## Solutions to Problems

1. Form the matrix equation $\underline{Y V}=\underline{I}$ for the network below.


Answer:

$$
\left[\begin{array}{ccc}
-j 4 & j 3 & j 1 \\
j 3 & -j 4 & j 2 \\
j 1 & j 2 & -j 3
\end{array}\right]\left[\begin{array}{l}
v_{1} \\
v_{2} \\
v_{3}
\end{array}\right]=\left[\begin{array}{c}
0.2 \\
0 \\
-0.2
\end{array}\right]
$$

2. Form the Y-bus for the following two-bus power system, in both rectangular and polar notation. The reactance $X_{12}$ is given in per unit.


Answer:

$$
\begin{gathered}
y_{12}=\frac{1}{x_{12}}=\frac{1}{j 0.1}=-j 10 \\
Y_{\text {bus }}=\left[\begin{array}{ll}
Y_{12} & Y_{12} \\
Y_{21} & Y_{22}
\end{array}\right]=\left[\begin{array}{cc}
-j 10 & j 10 \\
j 10 & -j 10
\end{array}\right]=\left[\begin{array}{cc}
10 \angle-90^{\circ} & 10 \angle 90^{\circ} \\
10 \angle 90^{\circ} & 10 \angle-90^{\circ}
\end{array}\right]
\end{gathered}
$$

3. Draw the network, in terms of branches only, for the following admittance matrix

$$
\underline{Y}=\left[\begin{array}{cc}
-\frac{j 7}{4} & \frac{j 10}{4} \\
\frac{j 10}{4} & -j 2
\end{array}\right]
$$

## Answer:

$$
\begin{array}{ll}
Y_{11}=y_{12}+y_{1} & Y_{22}=y_{21}+y_{2} \\
\frac{-j 7}{4}=\frac{-j 10}{4}+y_{1} & -j 2=\frac{-j 10}{4}+y_{2} \\
y_{1}=\frac{j 3}{4} & y_{2}=\frac{j}{2}
\end{array}
$$


7. George and Clara read the output from an industrial-grade power flow program; part of it is as follows:

| From bus no. | To bus no. | Pflow (MW) | Qflow (MVAR) |
| :---: | :---: | :---: | :---: |
| 6 | 8 | 53.8 | 2.4 |

The voltages and transmission line admittance parameters are given on the diagram below. All parameters are on a 100 MVA base.


They thought the power flow program had a "bug" because they did the following per unit calculations and compared the result to the Qflow of the power flow program output.

$$
\begin{aligned}
& I_{6,8}=\left(V_{6}-V_{8}\right)(G-j B)=\left(1.05 \angle 4.5^{\circ}-1.03 \angle 2.0^{\circ}\right)(3-j 10)=0.518 \angle-3.83^{\circ} \\
& S_{6,8}=V_{6} I_{6,8}{ }^{*}=\left(1.05 \angle 4.5^{\circ}\right)\left(0.518 \angle 3.83^{\circ}\right)=0.544 \angle 8.33^{\circ}=0.538+j 0.079 \\
& \Rightarrow P_{6,8}=53.8 \mathrm{MW}, \quad Q_{6,8}=7.9 \mathrm{MVARS}
\end{aligned}
$$

Were George and Clara correct? Explain.

## Answer:

George and Clara are computing $\mathrm{S}=\mathrm{P}+\mathrm{jQ}$ without including the effects of the reactive power injection from the charging capacitance. $\mathrm{C}_{\text {charge }}=(100)(0.05)(1.05)^{2}=5.5125$

