

Module T7

Power Flow



Read Module T7.

Work problems 1, 2, 3, 7 at end of Module T7.

Turn in on Tuesday, April 9.



Learning the power flow problem.

1. High-Level View

Program Output & Use (project)

9. Problem solution

8. Problem formulation

FOUNDATION

7. # of equations,
of variables

6. PF equations
for injections

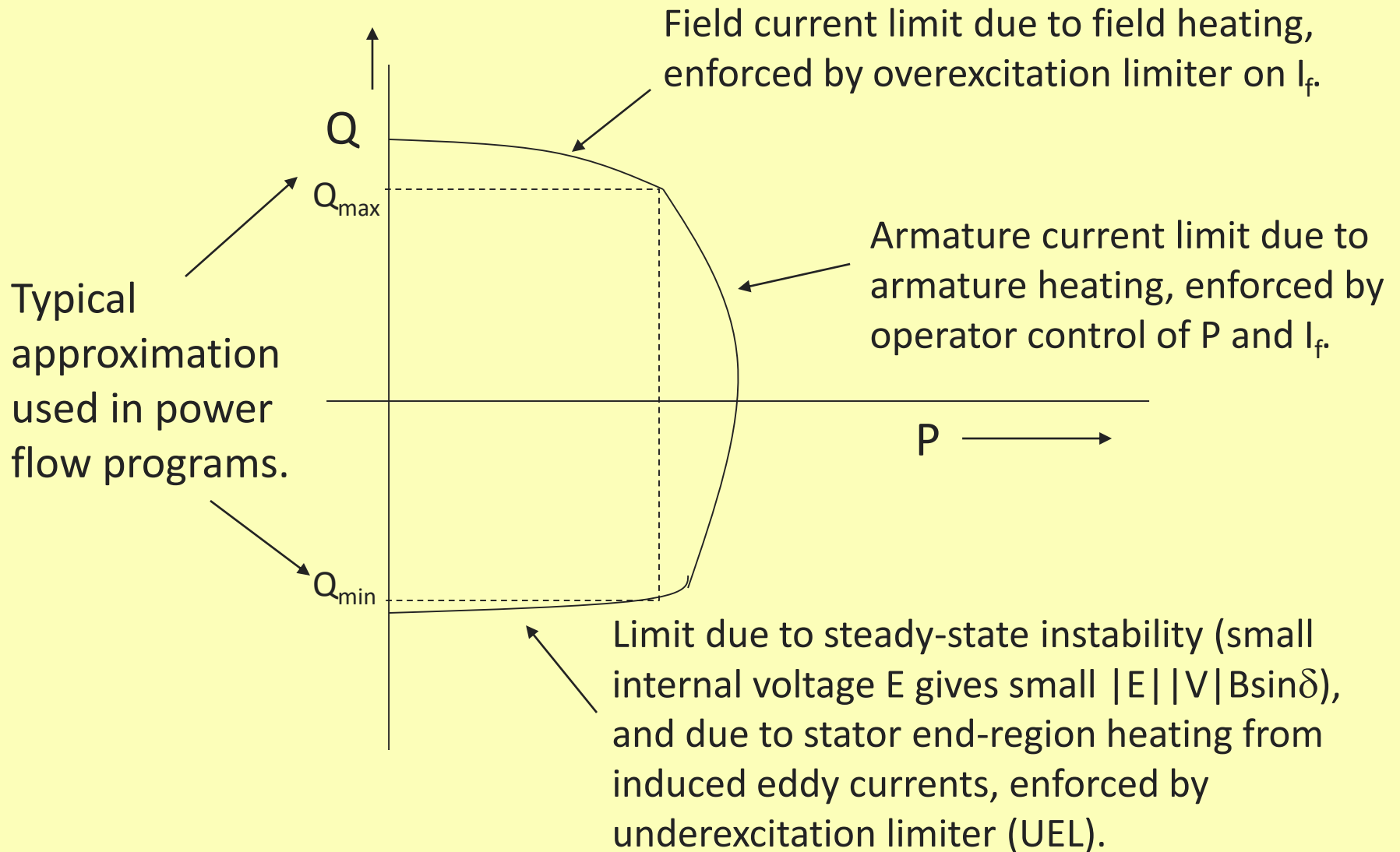
5. Admittance
matrix

4. Bus types

3. Injections

2. Gen React
Cap Curves

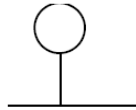
2. Gen reactive capability curve



3. Injections

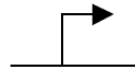
An *injection* is the power, either real or reactive, that is being injected into or withdrawn from a bus by an element having its other terminal (in the per-phase equivalent circuit) connected to ground. Such an element would be either a generator or a load. We define a positive injection as one where power is flowing from the element into the bus (i.e., into the network); a negative injection is then when power is flowing from the bus (i.e., from the network) into the element. Generators normally have positive real power injections, although they may also be assigned negative real power injections, in which case they are operating as a motor. Generators may have either positive or negative reactive power injections: positive if the generator is operating lagging and delivering reactive power to the bus, negative if the generator is operating leading and absorbing reactive power from the bus, and zero if the generator is operating at unity power factor. Loads normally have negative real and reactive power injections, although they may also be assigned positive real power injections in the case of very special modeling needs. Figure T7.3 (a) and (b) illustrate the two most common possibilities. Figure T.7.3 (c) illustrates that we must compute a net injection as the algebraic sum when a bus has both load and generation; in this case, the net injection for both real and reactive power is positive (into the bus). Thus, the net real power injection is $P_k = P_{gk} - P_{dk}$, and the net reactive power injection is $Q_k = Q_{gk} - Q_{dk}$. We may also refer to the net complex power injection as $S_k = S_{gk} - S_{dk}$, where $S_k = P_k + jQ_k$.

$$P_k = 100$$
$$Q_k = 30$$



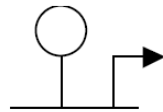
(a)

$$P_k = -40$$
$$Q_k = -20$$



(b)

$$P_k = 100 + (-40) = 60$$
$$Q_k = 30 + (-20) = 10$$



(c)

4. Bus types

First classification:

- Gen
- Load
- Both

2nd classification (in terms of info known at the buses).

What is possible info?

- V
- θ
- P_{inj}
- Q_{inj}

Bus type PV (usually gen)

- Knowns:
 - P_{inj}
 - V
- Unknowns:
 - Q_{inj}
 - θ

Bus type PQ (usually load)

- Knowns:
 - P_{inj}
 - Q_{inj}
- Unknowns:
 - V
 - θ

4. Bus types

One more type....

This type is motivated by a power specification issue inherent to solving the power flow problem:

- We need to specify all MW gen and load.
- To do this, we need to know losses.
- Losses are a function of line flows.
- We cannot know line flows until we specify all MW gen & load.
- So what do we do?

$$\underbrace{\sum_{k \in \text{Gens}} P_k}_{\text{TotalMWGen}} + \underbrace{\sum_{k \in \text{Loads}} P_k}_{\text{TotalMWLoad}} = P_{\text{Loss}}$$

4. Bus types

$$\underbrace{\sum_{k \in Gens} P_k}_{TotalMWGen} + \underbrace{\sum_{k \in Loads} P_k}_{TotalMWLoad} = P_{Loss}$$

Pull out one gen:

$$P_1 + \sum_{\substack{k \in Gens \\ k \neq 1}} P_k + \sum_{k \in Loads} P_k = P_{Loss}$$

