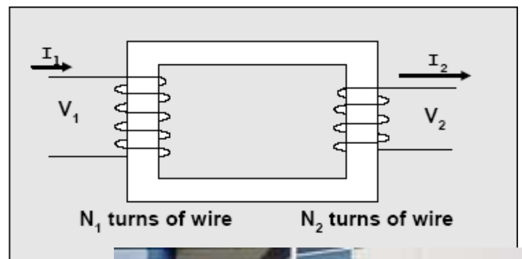
Power Transformers

1. Download/read notes on transformers from website.
2. Download HW4 on website; I will give due-date next week.
3. Read Chapters 5 and 6 in Kirtley's text



Power Industry Uses of Transformers

Instrument Transformers:

Current (CT) and potential (PT) transformers: Step down quantity from power system level (gen, trans, dist) so that quantity is compatible at the instrument level, in order to perform protective relaying (you need to take EE 457 to learn about these).

Power Transformers:

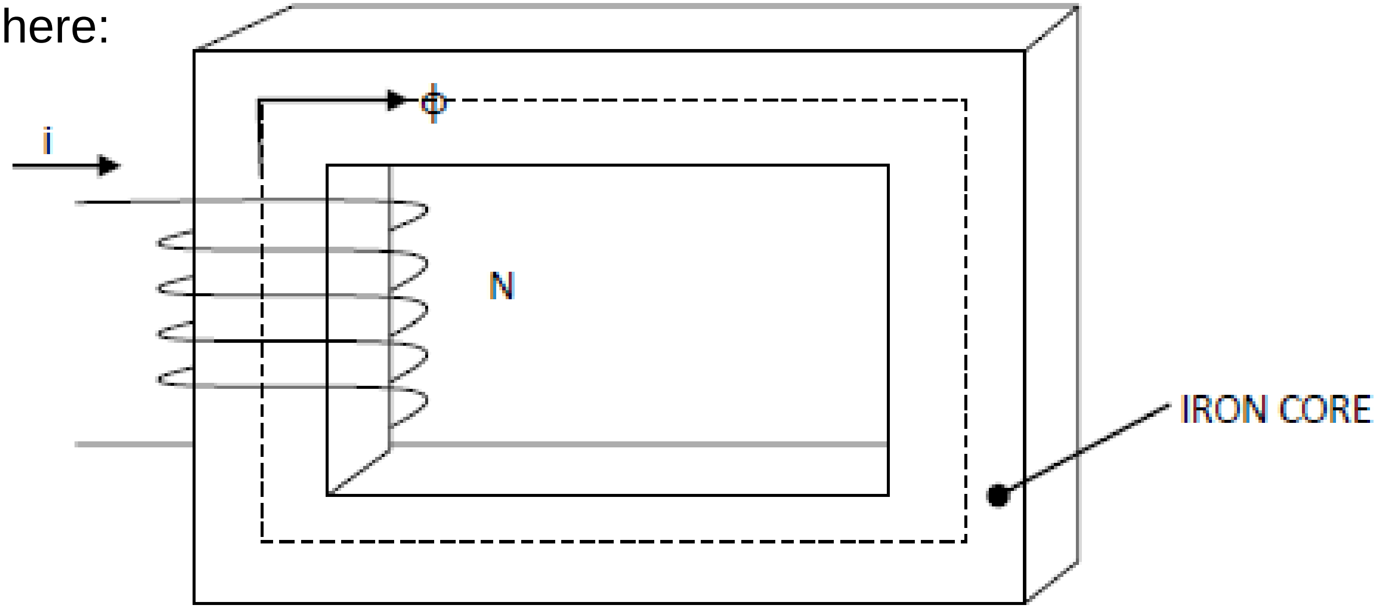
1. Step up voltage from generator to transmission (GSU)
2. Step down voltage from transmission to distribution primary levels
3. Step down voltage from distribution primary to distribution secondary
4. Interconnecting different system voltage levels in HV and EHV systems

Magnetic circuits

Ampere's Law: $\oint \vec{H} \bullet d\vec{L} = I$

➔Line integral of mag fld intensity about a closed path equals current enclosed.

Apply it here:



- Make the path the dotted line.
- H is along direction of ϕ, which is same direction as dL
- Let l be length of dotted path, therefore LHS is Hl
- RHS is current enclosed: Ni.

$$Hl = Ni$$

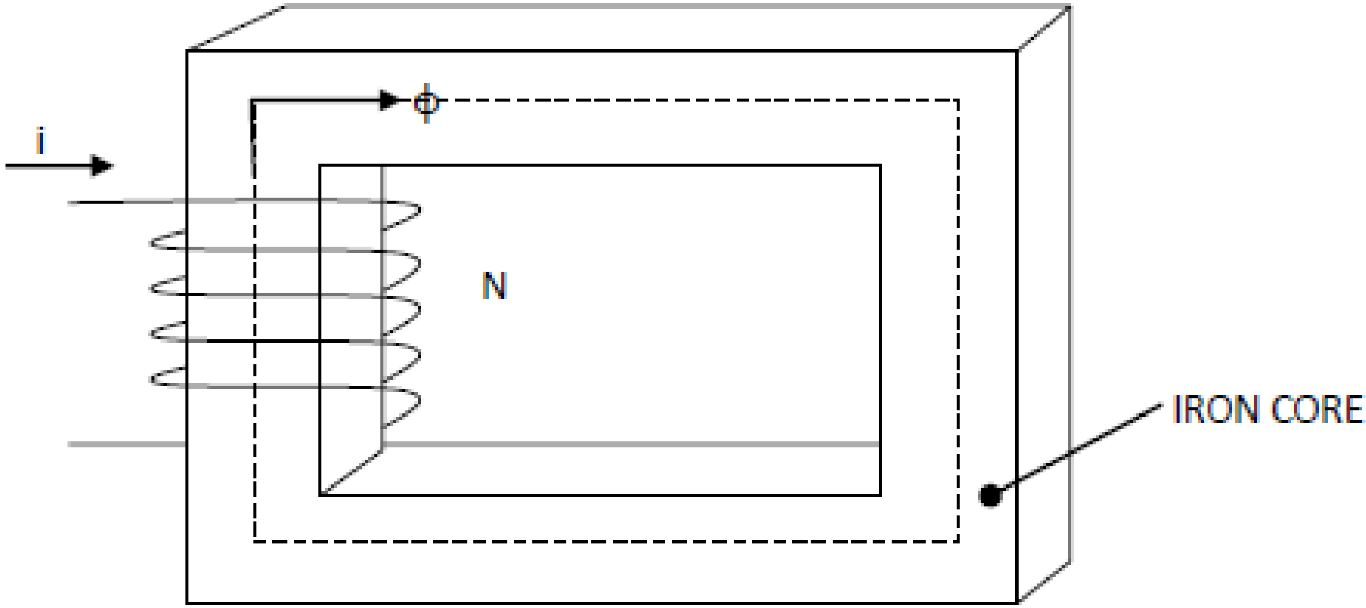
Magnetic circuits

$$Hl = Ni$$

Recall

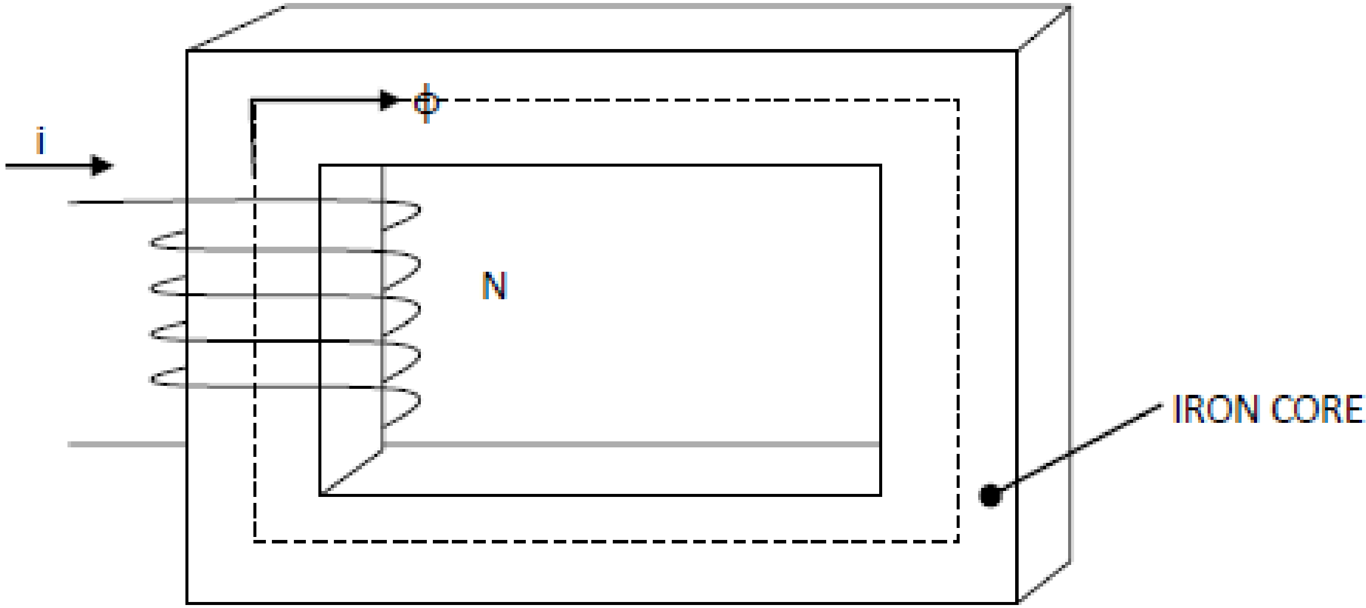
$$B = \mu H$$

$$\phi = BA$$



$$\Rightarrow \quad \frac{\phi}{\mu A} l = Ni \quad \Rightarrow \quad \phi = \frac{\mu A}{l} Ni$$

