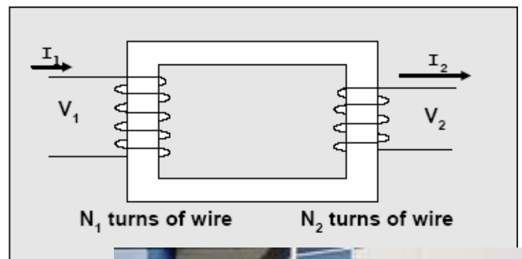
Three-Phase Transformers

Section 9 of transformer notes on website provide only first part of the material in these slides, the rest is covered only in these slides.

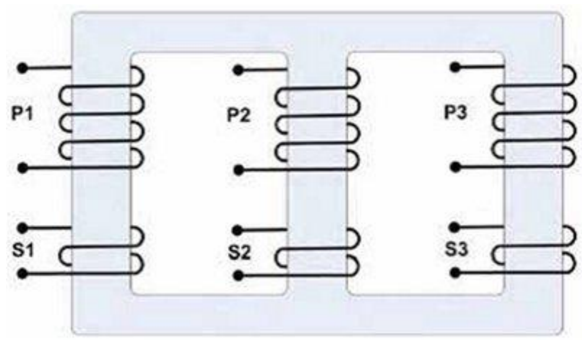
Quiz Thursday over HW5 and what we cover today.



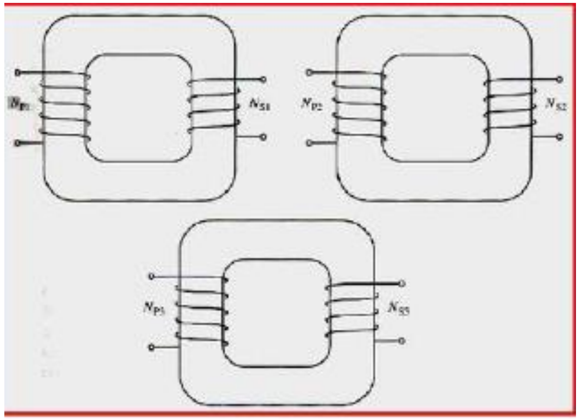
Three-Phase Transformers - Types

- Three-phase xfmrs have 6 windings:
  - Phases A, B, and C on primary
  - Phases a, b, and c on secondary
- Two approaches for developing a 3phase xfmr:

One "three-phase bank" on a single core (a single magnetic circuit)



Interconnect 3 single-phase xfmrs

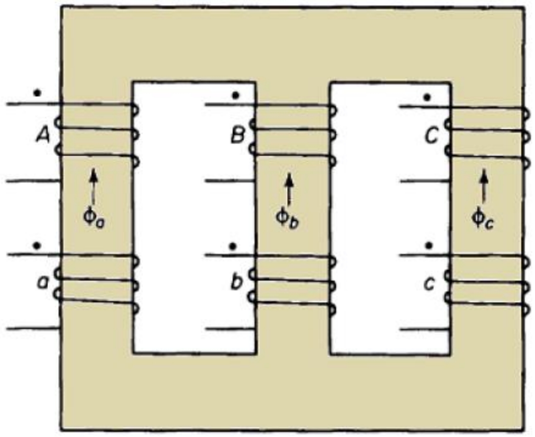


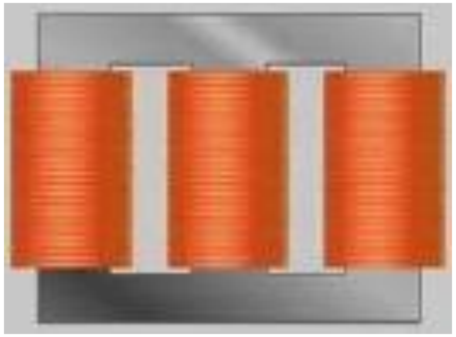
Three-phase banks

One "three-phase bank" on a single core (a single magnetic circuit)

"Core"-type construction

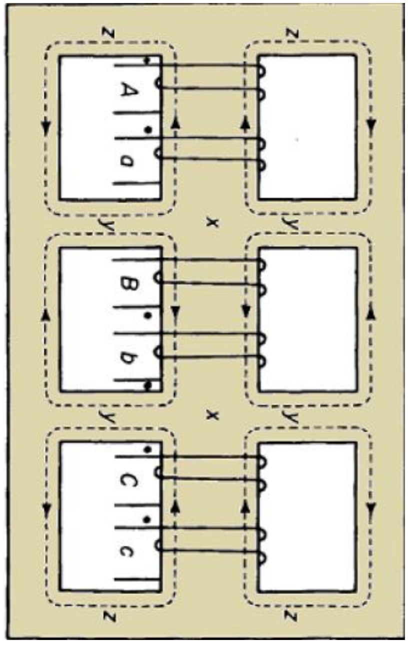
primary and secondary windings are wound outside and surround their leg





"Shell"-type construction

windings pass inside core, so that core forms a shell around the windings.

