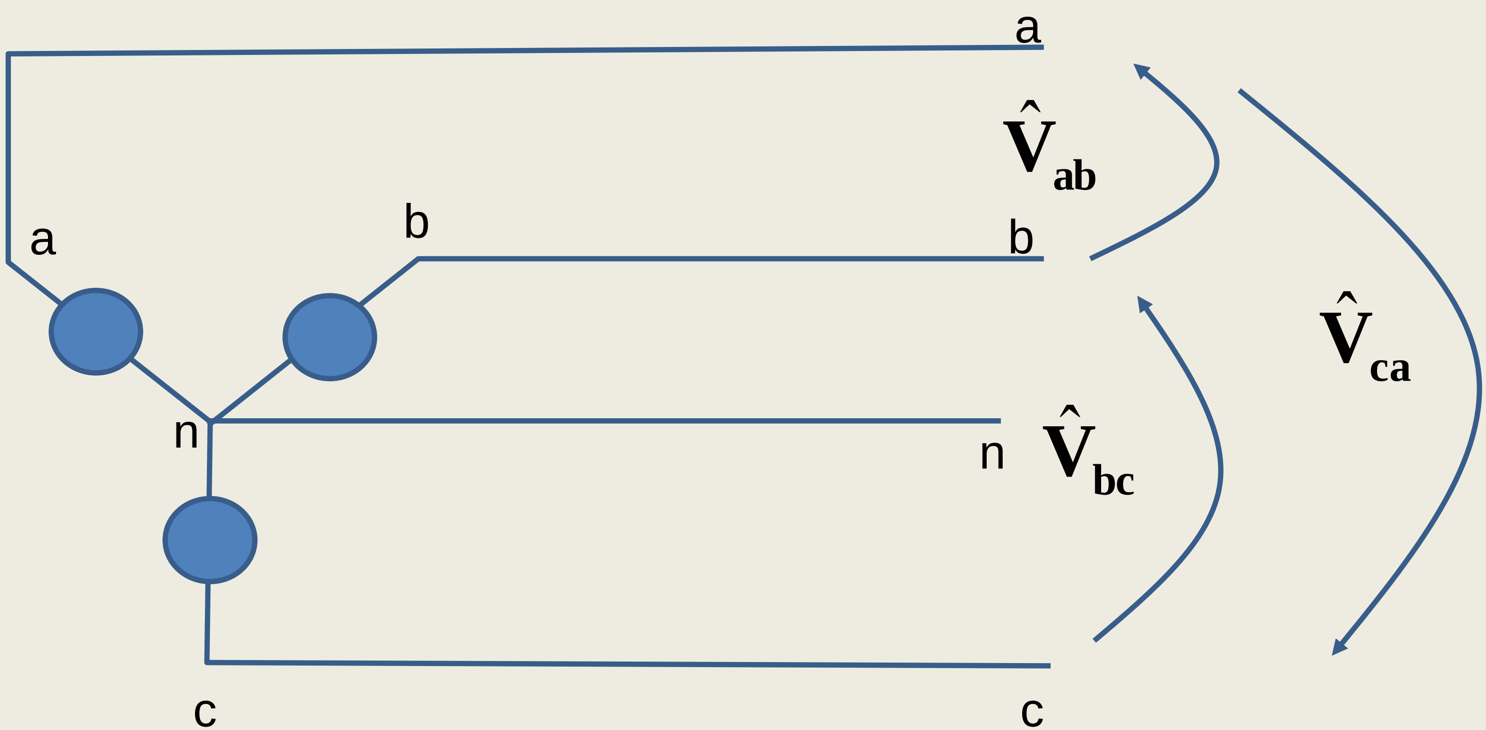
AC generators on the grid supply 3-phase power. A circuit diagram for the stator of a typical 3-phase generator is provided in the next two slides.