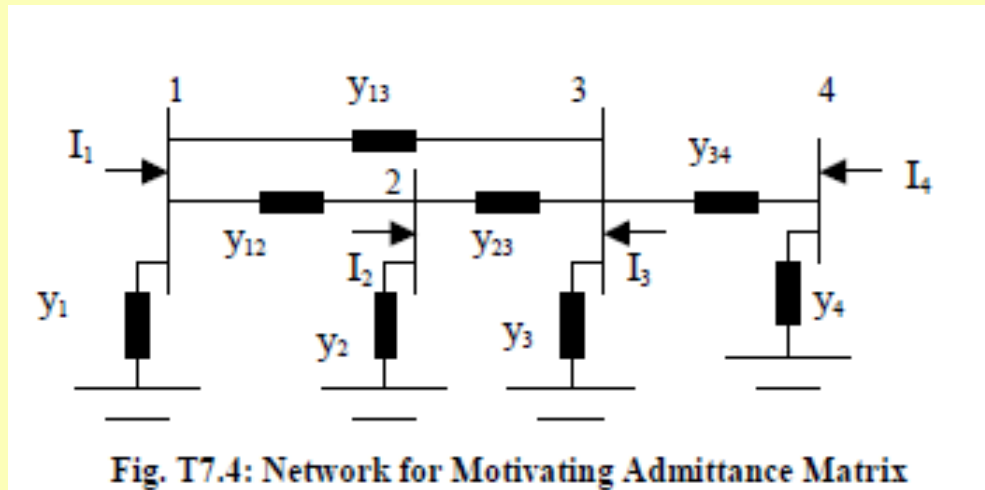
$$P_1 = P_{Loss} - \sum_{\substack{k \in Gens \\ k \neq 1}} P_k - \sum_{k \in Loads} P_k$$

And same for reactive power.
This one gen is called the swing bus, slack or reference bus.

Bus type “Swing”

- Knowns:
 - V
 - θ
- Unknowns:
 - P_{inj}
 - Q_{inj}

4. Admittance matrix



KCL $I_1 = (V_1 - V_2)y_{12} + (V_1 - V_3)y_{13} + V_1 y_1$

Add bus 4 connection
 $I_1 = (V_1 - V_2)y_{12} + (V_1 - V_3)y_{13} + (V_1 - V_4)y_{14} + V_1 y_1$

Gather terms $I_1 = V_1(y_1 + y_{12} + y_{13} + y_{14}) + V_2(-y_{12}) + V_3(-y_{13}) + V_4(-y_{14})$

Do same for other four buses and then put in matrix form

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} y_1 + y_{12} + y_{13} + y_{14} & -y_{12} & -y_{13} & -y_{14} \\ -y_{21} & y_2 + y_{21} + y_{23} + y_{24} & -y_{23} & -y_{24} \\ -y_{31} & -y_{32} & y_3 + y_{31} + y_{32} + y_{34} & -y_{34} \\ -y_{41} & -y_{42} & -y_{43} & y_4 + y_{41} + y_{42} + y_{43} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Off-diagonals: negative of admittance connecting corresponding two buses.

Diagonals: sum of admittances for all lines & shunts connected to corresponding bus.

4. Admittance matrix

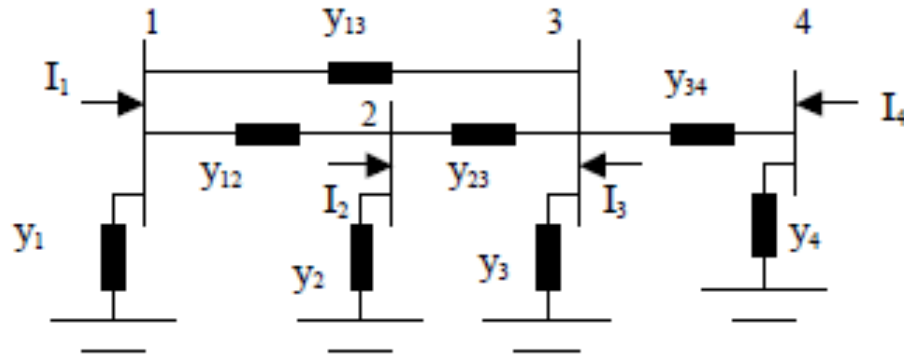


Fig. T7.4: Network for Motivating Admittance Matrix

Admittance matrix (or “Y-bus”):

$$\underline{Y} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} y_1 + y_{12} + y_{13} + y_{14} & -y_{12} & -y_{13} & -y_{14} \\ -y_{21} & y_2 + y_{21} + y_{23} + y_{24} & -y_{23} & -y_{24} \\ -y_{31} & -y_{32} & y_3 + y_{31} + y_{32} + y_{34} & -y_{34} \\ -y_{41} & -y_{42} & -y_{43} & y_4 + y_{41} + y_{42} + y_{43} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