Magnetic circuits



$$\phi = \frac{\mu A}{l} Ni$$

Define: magnetomotive force (MMF): $\mathscr{F} = Ni$

reluctance: $\mathscr{R} = \frac{l}{\mu A}$

$$\Rightarrow \varphi = \frac{\mathscr{F}}{\mathscr{R}}$$

This looks suspiciously familiar. What does this remind you of?

Write down the relations for F, R, and ϕ.

Magnetic circuits

$$I = \frac{V}{R} \qquad \varphi = \frac{\mathcal{F}}{\mathcal{R}}$$

- I → ϕ (flux "flows" like current)
- V → F (MMF provides the "push" like voltage)
- R → R (Reluctance "resists" like resistance)

<u>**Units**</u>:

ϕ webers (kg-m^2/sec^2/Amp)

B webers/m^2=tesla

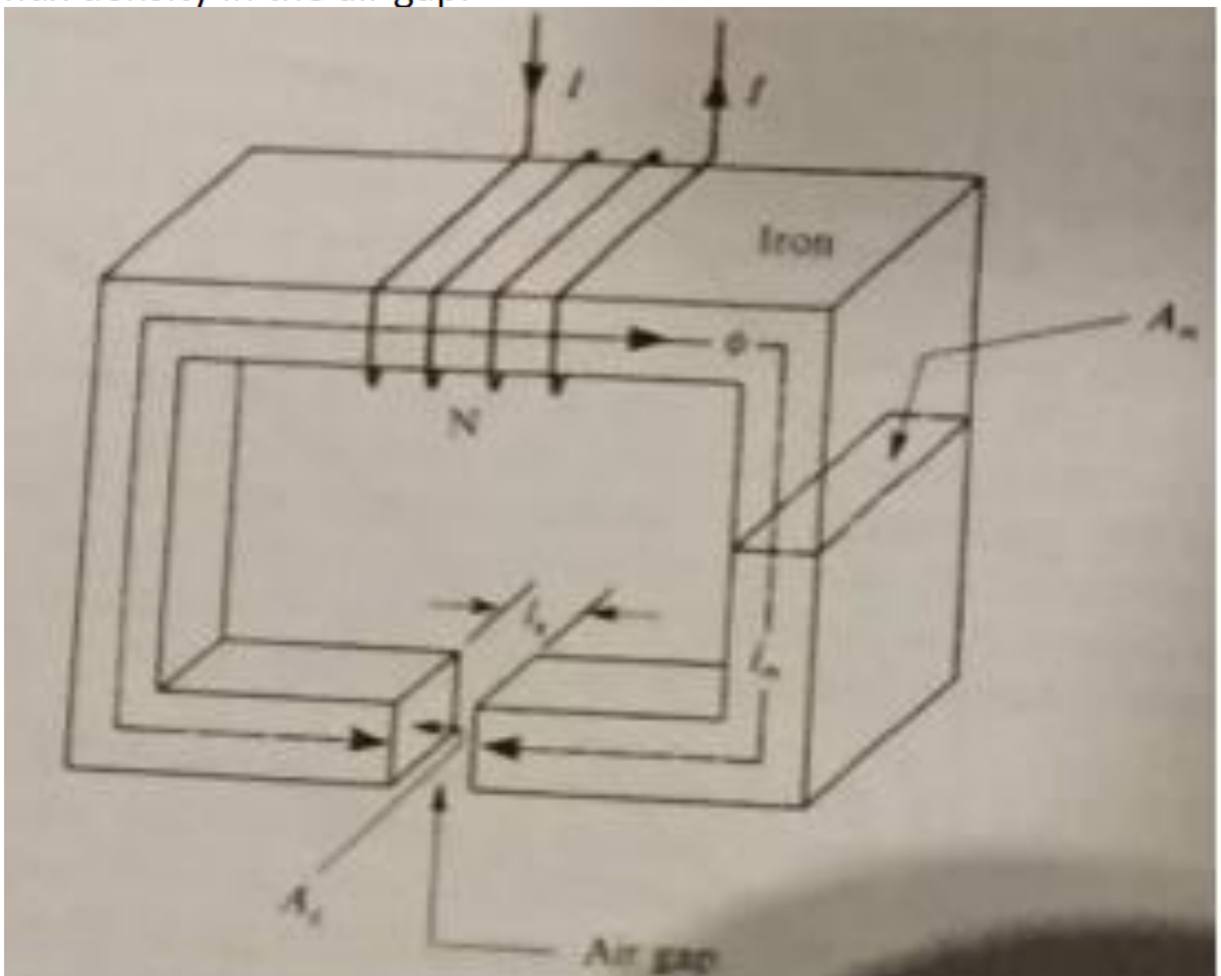
F ampere-turns

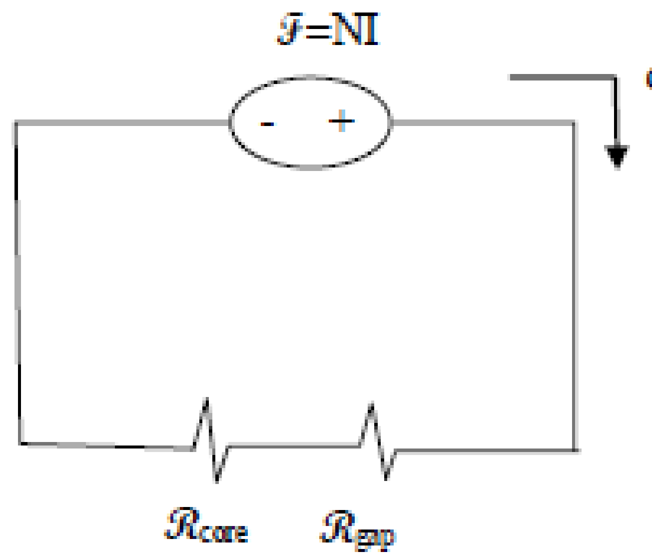
R amperes/weber

μ $= \mu_r\mu_0 = \mu_r(4\pi\times10^{-7})$Ntn/$Amp^2$

Example

Example 1 [1]: The magnetic circuit shown in the below figure has N=100 turns, a cross-section area of $A_m=A_g=40cm^2$, an air gap length of $l_g=0.5mm$, and a mean core length of $l_c=1.2m$. The relative permeability of the iron is $\mu_r=2500$. The current in the coil is $I_{DC}=7.8$ amperes. Determine the flux and flux density in the air gap.

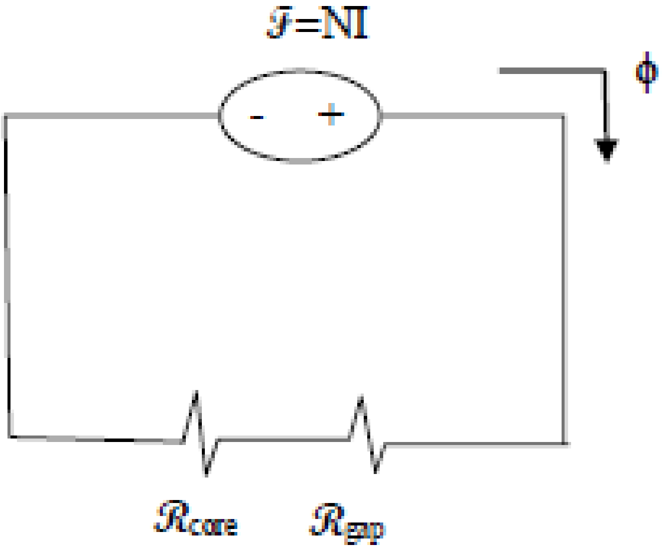




Compute ϕ.