Line-to-neutral (phase) voltages



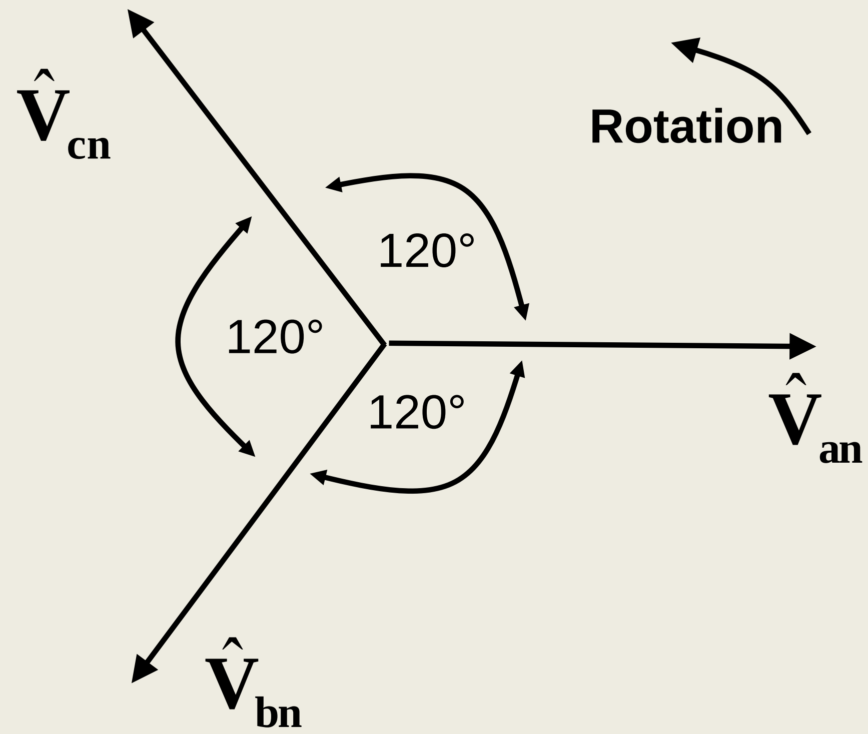
The identified voltages are referred to as "line-to-neutral voltages," or "phase voltages."

Line-to-line (line) voltages



The identified voltages are referred to as “line-to-line voltages,” or just “line voltages.”

Phasor diagram for line-neutral (phase) voltages



What is rotating?

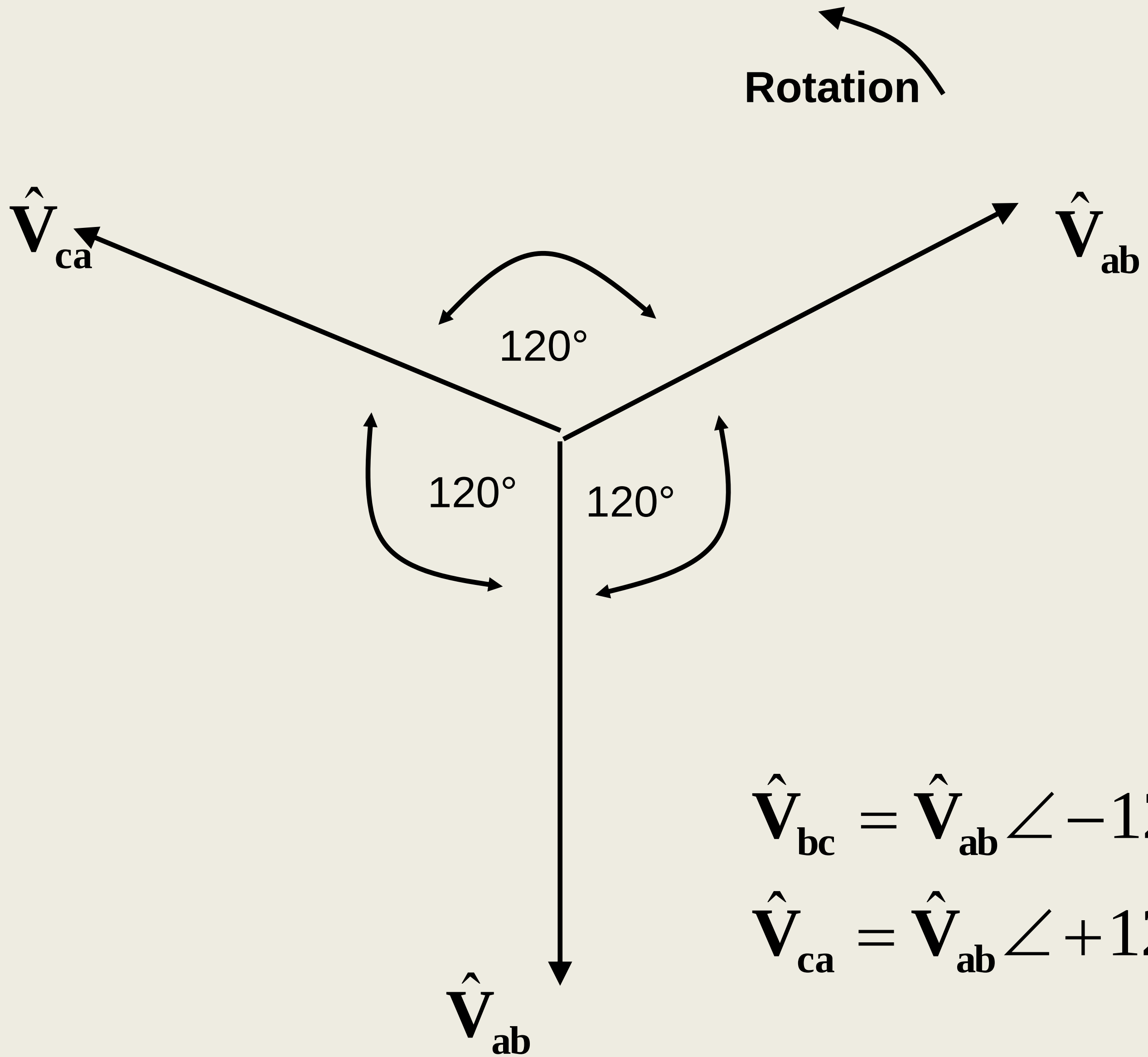
➔The peak value of the sinusoid; this peak value is projected onto the positive horizontal axis to obtain the instantaneous value of the quantity, a concept equivalent to writing $v_{an}(t)=V_{peak}\sin\omega t$.