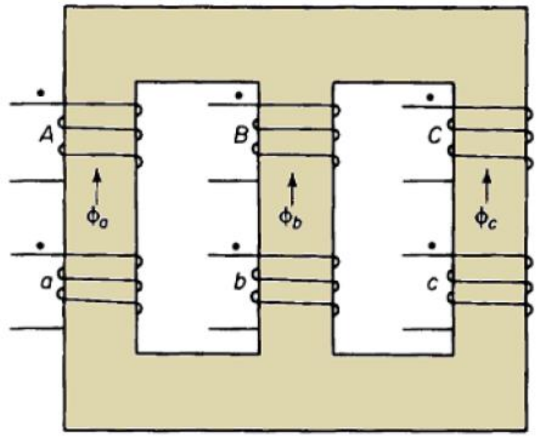


Three-phase banks

One “three-phase bank” on a single core (a single magnetic circuit)

“Core”-type construction

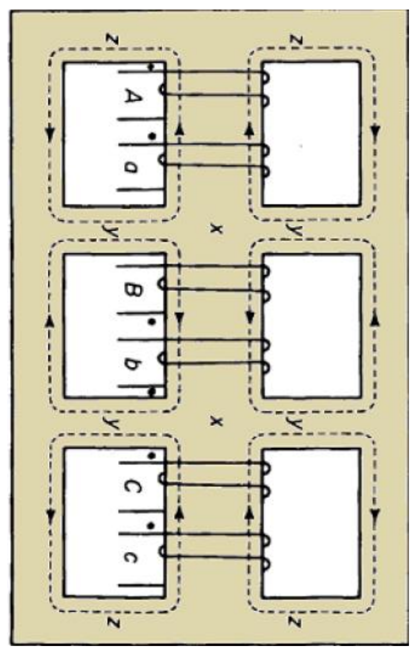
primary and secondary windings are wound outside and surround their leg



The shell-type requires more ferromagnetic material than does the core-type and so is typically more costly on a per-MVA basis.

“Shell”-type construction

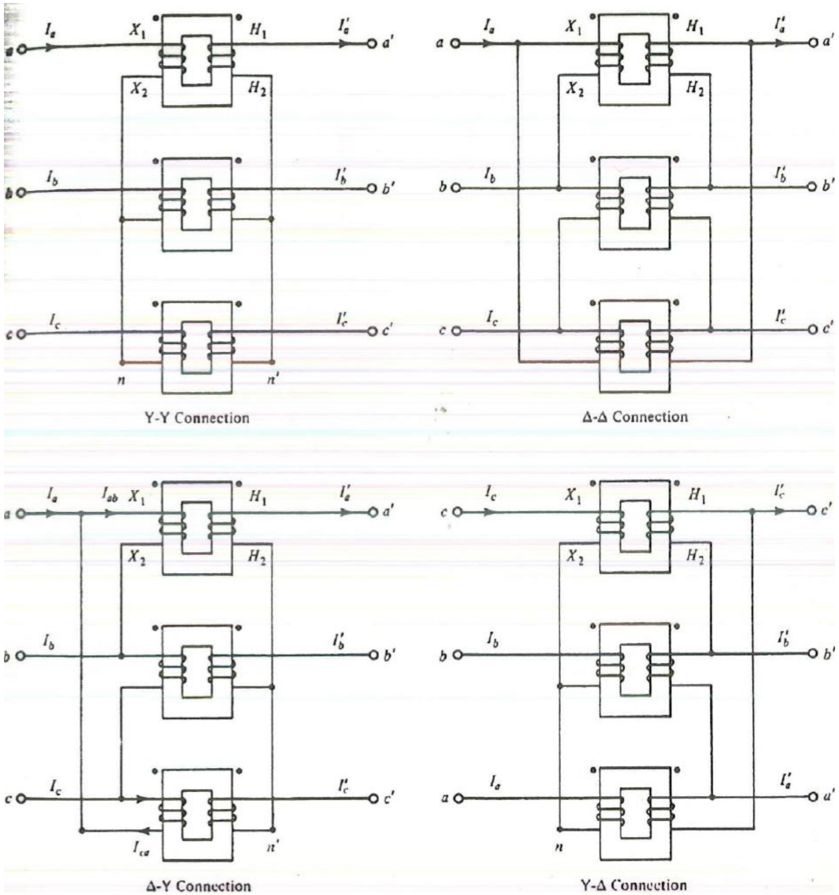
windings pass inside core, so that core forms a shell around the windings.