Example

Example 1 [1]: The magnetic circuit shown in the below figure has N=100 turns, a cross-section area of $A_m=A_g=40cm^2$, an air gap length of $l_g=0.5mm$, and a mean core length of $l_c=1.2m$. The relative permeability of the iron is $\mu_r=2500$. The current in the coil is $I_{DC}=7.8$ amperes. Determine the flux and flux density in the air gap.



$$\varphi = \frac{\mathcal{F}}{\mathcal{R}}$$

$$\mathcal{F} = NI = 100 * 7.8 = 780 \text{ ampere-turns}$$

$$\mathcal{R}_{gap} = \frac{l_g}{\mu A_g} = \frac{0.5/1000}{(4\pi \times 10^{-7})(40/100^2)} = 99,472 \text{amperes/Weber}$$

$$\mathcal{R}_{core} = \frac{l_c}{\mu A_c} = \frac{1.2}{2500(4\pi \times 10^{-7})(40/100^2)} = 95,492 \text{amperes/Weber}$$

$$\varphi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{780}{99,472 + 95,492} = 0.004\text{Wb}$$

$$B = \frac{\varphi}{A} = \frac{0.004}{40/(100)^2}$$

$$= 1 \text{ Weber/m}^2 = 1\text{Tesla}$$

Inductance

Recall: $$\phi = \frac{\mu A}{l} N i$$

$$\Rightarrow \quad N\phi = \frac{\mu A}{l} N^2 i$$

Define:

Flux Linkage: $$\lambda = N\phi$$

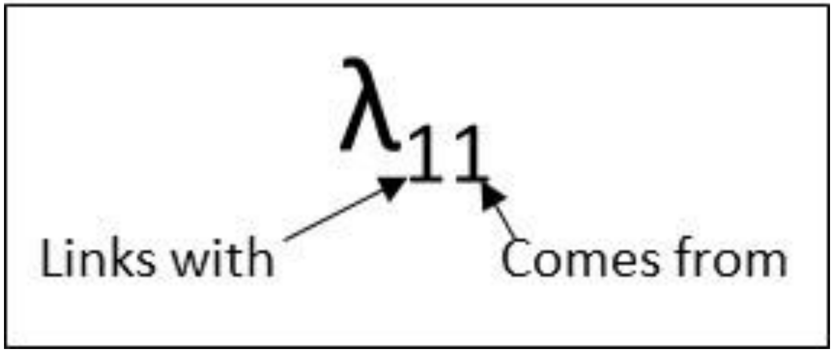
Self Inductance: $$L = \frac{\mu A N^2}{l} = \frac{N^2}{\mathcal{R}}$$

$$\Rightarrow \quad \lambda = Li$$

Inductance

Some notation that will be useful later:

$$L_{11} = \frac{\lambda_{11}}{i_1}$$

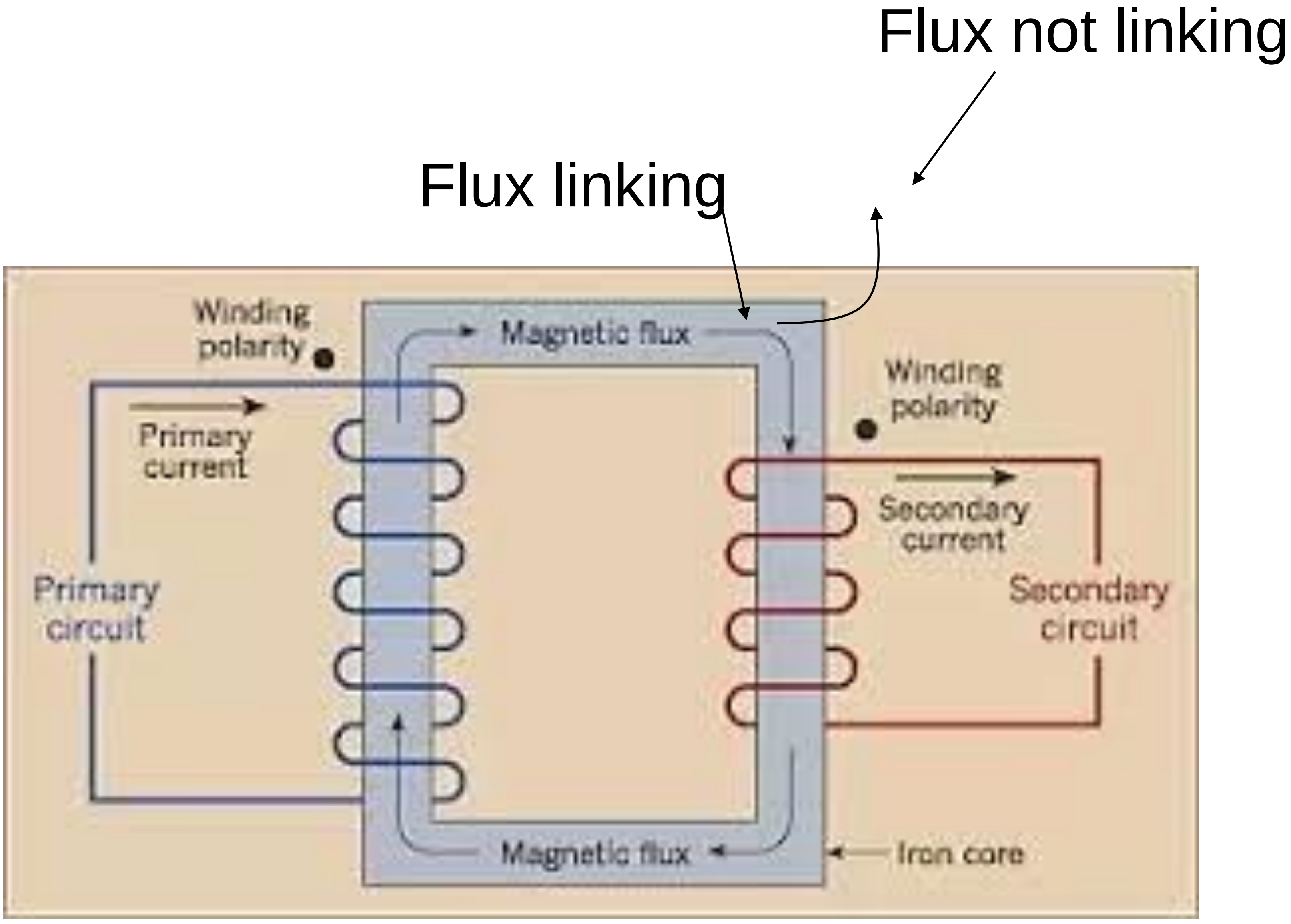


The self inductance L_{11} is the ratio of

- the flux from coil 1 linking with coil 1, λ_{11}
- to the current in coil 1, i_1

Linking? … Passing through the coil interior

Inductance

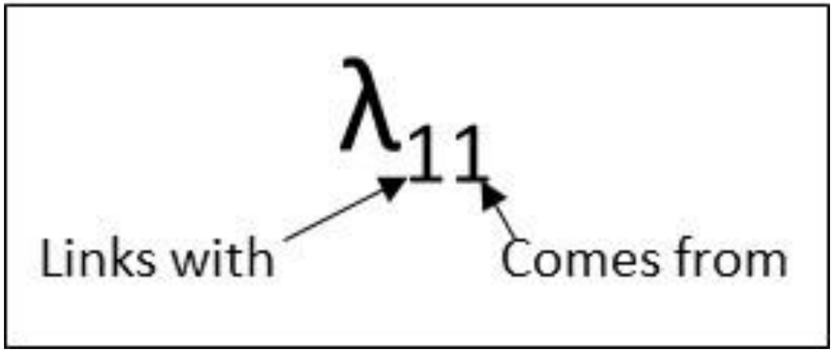


Flux not linking
Flux linking
Winding polarity
Magnetic flux
Primary current
Primary circuit
Winding polarity
Secondary current
Secondary circuit
Magnetic flux
Iron core

Inductance

Some notation that will be useful later:

$$L_{11} = \frac{\lambda_{11}}{i_1}$$



The self inductance L_{11} is the ratio of

- the flux from coil 1 linking with coil 1, λ_{11}
- to the current in coil 1, i_1

Inductance L_{11}? The ability of a current in coil 1, i_1, to create flux φ_{11} that links with coil 1.