$$\hat{\mathbf{V}}_{bn} = \hat{\mathbf{V}}_{an}\angle -120°$$

$$\hat{\mathbf{V}}_{cn} = \hat{\mathbf{V}}_{an}\angle +120°$$

www.animations.physics.unsw.edu.au/jw/phasor-addition.html

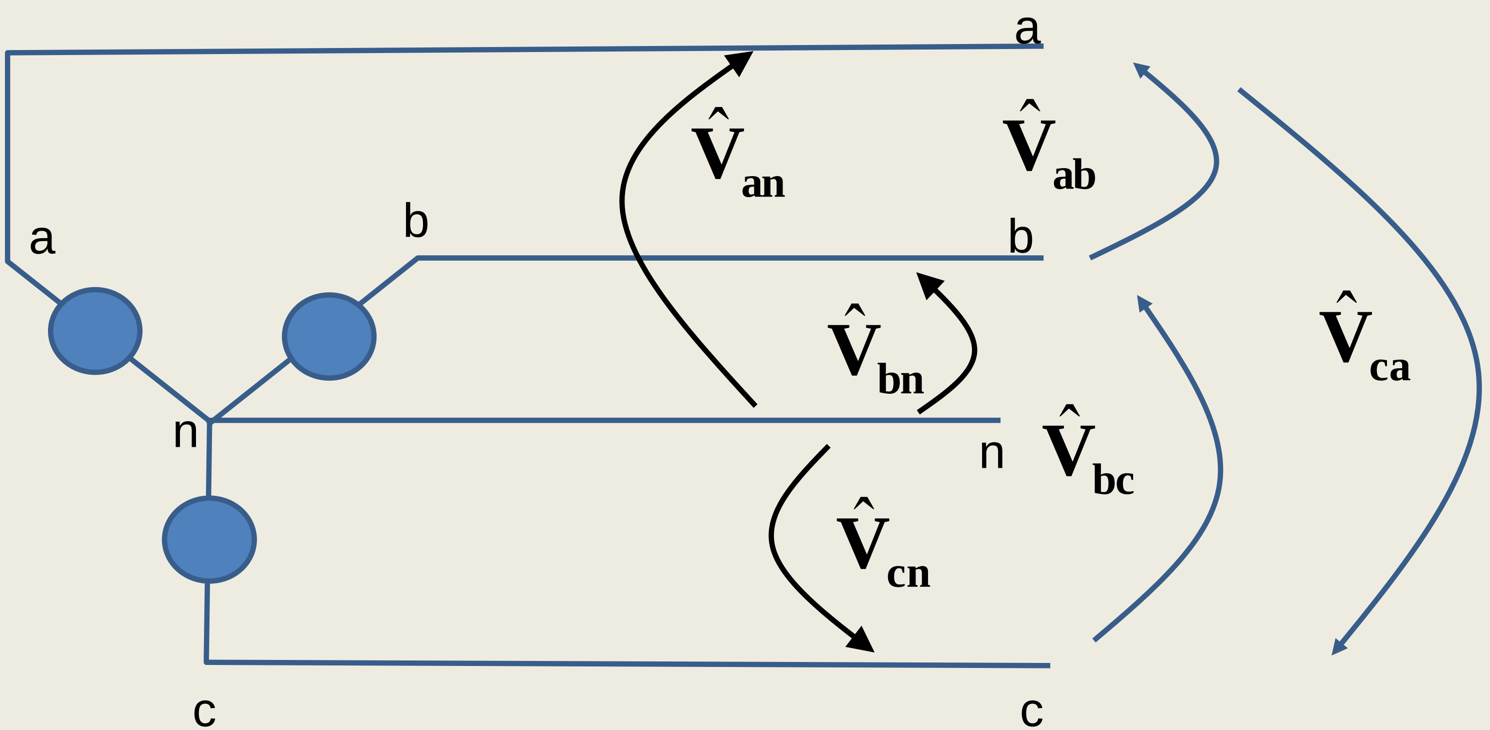
Phasor diagram for line-line (line) voltages



$$\hat{\mathbf{V}}_{\mathbf{bc}} = \hat{\mathbf{V}}_{\mathbf{ab}} \angle -120^\circ$$

$$\hat{\mathbf{V}}_{\mathbf{ca}} = \hat{\mathbf{V}}_{\mathbf{ab}} \angle +120^\circ$$