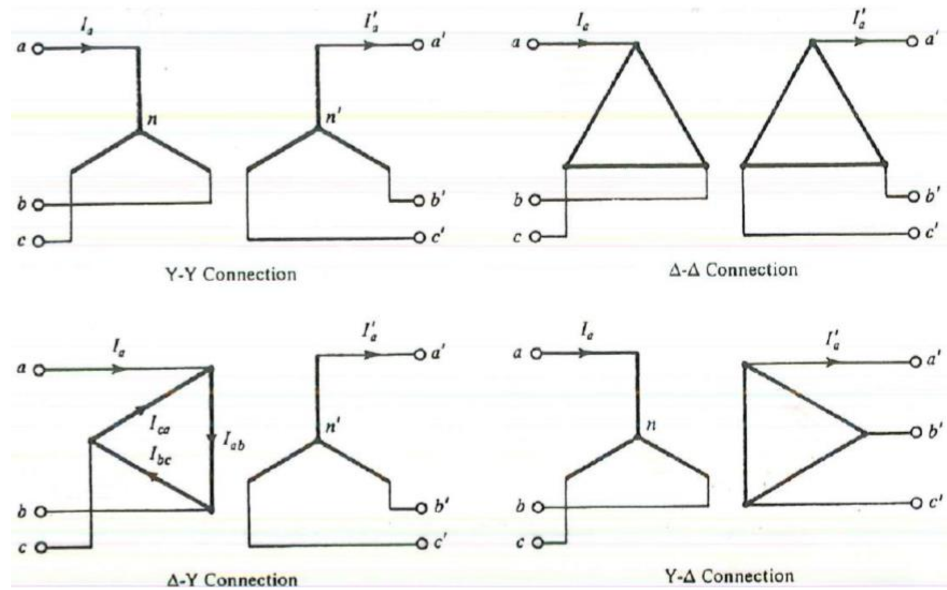
However, its mag circuit provides multiple paths for flux to flow; this reduces flux density seen by each leg which is advantageous for short-circuit performance (thermal & mechanical); so shell-types are used more frequently when high capacity is required.

3 single-phase banks – connection types

Physical connections



Circuit diagrams



There is no connection between intersecting wires unless there is a “dot” at their intersection.

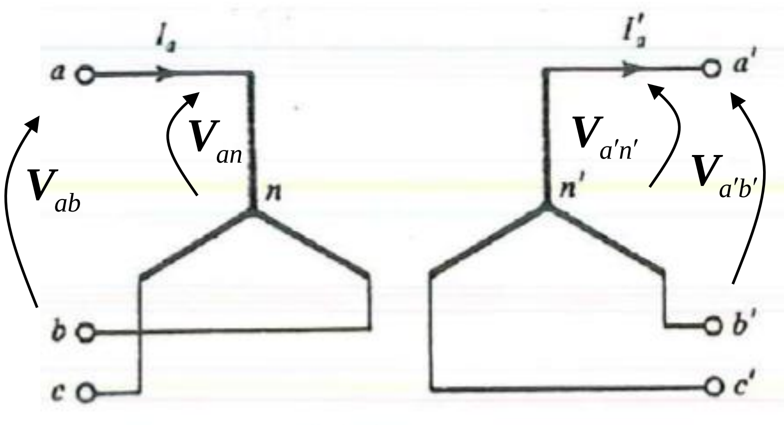
Each phase of above circuit diagrams contain a winding in them.

From now on, we will simply refer to “3 phase xfmrs” without regards to whether they are a three-phase bank or 3 single phase units. We will usually, however, need to indicate their type of connection (Y-Y, Y-Δ, Δ-Y, or Δ-Δ).

An important concept

For a Y-Y connected 3-phase transformer, the ratio of the line-to-neutral voltages on either side is the same as the ratio of the line-to-line voltages on either side. Likewise, the ratio of the line-currents is the same as the ratio of the phase currents.

<u>Proof</u>:



Y-Y Connection

$$\frac{\boldsymbol{V}_{an}}{\boldsymbol{V}_{a'n'}} = \frac{\frac{1}{\sqrt{3}}\boldsymbol{V}_{ab}\angle -30^\circ}{\frac{1}{\sqrt{3}}\boldsymbol{V}_{a'b'}\angle -30^\circ} = \frac{\boldsymbol{V}_{ab}}{\boldsymbol{V}_{a'b'}}$$

$$\frac{\boldsymbol{I}_{a}}{\boldsymbol{I}_{a'}} = \frac{\boldsymbol{I}_{a}}{\boldsymbol{I}_{a'}}$$

This means that the ratio of the line-to-line voltages on either side equals the ratio of the line-to-neutral voltages on either side if we can convert the transformer to an "equivalent Y-Y."