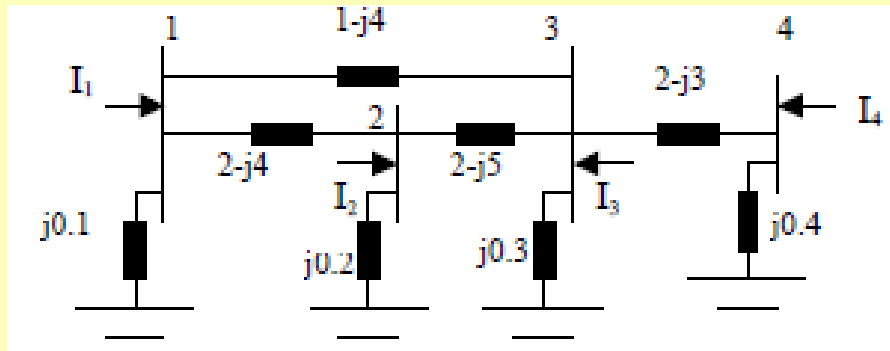
$$I_k = \sum_{j=1}^N Y_{kj} V_j$$

$$\underline{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$\underline{I} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

$$\underline{I} = \underline{YV}$$

4. Admittance matrix



Admittance matrix (or “Y-bus”):

$$\underline{Y} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} 3-j7.9 & -2+j4 & -1+j4 & 0 \\ -2+j4 & 4-j8.8 & -2+j5 & 0 \\ -1+j4 & -2+j5 & 5-j11.7 & -2+j3 \\ 0 & 0 & -2+j3 & 2-j2.6 \end{bmatrix}$$

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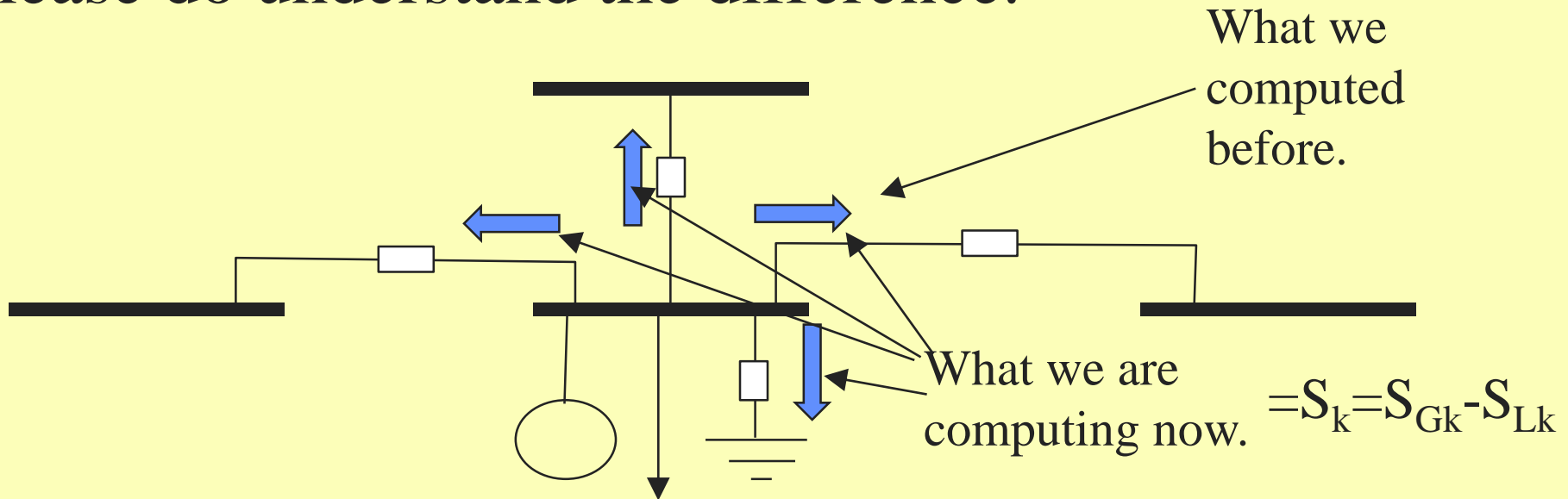
3. Injections

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6. PF Equations

Previously, we wrote the power flow equations for the power flowing over a certain line. Now, we want to write the PF equations for the *power injection*.