Relating phase and line voltages



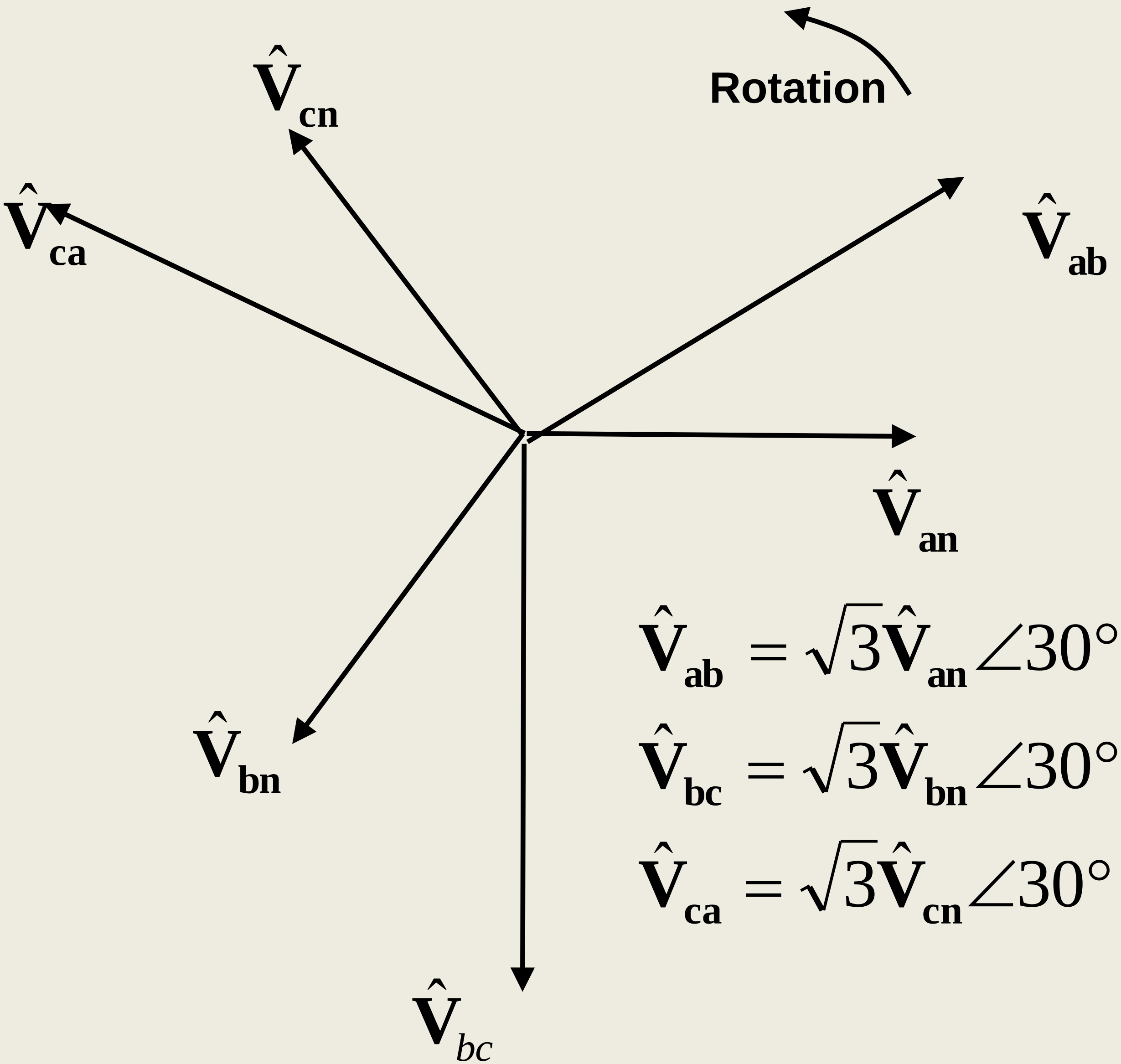
$$\hat{\mathbf{V}}_{ab} - \hat{\mathbf{V}}_{an} + \hat{\mathbf{V}}_{bn} = 0 \Rightarrow \hat{\mathbf{V}}_{ab} - \hat{\mathbf{V}}_{an} + \hat{\mathbf{V}}_{an}\angle -120° = 0$$

$$\Rightarrow \hat{\mathbf{V}}_{ab} - \hat{\mathbf{V}}_{an}(1 - 1\angle -120°) = 0$$