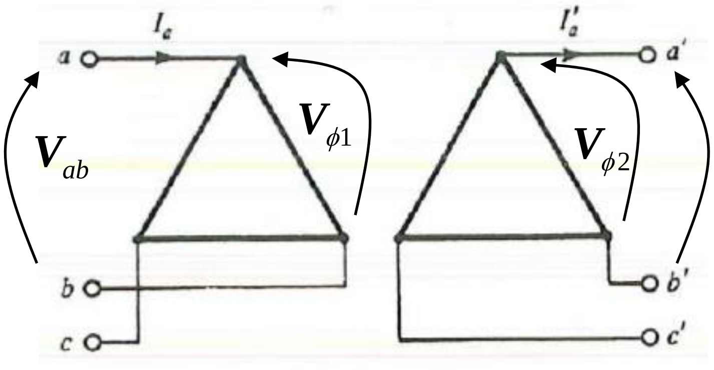
And this means that if we want the ratio of the "equivalent Y-Y", all we need to get is the ratio of the actual line-to-line voltages!

Fact: Getting a per-phase equivalent cct depends on getting an equivalent Y-Y connected transformer.

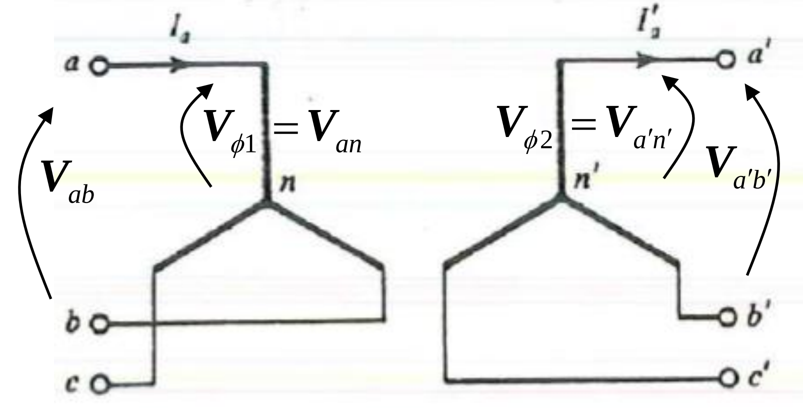
Voltage & current transformation ratios

Two concepts:

1. The voltage transformation ratio for 3-phase xfmrs is always given as the ratio of the line-to-line voltage magnitude on either side, $\mathbf{V}_{ab1}/\mathbf{V}_{ab2}$.
2. The voltage transformation of the phase voltages is always $\mathbf{V}_{\phi1}/\mathbf{V}_{\phi2} = N_1/N_2$ (phase voltages are across the windings).



Δ-Δ Connection



Y-Y Connection

$$\frac{\boldsymbol{V}_{ab}}{\boldsymbol{V}_{a'b'}} = \frac{\boldsymbol{V}_{\phi1}}{\boldsymbol{V}_{\phi2}} = \frac{N_1}{N_2}$$

This is ratio across phases

$$\frac{\boldsymbol{V}_{ab}}{\boldsymbol{V}_{a'b'}} = \frac{\sqrt{3}\boldsymbol{V}_{an}\angle 30^\circ}{\sqrt{3}\boldsymbol{V}_{a'n'}\angle 30^\circ} = \frac{\sqrt{3}\boldsymbol{V}_{\phi1}\angle 30^\circ}{\sqrt{3}\boldsymbol{V}_{\phi2}\angle 30^\circ} = \frac{\boldsymbol{V}_{\phi1}}{\boldsymbol{V}_{\phi2}} = \frac{N_1}{N_2}$$

This is ratio across phases

For Y-Y or Δ-Δ, the ratio of the **line-to-line voltages** *is* the ratio of the **phase voltages**. Likewise:

$$\frac{\boldsymbol{I}_a}{\boldsymbol{I}_{a'}} = \frac{\sqrt{3}\boldsymbol{I}_{ab}\angle -30^\circ}{\sqrt{3}\boldsymbol{I}_{a'b'}\angle -30^\circ} = \frac{\sqrt{3}\boldsymbol{I}_{\phi1}\angle -30^\circ}{\sqrt{3}\boldsymbol{I}_{\phi2}\angle -30^\circ} = \frac{\boldsymbol{I}_{\phi1}}{\boldsymbol{I}_{\phi2}} = \frac{N_2}{N_1}$$

$$\frac{\boldsymbol{I}_a}{\boldsymbol{I}_{a'}} = \frac{\boldsymbol{I}_{\phi1}}{\boldsymbol{I}_{\phi2}} = \frac{N_2}{N_1}$$

Per-phase equivalent circuit for Y-Y or Δ-Δ

When analyzing balanced 3-phase ccts with xfmrs, you must obtain the per-phase equivalent cct of the xfmr. The effective turns ratio of this per-phase equivalent xfmr cct will be the ratio of the phase-to-phase voltages for the *equivalent* Y-Y connected transformer, which will be the same as the ratio of the line-to-line voltages of the actual Y-Y connected transformer.