Inductance

$$L_{11} = \frac{\mu A N_1^2}{l}$$

To make large self-inductance, we need to

- make N_1, μ, and A large;
- make l small

And so a large L_{11} results from

- many turns (N_1)
- large μ (core made of iron)
- large cross section (A)
- compact construction (small l)

And so a large L_{11} results from

- many turns (N_1)
- large μ (core made of iron)
- large cross section (A)
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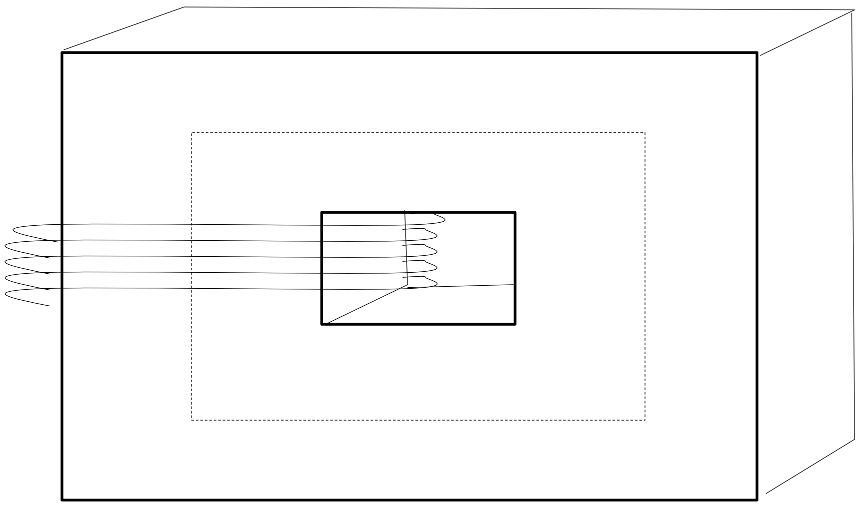


Recalling

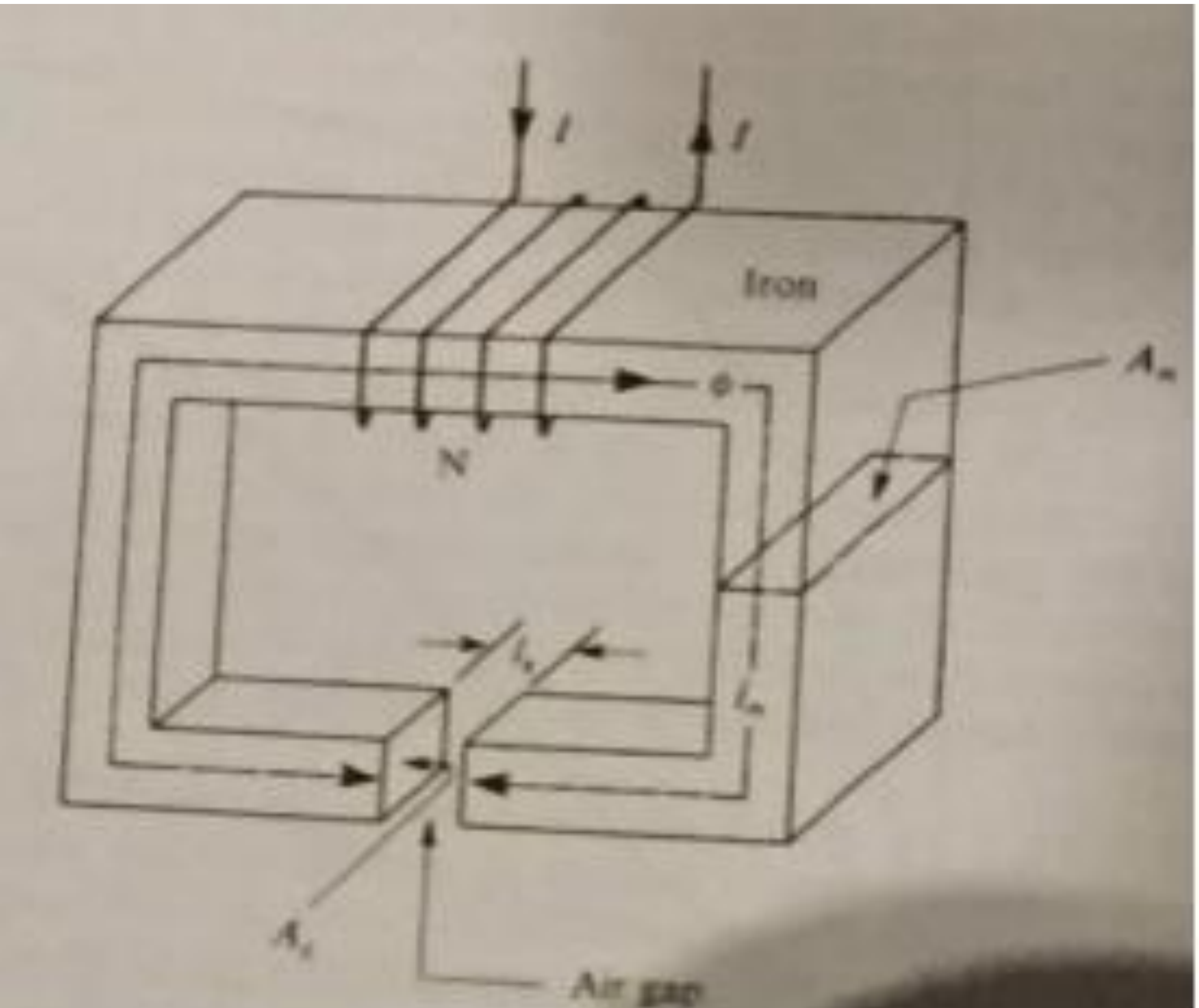
$$\mathcal{R} = \frac{l}{\mu A} \qquad L_{11} = \frac{\mu A N_1^2}{l}$$

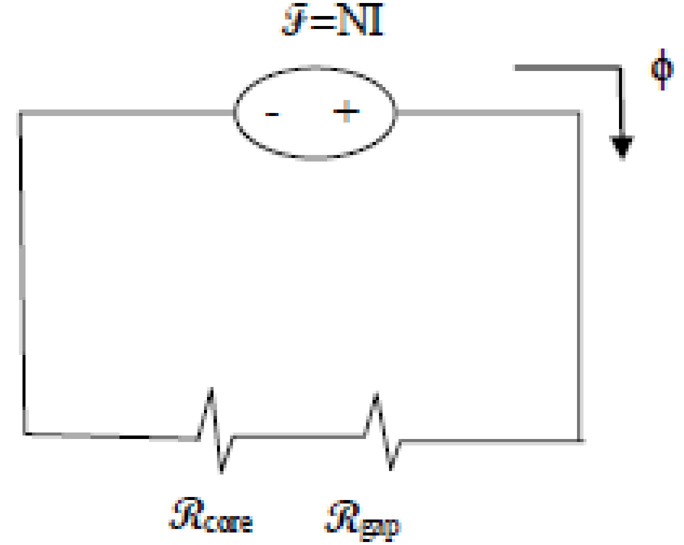
$$L_{11} = \frac{N_1^2}{\mathcal{R}}$$

we see that a magnetic circuit characterized by a large self-inductance will have a small magnetic path reluctance.

Example

Example 2: Compute the self-inductance of the magnetic circuit given in Ex 1.





We just found that

$$L_{11} = \frac{N_1^2}{\mathcal{R}}$$

Alternative derivation:

$$L_{11} = \frac{\lambda_{11}}{i_1} = \frac{N_1\phi_{11}}{i_1} = \frac{N_1\left(\frac{N_1 i_1}{\mathcal{R}_c + \mathcal{R}_g}\right)}{i_1}$$

$$= \frac{N_1^2}{\mathcal{R}_c + \mathcal{R}_g}$$

Example

Example 2: Compute the self-inductance of the magnetic circuit given in Ex 1.