$$\Rightarrow \hat{\mathbf{V}}_{ab} = \hat{\mathbf{V}}_{an}(1 - 1\angle -120°)$$

$$\Rightarrow \hat{\mathbf{V}}_{ab} = \hat{\mathbf{V}}_{an}\sqrt{3}\angle 30°$$

Relating phase and line voltages

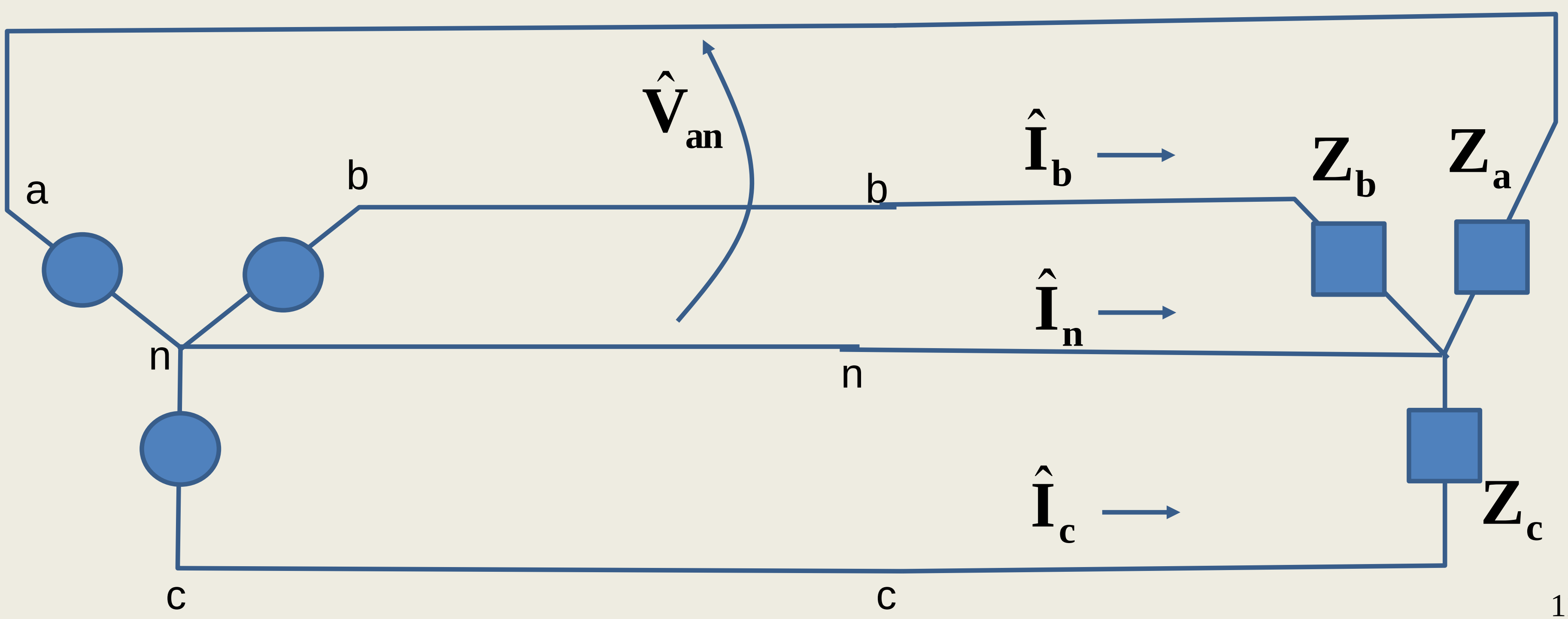


$$\hat{\mathbf{V}}_{\mathbf{ab}} = \sqrt{3}\hat{\mathbf{V}}_{\mathbf{an}}\angle 30^\circ$$

$$\hat{\mathbf{V}}_{\mathbf{bc}} = \sqrt{3}\hat{\mathbf{V}}_{\mathbf{bn}}\angle 30^\circ$$

$$\hat{\mathbf{V}}_{\mathbf{ca}} = \sqrt{3}\hat{\mathbf{V}}_{\mathbf{cn}}\angle 30^\circ$$