- If the actual connection is Y-Y, then the effective turns ratio is $[N_1/N_2]_{eff}=N_1/N_2$ of the windings (easy!).

$$\frac{\boldsymbol{V}_{ab}}{\boldsymbol{V}_{a'b'}}=\frac{\sqrt{3}\boldsymbol{V}_{an}\angle 30^\circ}{\sqrt{3}\boldsymbol{V}_{a'n'}\angle 30^\circ}=\frac{\sqrt{3}\boldsymbol{V}_{\phi 1}\angle 30^\circ}{\sqrt{3}\boldsymbol{V}_{\phi 2}\angle 30^\circ}=\frac{\boldsymbol{V}_{\phi 1}}{\boldsymbol{V}_{\phi 2}}=\frac{N_1}{N_2}$$

$$\frac{\boldsymbol{I}_a}{\boldsymbol{I}_{a'}}=\frac{\boldsymbol{I}_{\phi 1}}{\boldsymbol{I}_{\phi 2}}=\frac{N_2}{N_1}$$

- If the actual connection is Δ-Δ, then the effective turns ratio is $[N_1/N_2]_{eff}=N_1/N_2$ of the windings (easy!).

$$\frac{\boldsymbol{V}_{ab}}{\boldsymbol{V}_{a'b'}}=\frac{\boldsymbol{V}_{\phi 1}}{\boldsymbol{V}_{\phi 2}}=\frac{N_1}{N_2}$$

$$\frac{\boldsymbol{I}_a}{\boldsymbol{I}_{a'}}=\frac{\sqrt{3}\boldsymbol{I}_{ab}\angle -30^\circ}{\sqrt{3}\boldsymbol{I}_{a'b'}\angle -30^\circ}=\frac{\sqrt{3}\boldsymbol{I}_{\phi 1}\angle -30^\circ}{\sqrt{3}\boldsymbol{I}_{\phi 2}\angle -30^\circ}=\frac{\boldsymbol{I}_{\phi 1}}{\boldsymbol{I}_{\phi 2}}=\frac{N_2}{N_1}$$

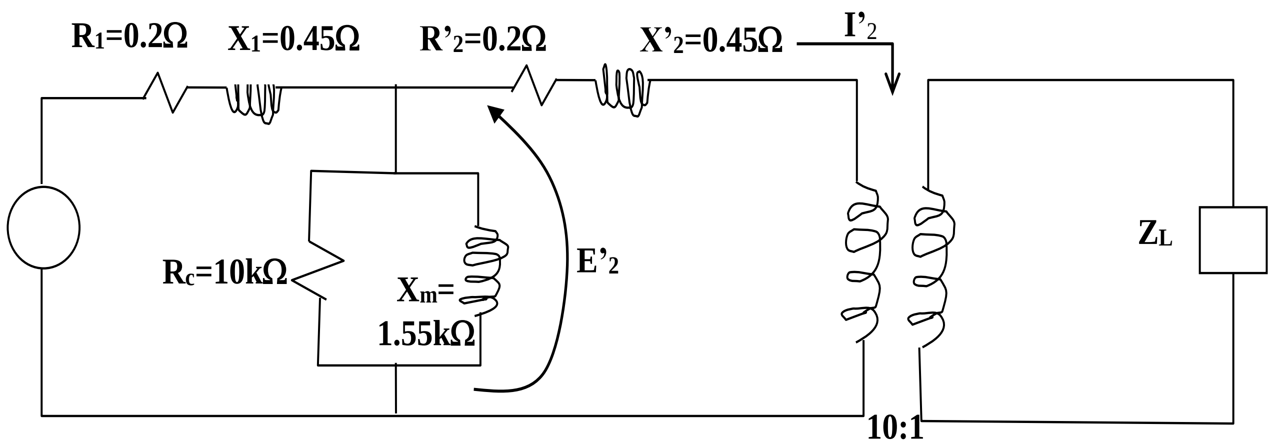
Per-phase equivalent circuit for Y-Y or Δ-Δ

The "exact" equivalent circuit parameters of a 150-kVA, 2400volt/240volt single-phase transformer are R_1=0.2Ω, R_2=2mΩ, X_1=0.45Ω, X_2=4.5mΩ, R_c=10kΩ, and X_m=1.55kΩ. R_1, X_1, R_c, and X_m are given referred to the primary side; R_2 and X_2 are given referred to the secondary side. A company purchases three of these single-phase transformers and connects them in a Y-Y configuration to a three-phase Y-connected load Z_L.

a. (5 pts) Draw per-phase "exact" equivalent of the circuit (transformer and load) with all elements, except the load, referred to primary side.
b. (5 pts) Label all impedance elements on the diagram with their ohmic value (use letters and numerical value).
c. (5 pts) Identify on the diagram the turns ratio $(N_1/N_2)_{eff}$ to be used in the per-phase circuit (identify numerical value).
d. (5 pts) Label the secondary current referred to the primary, $\mathbf{I}'_2$ (do not need numerical value, just location & directionality).
e. (5 pts) Label the voltage across the secondary winding, referred to the primary, $\mathbf{E}'_2$ (do not need numerical value, just location & directionality).
f. Would the effective turns ratio to be used in the per-phase circuit change from that identified in (c) if:
    i. the transformers were connected Δ- Δ?
    ii. the transformers were connected Y-Δ?
    iii. the transformers were connected Δ-Y?