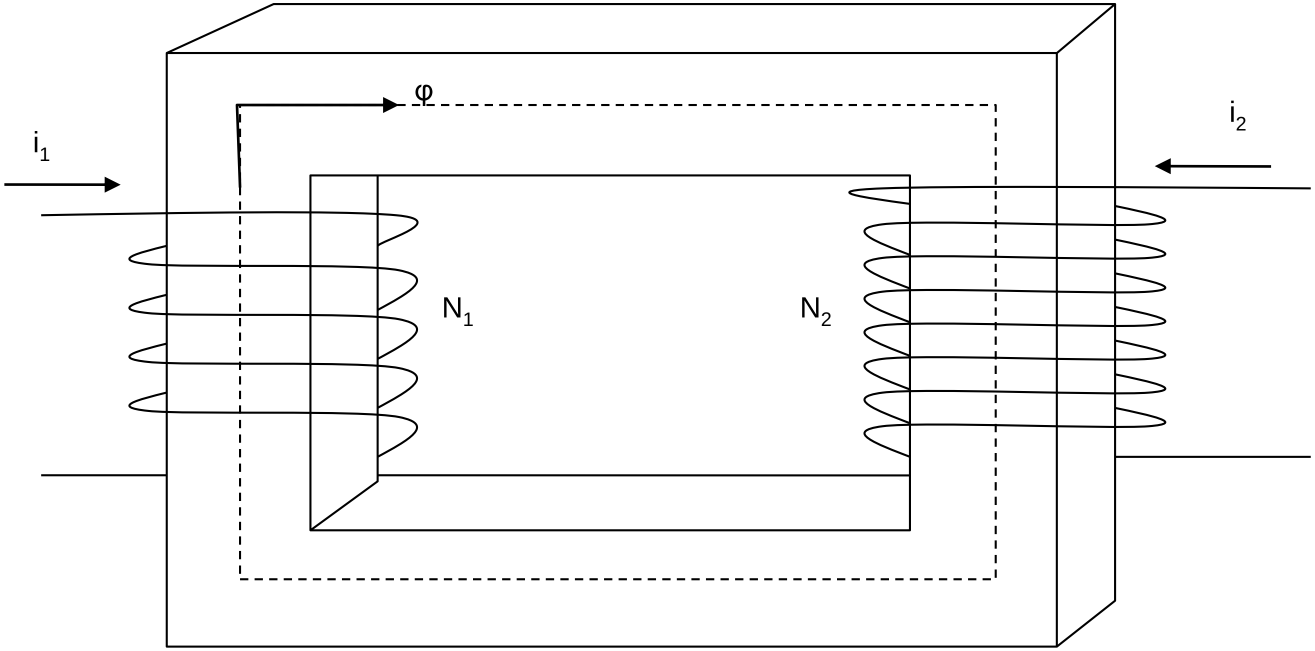
$$L_{11} = \frac{N_1^2}{\mathcal{R}} = \frac{N_1^2}{\mathcal{R}_c + \mathcal{R}_g}$$

From Ex 1, N_1=100 and

$\mathcal{R}_c = 95,492 \text{amperes/Weber}$ $\mathcal{R}_g = 99,472 \text{amperes/Weber}$

$$\Rightarrow L_{11} = \frac{100^2}{95492 + 99472} = 0.0513 \text{ henries}$$

Mutual inductance



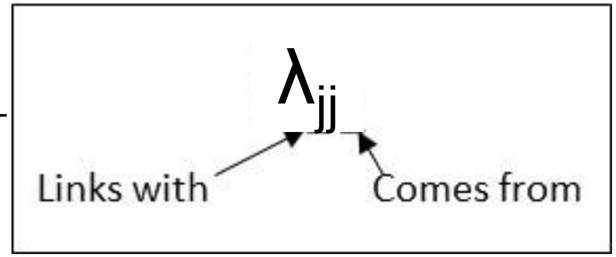
From our self-inductance work, we express for each coil

$$L_{11} = \frac{\lambda_{11}}{i_1} \qquad L_{22} = \frac{\lambda_{22}}{i_2}$$

where we recall the self inductance L_{jj} is the ratio of

- the flux from coil j linking with coil j, λ_{jj}
- to the current in coil j, i_j

Mutual inductance

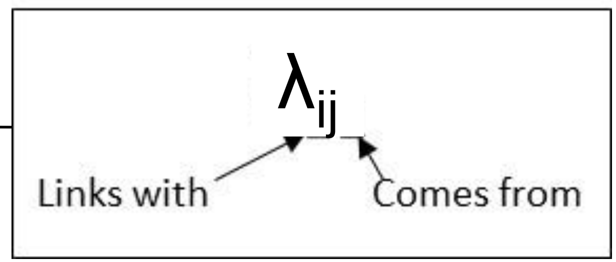


The self inductance L_{jj} is the ratio of

- the flux from coil j linking with coil j, λ_{jj}
- to the current in coil j, i_j

$$L_{11} = \frac{\lambda_{11}}{i_1}$$

$$L_{22} = \frac{\lambda_{22}}{i_2}$$

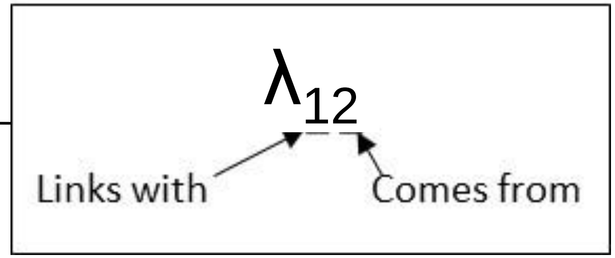


Likewise, mutual inductance L_{ij} is the ratio of

- the flux from coil j linking with coil i, λ_{ij}
- to the current in coil j, i_j

$$L_{ij} = \frac{\lambda_{ij}}{i_j}$$

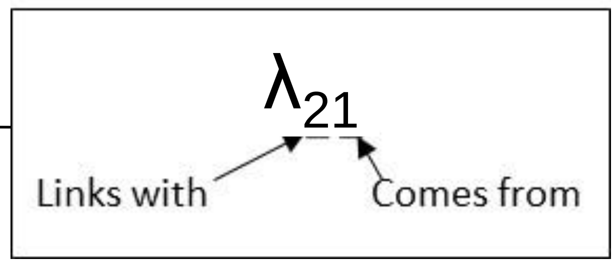
Mutual inductance



Mutual inductance L_{12} is the ratio of

- the flux from coil 2 linking with coil 1, λ_{12}
- to the current in coil 2, i_2

$$L_{12} = \frac{\lambda_{12}}{i_2}$$



Mutual inductance L_{21} is the ratio of

- the flux from coil 1 linking with coil 2, λ_{21}
- to the current in coil 1, i_1

$$L_{21} = \frac{\lambda_{21}}{i_1}$$

Mutual inductance

Recall that

$$\lambda = N\varphi \Rightarrow \lambda_{11} = N_1\varphi_{11}$$

Likewise

$$\lambda_{12} = N_1\varphi_{12}$$

$$\lambda_{21} = N_2\varphi_{21}$$

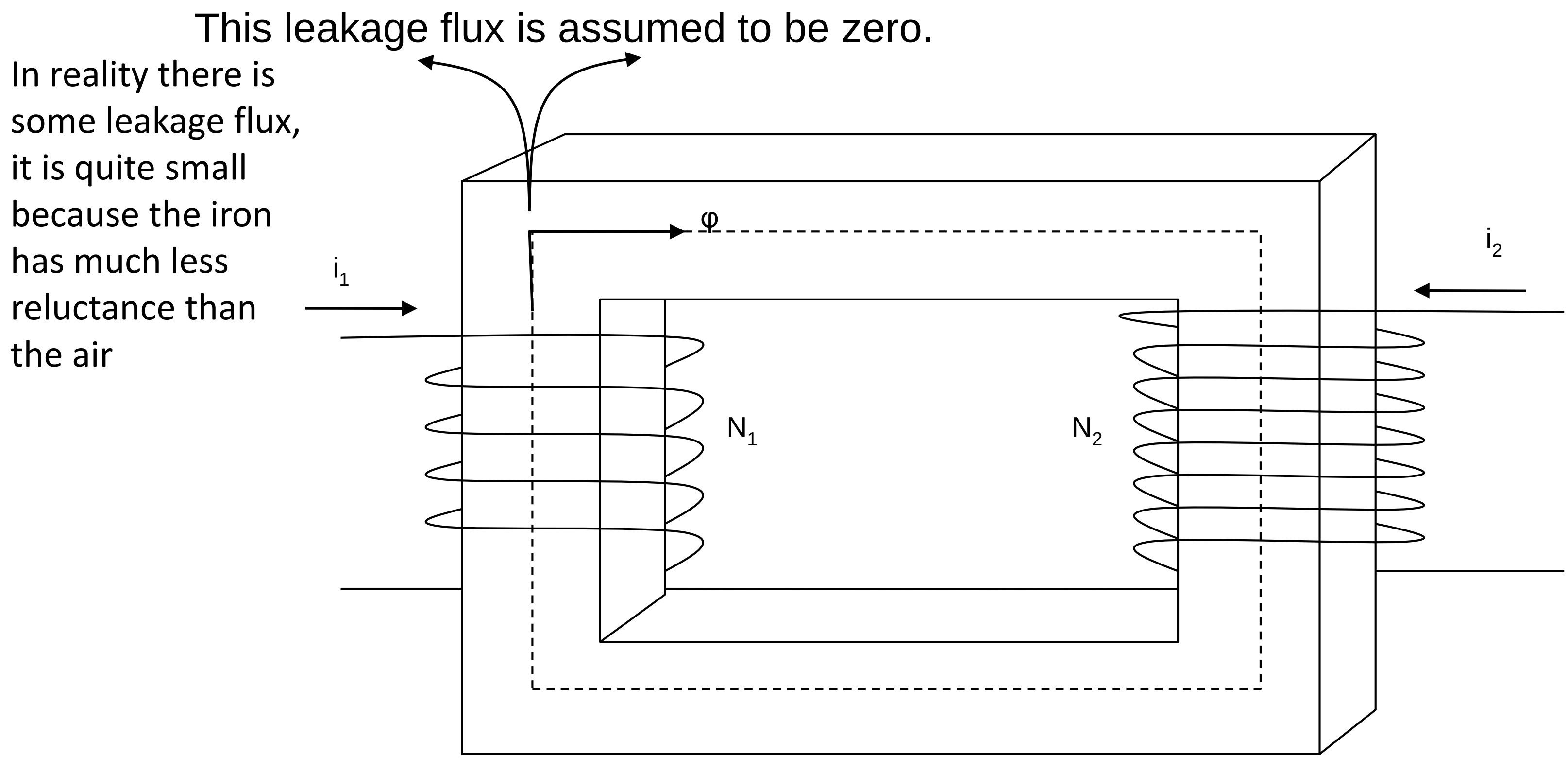
And the mutual inductances become

$$L_{12} = \frac{\lambda_{12}}{i_2} = \frac{N_1\varphi_{12}}{i_2}$$

$$L_{21} = \frac{\lambda_{21}}{i_1} = \frac{N_2\varphi_{21}}{i_1}$$

Mutual inductance

Assumption: All flux produced by each coil links with the other coil. This implies there is no leakage flux.