Wye-connected sources and loads



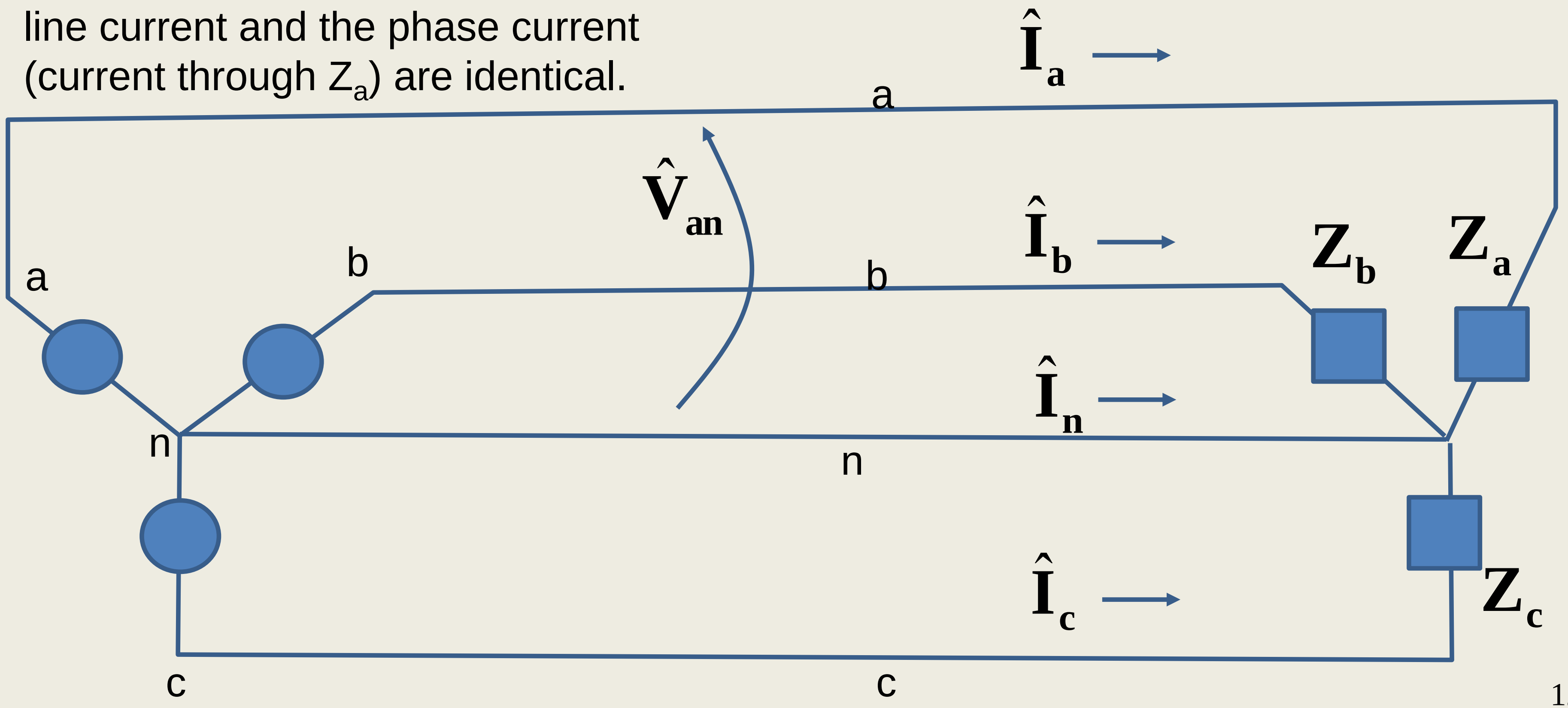
Balanced conditions

Balanced 3-phase conditions have:

- Line and phase voltages related as in previous slides.
- $Z_a = Z_b = Z_c$

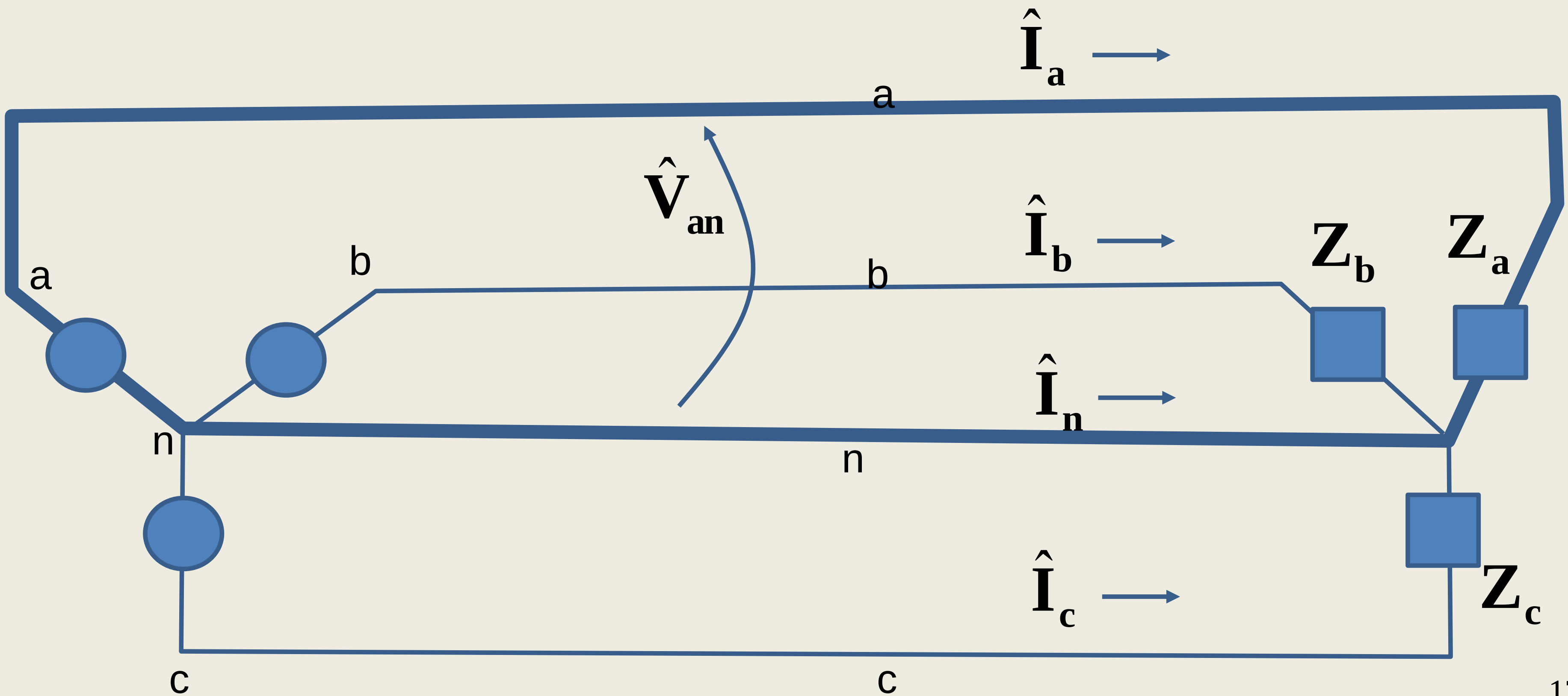
This results in: $\hat{\mathbf{I}}_{\mathbf{b}} = \hat{\mathbf{I}}_{\mathbf{a}} \angle -120°, \qquad \hat{\mathbf{I}}_{\mathbf{c}} = \hat{\mathbf{I}}_{\mathbf{a}} \angle +120°, \qquad \hat{\mathbf{I}}_{\mathbf{n}} = 0$