The procedure is the same in both cases.
Please do understand the difference.



6. PF Equations

$$S_k = V_k I_k^*$$

$$I_k = \sum_{j=1}^N Y_{kj} V_j$$

$$S_k = V_k \left(\sum_{j=1}^N Y_{kj} V_j \right)^* = V_k \sum_{j=1}^N Y_{kj}^* V_j^*$$

$$V_k = |V_k| \angle \theta_k$$

$$Y_{kj} = G_{kj} + jB_{kj}$$

$$\begin{aligned} S_k &= V_k \sum_{j=1}^N Y_{kj}^* V_j^* = |V_k| \angle \theta_k \sum_{j=1}^N (G_{kj} + jB_{kj})^* (|V_j| \angle \theta_j)^* = |V_k| \angle \theta_k \sum_{j=1}^N (G_{kj} - jB_{kj}) (|V_j| \angle -\theta_j) \\ &= \sum_{j=1}^N |V_k| \angle \theta_k (|V_j| \angle -\theta_j) (G_{kj} - jB_{kj}) = \sum_{j=1}^N (|V_k| |V_j| \angle (\theta_k - \theta_j)) (G_{kj} - jB_{kj}) \end{aligned}$$

6. PF Equations

$$S_k = V_k I_k^*$$

$$I_k = \sum_{j=1}^N Y_{kj} V_j$$

$$S_k = V_k \left(\sum_{j=1}^N Y_{kj} V_j \right)^* = V_k \sum_{j=1}^N Y_{kj}^* V_j^*$$

$$V_k = |V_k| \angle \theta_k$$

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$$\begin{aligned} S_k &= V_k \sum_{j=1}^N Y_{kj}^* V_j^* = |V_k| \angle \theta_k \sum_{j=1}^N (G_{kj} + jB_{kj})^* (|V_j| \angle \theta_j)^* = |V_k| \angle \theta_k \sum_{j=1}^N (G_{kj} - jB_{kj}) (|V_j| \angle -\theta_j) \\ &= \sum_{j=1}^N |V_k| \angle \theta_k (|V_j| \angle -\theta_j) (G_{kj} - jB_{kj}) = \sum_{j=1}^N (|V_k| |V_j| \angle (\theta_k - \theta_j)) (G_{kj} - jB_{kj}) \end{aligned}$$

6. PF Equations

$$\begin{aligned} S_k &= V_k \sum_{j=1}^N Y_{kj}^* V_j^* = |V_k| \angle \theta_k \sum_{j=1}^N (G_{kj} + jB_{kj})^* (|V_j| \angle \theta_j)^* = |V_k| \angle \theta_k \sum_{j=1}^N (G_{kj} - jB_{kj}) (|V_j| \angle -\theta_j) \\ &= \sum_{j=1}^N |V_k| \angle \theta_k (|V_j| \angle -\theta_j) (G_{kj} - jB_{kj}) = \sum_{j=1}^N (|V_k| |V_j| \angle (\theta_k - \theta_j)) (G_{kj} - jB_{kj}) \end{aligned}$$

$$V = |V| \angle \theta = |V| \{ \cos \theta + j \sin \theta \}$$

$$\begin{aligned} S_k &= \sum_{j=1}^N (|V_k| |V_j| \angle (\theta_k - \theta_j)) (G_{kj} - jB_{kj}) \\ &= \sum_{j=1}^N |V_k| |V_j| (\cos(\theta_k - \theta_j) + j \sin(\theta_k - \theta_j)) (G_{kj} - jB_{kj}) \end{aligned}$$

$$P_k = \sum_{j=1}^N |V_k| |V_j| (G_{kj} \cos(\theta_k - \theta_j) + B_{kj} \sin(\theta_k - \theta_j))$$

$$Q_k = \sum_{j=1}^N |V_k| |V_j| (G_{kj} \sin(\theta_k - \theta_j) - B_{kj} \cos(\theta_k - \theta_j))$$