With no leakage flux, it must be the case that all flux developed by one coil must completely link with the other coil.

Mutual inductance

If all flux developed by one coil completely links with the other coil, then

- the flux from coil 2 linking with coil 1 is equal to the flux from coil 2 linking with coil 2, i.e.,

$$\phi_{12} = \phi_{22} = \frac{\mu A}{l} N_2 i_2$$

- the flux from coil 1 linking with coil 2 is equal to the flux from coil 1 linking with coil 1,

$$\phi_{21} = \phi_{11} = \frac{\mu A}{l} N_1 i_1$$

Substitute above into slide 20 inductance expressions:

$$L_{12} = \frac{N_1 \varphi_{12}}{i_2} = \frac{N_1 \frac{\mu A}{l} N_2 i_2}{i_2} = N_1 N_2 \frac{\mu A}{l} = \frac{N_1 N_2}{\mathcal{R}} \qquad L_{21} = \frac{N_2 \varphi_{21}}{i_1} = \frac{N_2 \frac{\mu A}{l} N_1 i_1}{i_1} = N_2 N_1 \frac{\mu A}{l} = \frac{N_2 N_1}{\mathcal{R}}$$

Mutual inductance

$$L_{12} = \frac{N_1 \varphi_{12}}{i_2} = \frac{N_1 \frac{\mu A}{l} N_2 i_2}{i_2} = N_1 N_2 \frac{\mu A}{l} = \frac{N_1 N_2}{\mathcal{R}} \qquad L_{21} = \frac{N_2 \varphi_{21}}{i_1} = \frac{N_2 \frac{\mu A}{l} N_1 i_1}{i_1} = N_2 N_1 \frac{\mu A}{l} = \frac{N_2 N_1}{\mathcal{R}}$$

Comparing the above leads to

$$L_{21} = L_{12} = \frac{N_1 N_2}{\mathcal{R}}$$

This says that the mutual inductances are reciprocal:

- The ratio of
  - flux from coil 2 linking with coil 1, λ_{12}, to i_2

$$L_{12} = \frac{\lambda_{12}}{i_2} = \frac{N_1 \varphi_{12}}{i_2}$$

- is the same as the ratio of
  - flux from coil 1 linking with coil 2, λ_{21}, to i_1

$$L_{21} = \frac{\lambda_{21}}{i_1} = \frac{N_2 \varphi_{21}}{i_1}$$

Mutual inductance

We showed on slide 14 that

$$L_{11} = \frac{N_1^2}{\mathscr{R}} \qquad L_{22} = \frac{N_2^2}{\mathscr{R}}$$

Solve for N_1 and N_2:

$$N_1 = \sqrt{L_{11}\mathscr{R}} \qquad N_2 = \sqrt{L_{22}\mathscr{R}}$$

Substitute into slide 22 mutual inductance expression:

$$L_{21} = L_{12} = \frac{N_1 N_2}{\mathscr{R}} = \frac{\sqrt{L_{11}\mathscr{R}}\sqrt{L_{22}\mathscr{R}}}{\mathscr{R}} = \sqrt{L_{11}L_{22}}$$

Mutual inductance

It is conventional to denote mutual inductance as M:

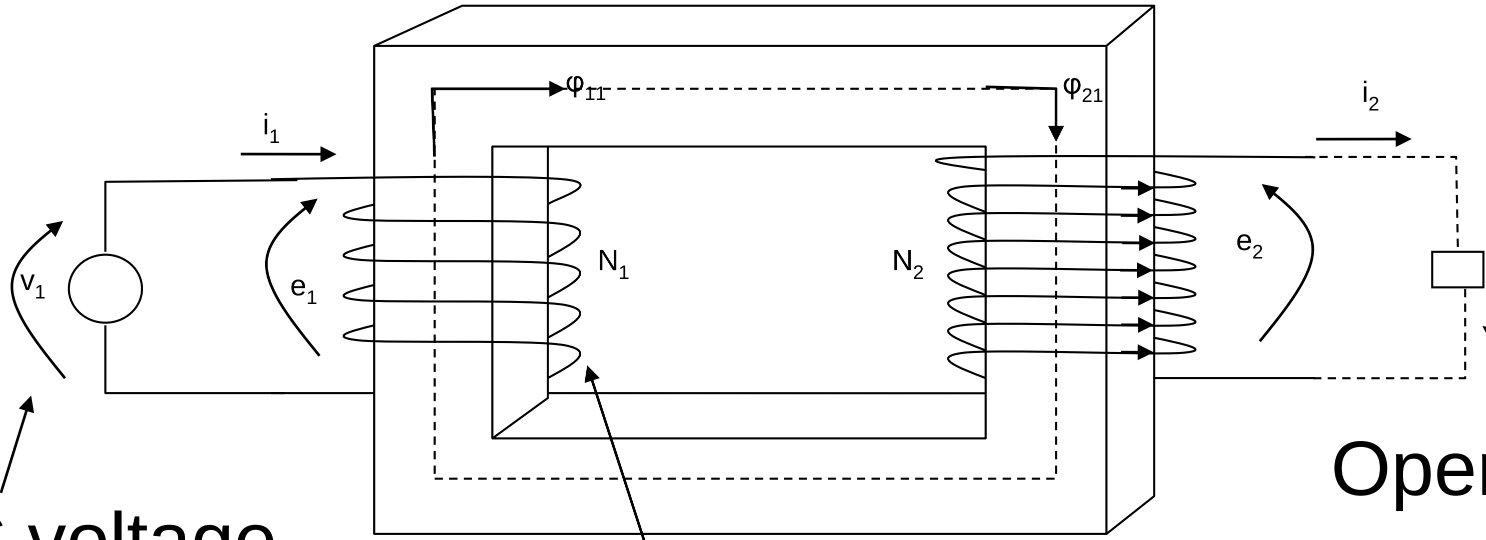
$$M = L_{21} = L_{12} = \frac{N_1 N_2}{\mathcal{R}} = \frac{\sqrt{L_{11}\mathcal{R}}\sqrt{L_{22}\mathcal{R}}}{\mathcal{R}} = \sqrt{L_{11}L_{22}}$$

Mutual inductance gives the ratio of:

- flux from coil k linking with coil j, λ_{jk}
- to the current in coil k, i_k,

$$M = \begin{cases} \dfrac{\lambda_{12}}{i_2} \\ \dfrac{\lambda_{21}}{i_1} \end{cases}$$