Note: In Wye-connected loads, the line current and the phase current (current through Z_a) are identical.

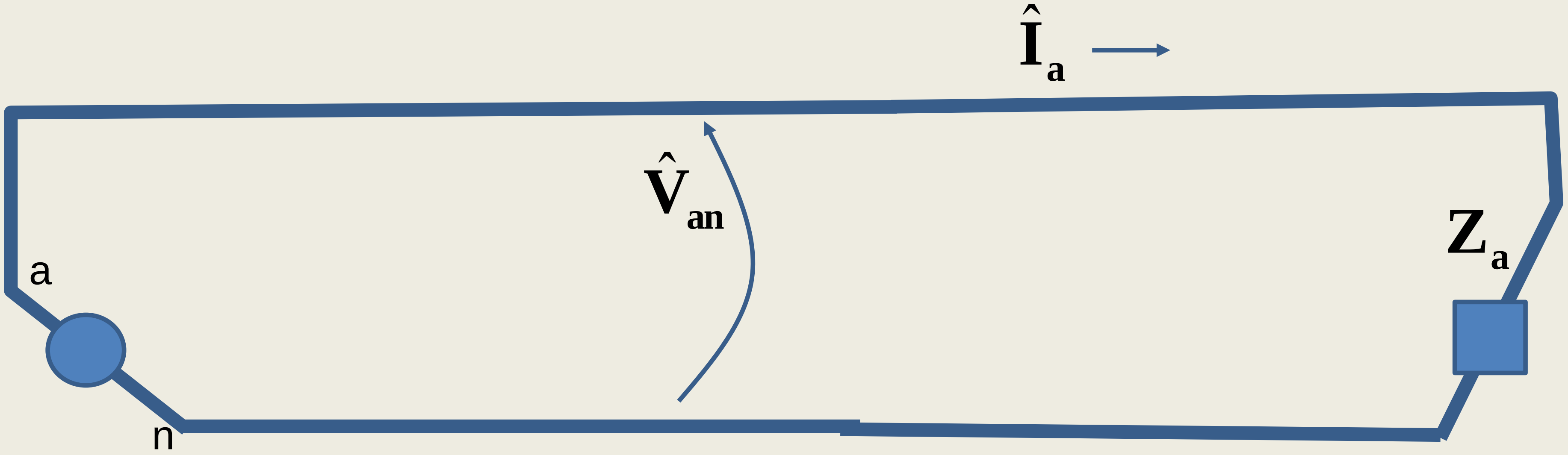


Per-phase analysis

Under balanced conditions, we may perform single-phase analysis on a “lifted-out” a-phase and neutral circuit, as shown below.