Per-phase equivalent circuit for Y-Y or Δ-Δ

(a, b, c, d, e)



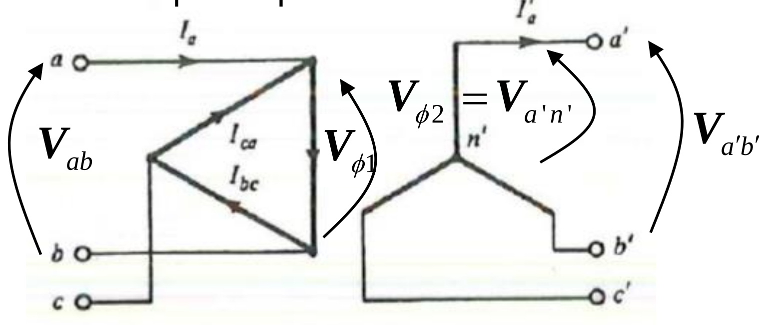
(f) Would the effective turns ratio to be used in the per-phase circuit change from that identified in (c) if:

i. the transformers were connected Δ- Δ? No.
ii. the transformers were connected Y-Δ? Yes.
iii. the transformers were connected Δ-Y? Yes.

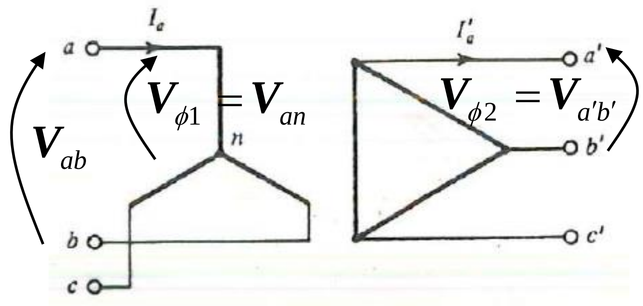
Voltage & current transformation ratios

Two concepts:

1. The voltage transformation ratio for 3-phase xfmrs is always given as the ratio of the line-to-line voltage magnitude on either side, $\mathbf{V}_{ab1}/\mathbf{V}_{ab2}$.
2. The voltage transformation of the phase voltages is always $\mathbf{V}_{\phi1}/\mathbf{V}_{\phi2} = N_1/N_2$ (phase voltages are across the windings).



Δ-Y Connection



Y-Δ Connection

$$\frac{\boldsymbol{V}_{ab}}{\boldsymbol{V}_{a'b'}} = \frac{\boldsymbol{V}_{\phi 1}}{\sqrt{3}\boldsymbol{V}_{a'n'}\angle 30^\circ} = \frac{\boldsymbol{V}_{\phi 1}}{\sqrt{3}\boldsymbol{V}_{\phi 2}\angle 30^\circ} = \frac{N_1}{N_2}\frac{1}{\sqrt{3}}\angle -30^\circ$$

$$\frac{\boldsymbol{V}_{ab}}{\boldsymbol{V}_{a'b'}} = \frac{\sqrt{3}\boldsymbol{V}_{an}\angle 30^\circ}{\boldsymbol{V}_{\phi 2}} = \frac{\sqrt{3}\boldsymbol{V}_{\phi 1}\angle 30^\circ}{\boldsymbol{V}_{\phi 2}} = \frac{\boldsymbol{V}_{\phi 1}}{\boldsymbol{V}_{\phi 2}}\sqrt{3}\angle 30^\circ = \frac{N_1}{N_2}\sqrt{3}\angle 30^\circ$$

For Δ-Y or Y-Δ, the ratio of the **line-to-line voltages** *is not* the ratio of the **phase voltages**. Likewise:

$$\frac{\boldsymbol{I}_{a}}{\boldsymbol{I}_{a'}} = \frac{\sqrt{3}\boldsymbol{I}_{ab}\angle -30^\circ}{\boldsymbol{I}_{a'}} = \frac{\sqrt{3}\boldsymbol{I}_{\phi 1}\angle -30^\circ}{\boldsymbol{I}_{\phi 2}} = \frac{\boldsymbol{I}_{\phi 1}}{\boldsymbol{I}_{\phi 2}}\sqrt{3}\angle -30^\circ = \frac{N_2}{N_1}\sqrt{3}\angle -30^\circ$$

$$\frac{\boldsymbol{I}_{a}}{\boldsymbol{I}_{a'}} = \frac{\boldsymbol{I}_{a}}{\sqrt{3}\boldsymbol{I}_{a'b'}\angle -30^\circ} = \frac{\boldsymbol{I}_{\phi 1}}{\sqrt{3}\boldsymbol{I}_{\phi 2}\angle -30^\circ} = \frac{\boldsymbol{I}_{\phi 1}}{\boldsymbol{I}_{\phi 2}}\frac{\angle 30^\circ}{\sqrt{3}} = \frac{N_2}{N_1}\frac{\angle 30^\circ}{\sqrt{3}}$$

Relative to voltage ratios, current ratios get inverse sqrt(3) but same angle.