Polarity & dot convention for coupled ccts



DC voltage

Open cct

We have a dial to increase v_1

Coil 1: very small resistance so in steady-state, $i_1 \neq \infty$

Increase voltage v_1 to some higher value

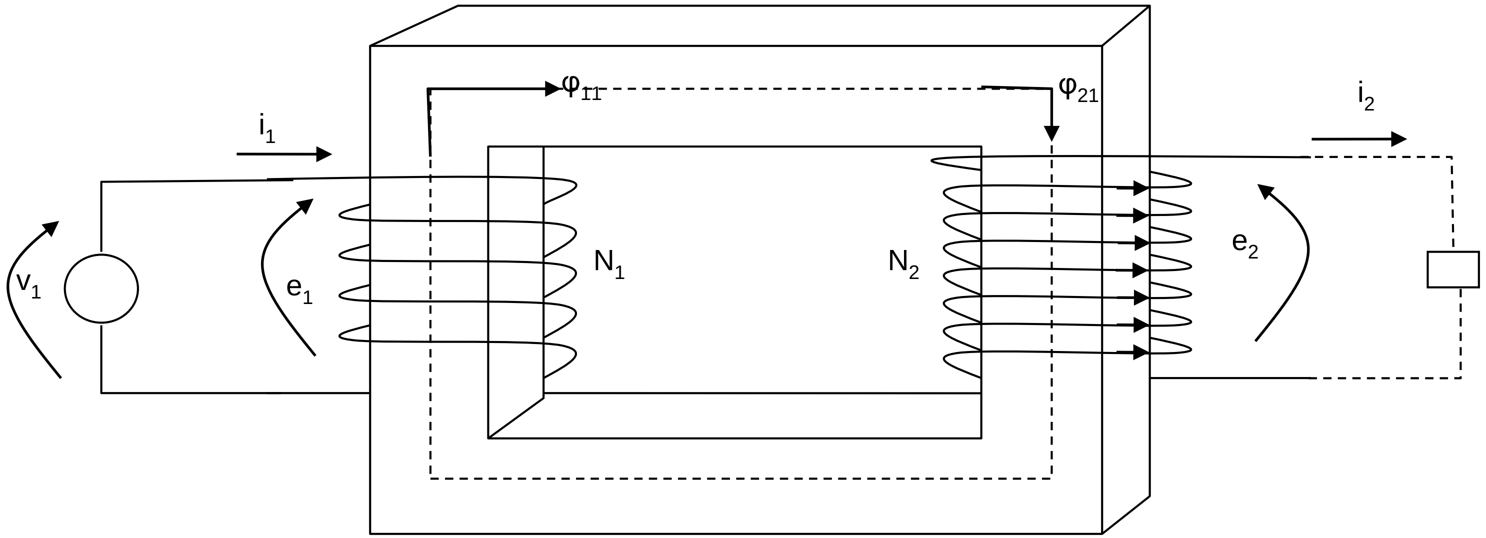
➔ current i_1 increases with time

➔ flux from coil 1, ϕ_{11}, increases with time

➔ flux linkages λ_{11} increases with time.

Who speaks when you have $d\lambda/dt$ $(=d(N\phi)/dt) \neq 0$?

Polarity & dot convention for coupled ccts



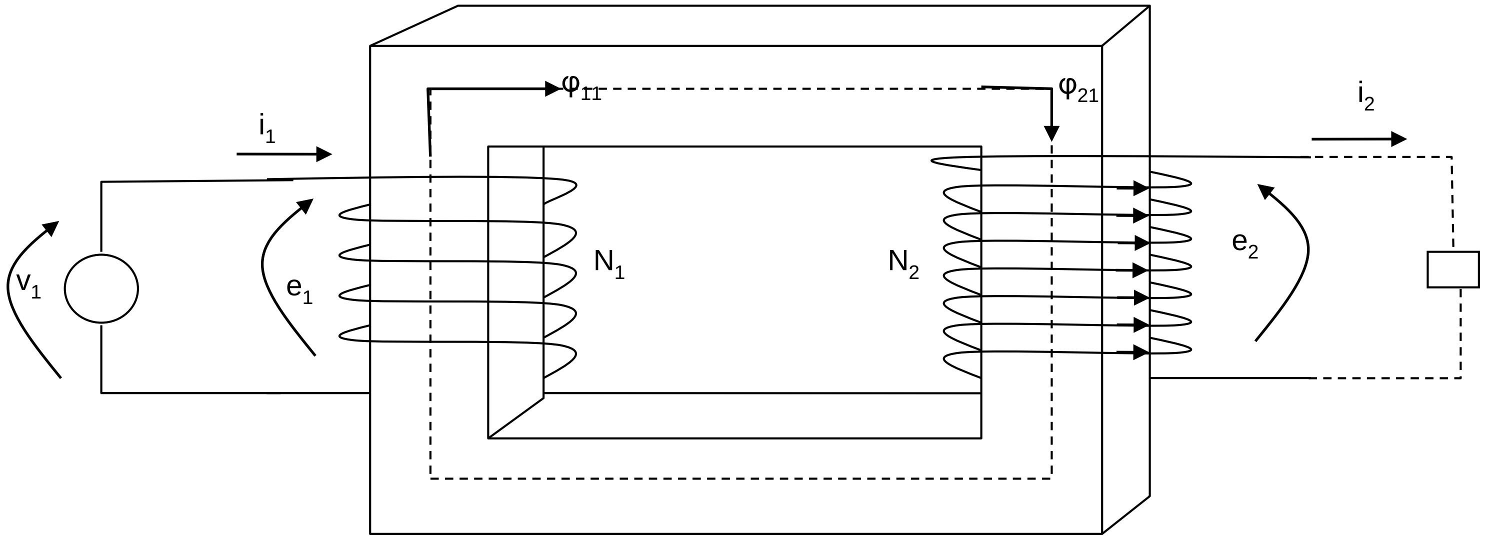
Who speaks when you have dλ/dt (=d(Nϕ)/dt)≠0?

Faraday! ➔ $$e_1 = N_1 \frac{d\phi_{11}}{dt} = \frac{d(N_1\phi_{11})}{dt} = \frac{d\lambda_{11}}{dt} = L_{11}\frac{di_1}{dt}$$

But what about the sign of the right-hand-side (RHS)?

Is it positive or negative?

Polarity & dot convention for coupled ccts



But what about the sign of the right-hand-side (RHS)?

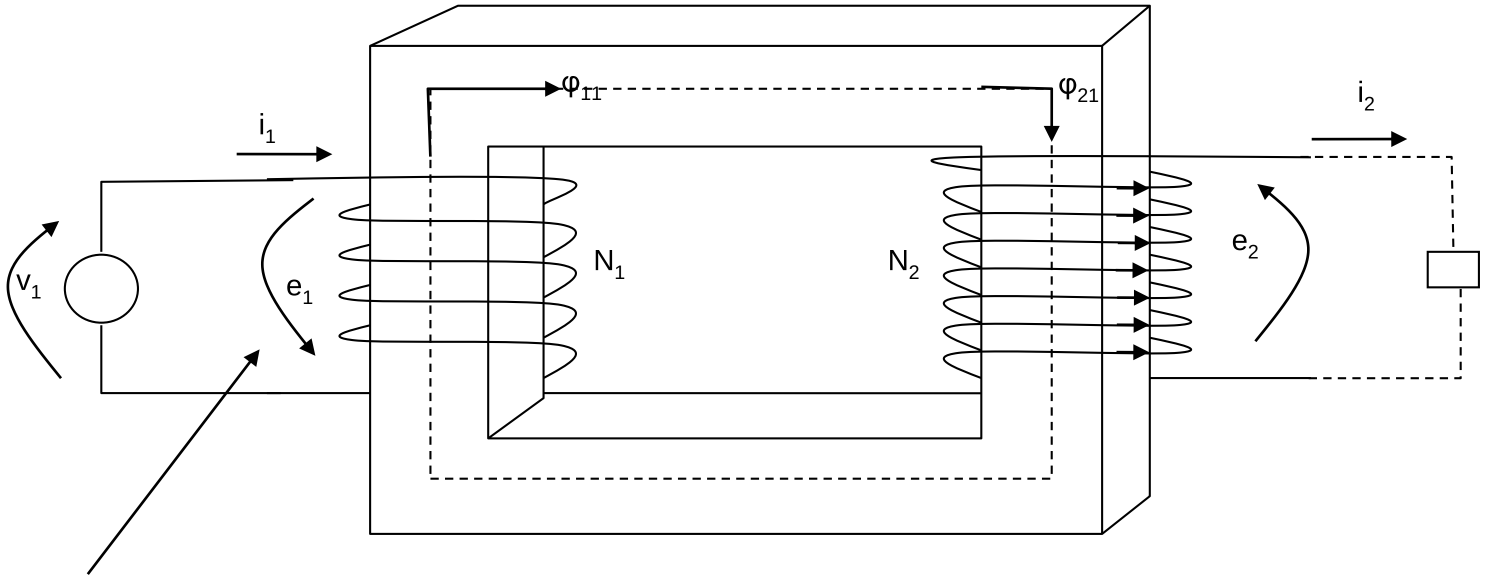
Is it positive or negative?

The sign of RHS is positive because self-induced voltage across a coil is always positive at terminal the current enters, & e_1 is defined positive at this terminal.

$$e_1 = +L_{11}\frac{di_1}{dt}$$

If e_1 would have been defined negative at terminal in which the current entered, then sign of RHS would be negative.