Per-phase analysis

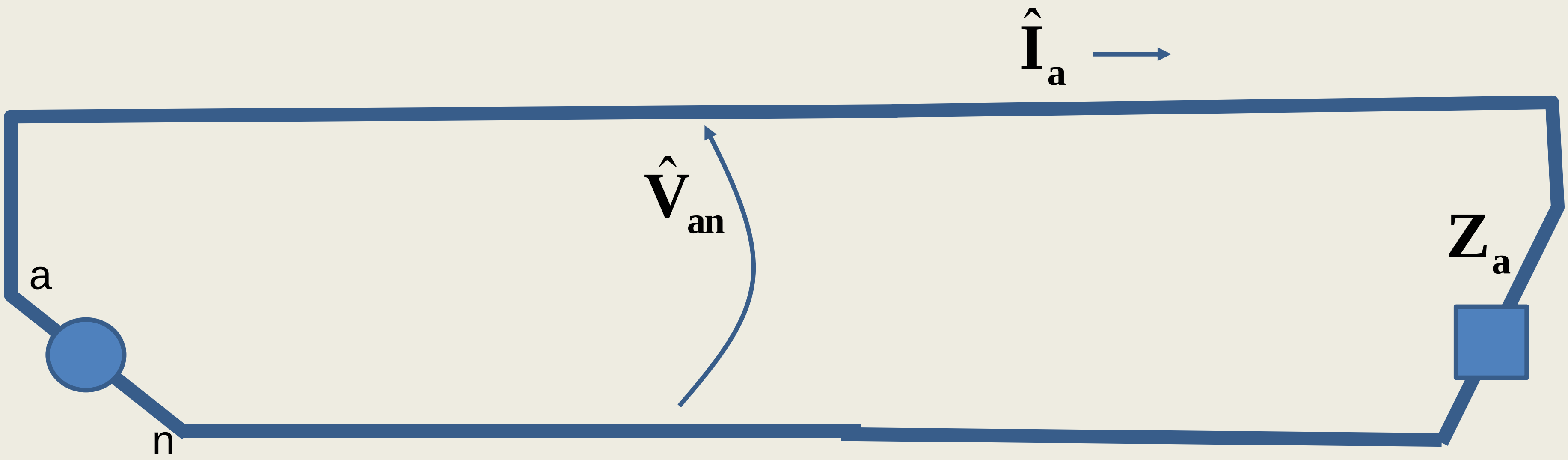


Now it is clear that:

$$\hat{\mathbf{I}}_{\mathbf{a}} = \frac{\hat{\mathbf{V}}_{\mathbf{an}}}{Z_a} \qquad \mathbf{S}_{1\varphi} = \hat{\mathbf{V}}_{\mathbf{an}}\hat{\mathbf{I}}_{\mathbf{a}}^{*} = P_{1\phi} + jQ_{1\phi}$$

Also, we still have: $P_{1\phi} = V_{an} I_a \cos\theta, \qquad Q_{1\phi} = V_{an} I_a \sin\theta$

Per-phase analysis



After we perform the single-phase analysis, we may then compute the 3-phase quantities according to:

$$\mathbf{S}_{3\varphi} = 3\mathbf{S}_{1\varphi} \Rightarrow P_{3\phi} = 3P_{1\phi}, \qquad Q_{3\phi} = 3Q_{1\phi}$$

Three phase power relations

The previous power relations utilize line-to-neutral voltages and line currents. Power may also be computed using line voltages, as developed in what follows:

$$P_{1\phi} = V_{an} I_a \cos\theta$$

$$\hat{\mathbf{V}}_{\mathbf{ab}} = \sqrt{3}\hat{\mathbf{V}}_{\mathbf{an}} \angle 30^\circ \Rightarrow V_{ab} = \sqrt{3} V_{an} \Rightarrow V_{an} = \frac{V_{ab}}{\sqrt{3}}$$

$$P_{1\phi} = \frac{V_{ab}}{\sqrt{3}} I_a \cos\theta = \frac{V_{ab}}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} I_a \cos\theta = \frac{V_{ab}\sqrt{3}}{3} I_a \cos\theta$$

$$P_{3\phi} = 3P_{1\phi} = 3\frac{V_{ab}\sqrt{3}}{3} I_a \cos\theta = \sqrt{3} V_{ab} I_a \cos\theta$$

Likewise, we may develop that

$$Q_{3\phi} = \sqrt{3} V_{ab} I_a \sin\theta$$

Three phase power relations

In summary:

$$\mathbf{S}_{3\varphi} = 3\mathbf{S}_{1\varphi} \Rightarrow P_{3\phi} = 3P_{1\phi}, \qquad Q_{3\phi} = 3Q_{1\phi}$$

$$P_{1\phi} = V_{an} I_a \cos\theta \qquad Q_{1\phi} = V_{an} I_a \sin\theta$$

$$P_{3\phi} = \sqrt{3} V_{ab} I_a \cos\theta \qquad Q_{3\phi} = \sqrt{3} V_{ab} I_a \sin\theta$$

Note 1: In Wye-connections, the power factor angle θ is the angle by which the line-to-neutral voltage $\hat{\mathbf{V}}_{\mathbf{an}}$ leads the phase current $\hat{\mathbf{I}}_{\mathbf{a}}$. It is not the angle by which the line-to-line voltage $\hat{\mathbf{V}}_{\mathbf{ab}}$ leads the phase current. More generally, the power factor angle at two terminals is the angle by which the voltage across those terminals leads the current into the positive terminal.

Note 2: You may see notation V_L or V_{LL} for V_{ab}.