Per-phase equivalent circuit for Y-Δ or Δ-Y

When analyzing balanced 3-phase ccts with xfmrs, you must obtain the per-phase equivalent cct of the xfmr. The effective turns ratio of this per-phase equivalent xfmr cct will be the ratio of the phase-to-phase voltages for the *equivalent* Y-Y connected transformer, which will be the same as the ratio of the line-to-line voltages of the actual Y-Y connected transformer.

- If the actual connection is Δ-Y, then this turns ratio is

$$\frac{\mathbf{V}_{ab}}{\mathbf{V}_{a'b'}} = \frac{\mathbf{V}_{\phi 1}}{\sqrt{3}\mathbf{V}_{a'n'}\angle 30^\circ} = \frac{\mathbf{V}_{\phi 1}}{\sqrt{3}\mathbf{V}_{\phi 2}\angle 30^\circ} = \frac{N_1}{N_2}\frac{1}{\sqrt{3}}\angle -30^\circ$$

$$\frac{\mathbf{I}_a}{\mathbf{I}_{a'}} = \frac{\sqrt{3}\mathbf{I}_{ab}\angle -30^\circ}{\mathbf{I}_{a'}} = \frac{\sqrt{3}\mathbf{I}_{\phi 1}\angle -30^\circ}{\mathbf{I}_{\phi 2}} = \frac{\mathbf{I}_{\phi 1}}{\mathbf{I}_{\phi 2}}\sqrt{3}\angle -30^\circ = \frac{N_2}{N_1}\sqrt{3}\angle -30^\circ$$

These are also the turns ratio of the equivalent Y-Y xfmr.

$$\frac{\mathbf{V}_{\phi 1Yeff}}{\mathbf{V}_{\phi 2}} = \frac{N_1}{N_2}\frac{1}{\sqrt{3}}\angle -30^\circ \Rightarrow \quad \mathbf{V}_{\phi 1Yeff} = \frac{N_1}{N_2}\frac{1}{\sqrt{3}}\angle -30^\circ \mathbf{V}_{\phi 2}$$

$$\frac{\mathbf{I}_{\phi 1Yeff}}{\mathbf{I}_{\phi 2}} = \frac{N_2}{N_1}\sqrt{3}\angle -30^\circ \Rightarrow \mathbf{I}_{\phi 1Yeff} = \frac{N_2}{N_1}\sqrt{3}\angle -30^\circ \mathbf{I}_{\phi 2}$$

$$Z_1 = \frac{\mathbf{V}_{\phi 1Yeff}}{\mathbf{I}_{\phi 1Yeff}} = \frac{\frac{N_1}{N_2}\frac{1}{\sqrt{3}}\angle -30^\circ \mathbf{V}_{\phi 2}}{\frac{N_2}{N_1}\sqrt{3}\angle -30^\circ \mathbf{I}_{\phi 2}} = \frac{N_1^2}{N_2^2}\frac{\mathbf{V}_{\phi 2}}{3\mathbf{I}_{\phi 2}} = \frac{N_1^2}{N_2^2}\frac{Z_2}{3}$$

- If the actual connection is Y-Δ, then this turns ratio is

$$\frac{\mathbf{V}_{ab}}{\mathbf{V}_{a'b'}} = \frac{\sqrt{3}\mathbf{V}_{an}\angle 30^\circ}{\mathbf{V}_{\phi 2}} = \frac{\sqrt{3}\mathbf{V}_{\phi 1}\angle 30^\circ}{\mathbf{V}_{\phi 2}} = \frac{\mathbf{V}_{\phi 1}}{\mathbf{V}_{\phi 2}}\sqrt{3}\angle 30^\circ = \frac{N_1}{N_2}\sqrt{3}\angle 30^\circ$$

$$\frac{\mathbf{I}_a}{\mathbf{I}_{a'}} = \frac{\mathbf{I}_a}{\sqrt{3}\mathbf{I}_{a'b'}\angle -30^\circ} = \frac{\mathbf{I}_{\phi 1}}{\sqrt{3}\mathbf{I}_{\phi 2}\angle -30^\circ} = \frac{\mathbf{I}_{\phi 1}}{\mathbf{I}_{\phi 2}}\frac{\angle 30^\circ}{\sqrt{3}} = \frac{N_2}{N_1}\frac{\angle 30^\circ}{\sqrt{3}}$$

These are also the turns ratio of the equivalent Y-Y xfmr.

$$\frac{\mathbf{V}_{\phi 1}}{\mathbf{V}_{\phi 2Yeff}} = \frac{N_1}{N_2}\sqrt{3}\angle 30^\circ \Rightarrow \mathbf{V}_{\phi 1} = \frac{N_1}{N_2}\sqrt{3}\angle 30^\circ \mathbf{V}_{\phi 2Yeff}$$

$$\frac{\mathbf{I}_{\phi 1}}{\mathbf{I}_{\phi 2Yeff}} = \frac{N_2}{N_1}\frac{\angle 30^\circ}{\sqrt{3}} \Rightarrow \quad \mathbf{I}_{\phi 1} = \frac{N_2}{N_1}\frac{\angle 30^\circ}{\sqrt{3}}\mathbf{I}_{\phi 2Yeff}$$

$$Z_1 = \frac{\mathbf{V}_{\phi 1}}{\mathbf{I}_{\phi 1}} = \frac{\frac{N_1}{N_2}\sqrt{3}\angle 30^\circ \mathbf{V}_{\phi 2Yeff}}{\frac{N_2}{N_1}\frac{\angle 30^\circ}{\sqrt{3}}\mathbf{I}_{\phi 2Yeff}} = \frac{N_1^2}{N_2^2}\frac{3\mathbf{V}_{\phi 2Yeff}}{\mathbf{I}_{\phi 2Yeff}} = \frac{N_1^2}{N_2^2}3Z_2$$

Observe: Referring voltages and currents gets 30° phase shift and factor of √3; referring impedances does not get phase shift but does have a factor of 3.

Per-phase equivalent circuit for Y-Δ

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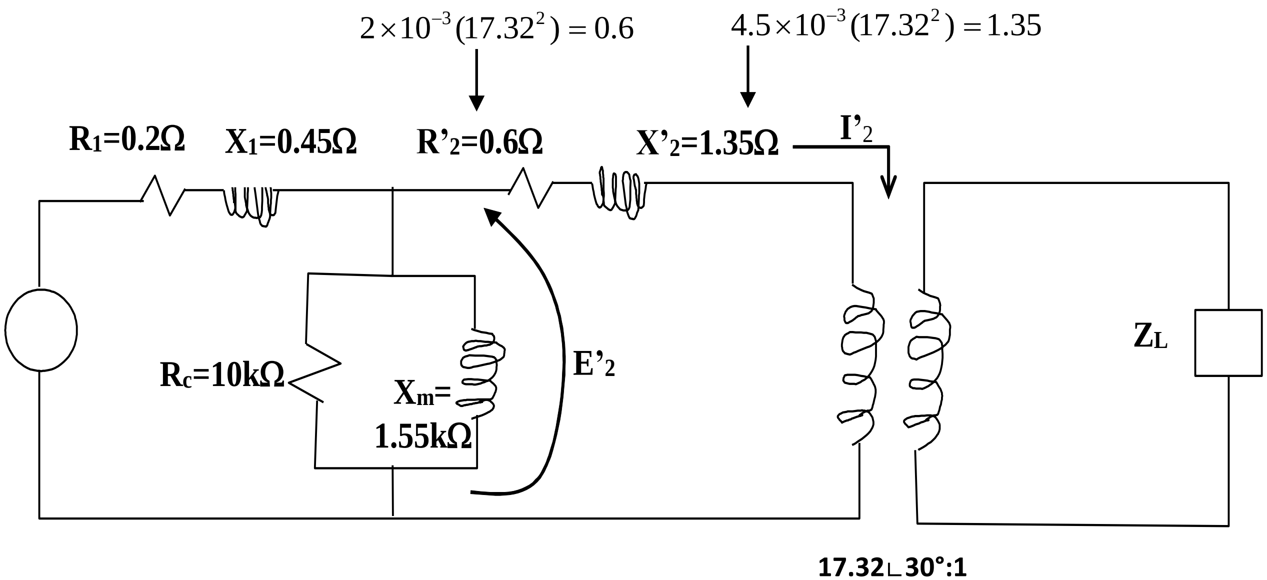
b. (5 pts) Label all impedance elements on the diagram with their ohmic value (use letters and numerical value).
c. (5 pts) Identify on the diagram the turns ratio $(N_1/N_2)_{eff}$ to be used in the per-phase circuit (identify numerical value).
d. (5 pts) Label the secondary current referred to the primary, $\mathbf{I}'_2$ (do not need numerical value, just location & directionality).
e. (5 pts) Label the voltage across the secondary winding, referred to the primary, $\mathbf{E}'_2$ (do not need numerical value, just location & directionality).

$$\frac{\boldsymbol{V}_{ab}}{\boldsymbol{V}_{a'b'}} = \frac{N_1}{N_2}\sqrt{3}\angle 30° = \frac{2400}{240}\sqrt{3}\angle 30° = 10\sqrt{3}\angle 30° = 17.32\angle 30°$$

Per-phase equivalent circuit for Y-Δ

(a, b, c, d, e)



Observe here that for referring impedances we need only the magnitude of the effective turns ratio (and not phase). Reason for this is shown on Slide 12.