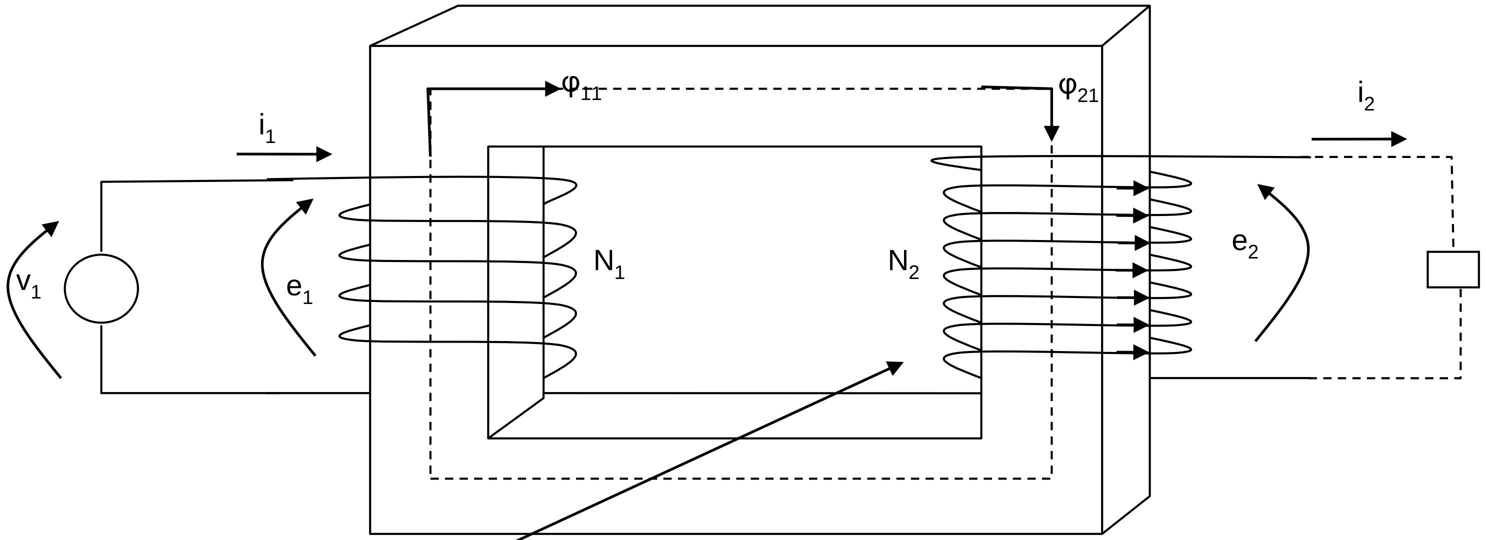
Polarity & dot convention for coupled ccts



If e_1 would have been defined negative at terminal in which the current entered, then sign of RHS would be negative.

$$e_1 = -L_{11}\frac{di_1}{dt}$$

Polarity & dot convention for coupled ccts

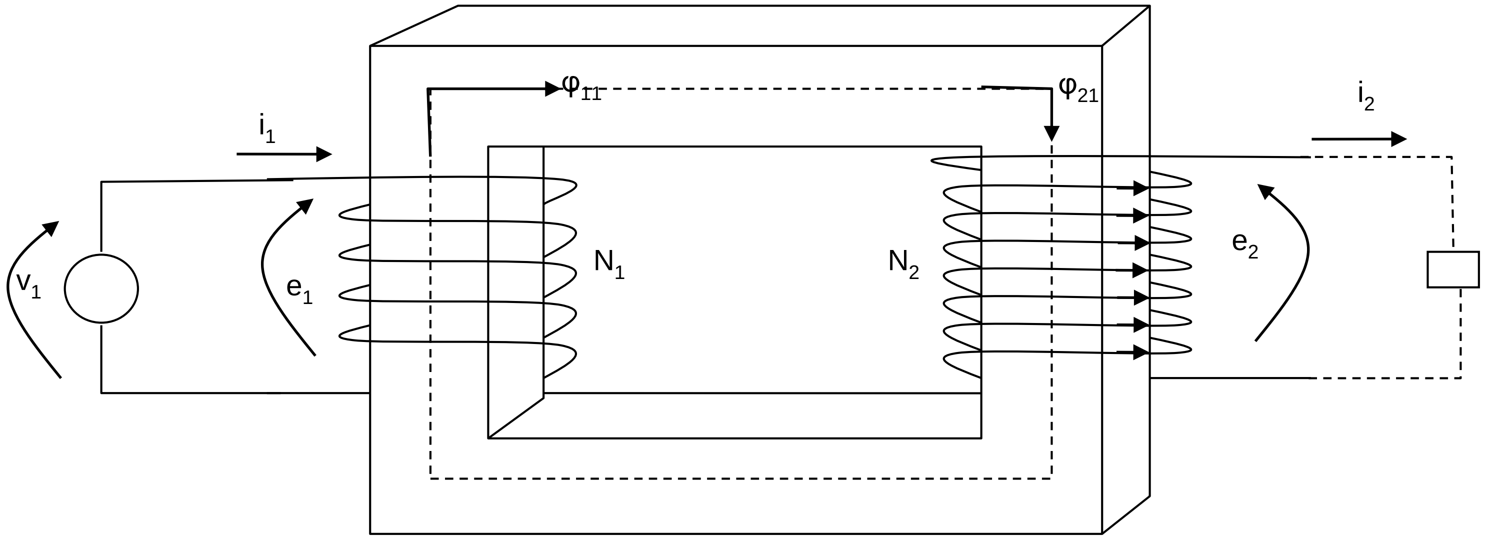


Now consider coil 2… it sees same flux that coil 1 sees which we denote by ϕ_{21} (and correspondingly, the flux linkages are denoted as λ_{21}). Considering our action of using the dial to increase v_1, we again have, by Faraday's Law,

$$e_2 = N_2 \frac{d\phi_{21}}{dt} = \frac{d(N_2\phi_{21})}{dt} = \frac{d\lambda_{21}}{dt} = L_{21}\frac{di_1}{dt} = M\frac{di_1}{dt}$$

Is the sign of RHS positive or negative?

Polarity & dot convention for coupled ccts



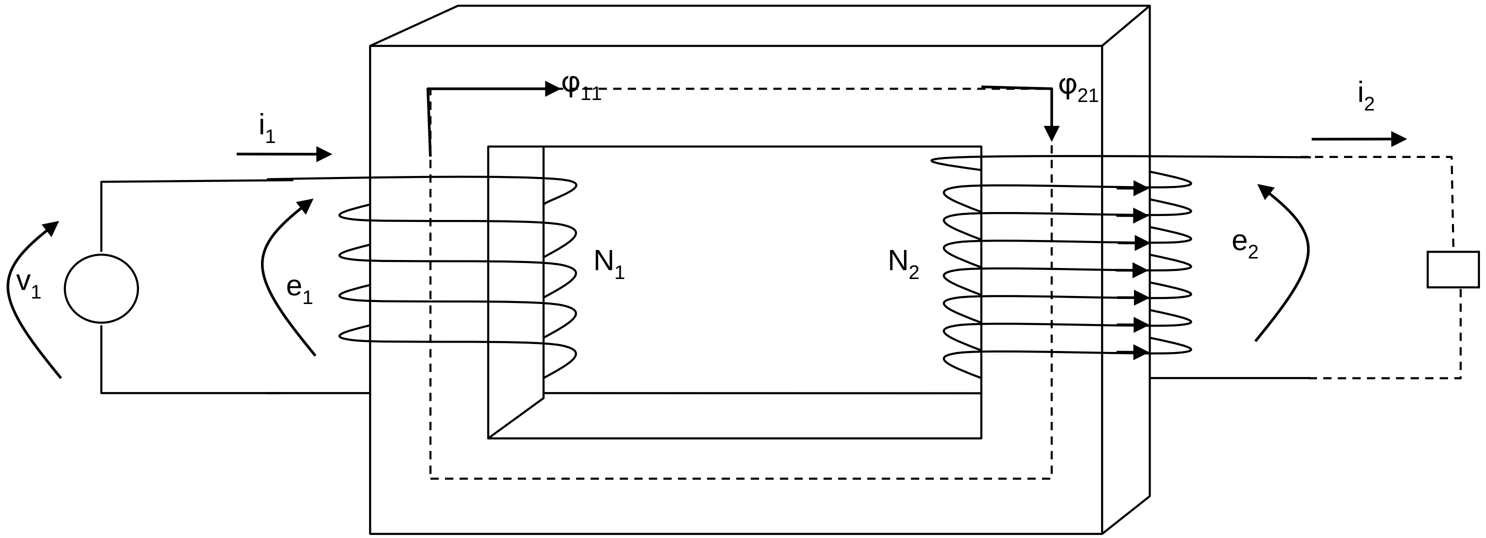
$$e_2 = N_2 \frac{d\phi_{21}}{dt} = \frac{d(N_2\phi_{21})}{dt} = \frac{d\lambda_{21}}{dt} = L_{21}\frac{di_1}{dt} = M\frac{di_1}{dt}$$

Is the sign of RHS positive or negative?

That is, how do we know which of below are correct?

$$e_2 = +M\frac{di_1}{dt} \qquad\qquad e_2 = -M\frac{di_1}{dt}$$

Polarity & dot convention for coupled ccts



That is, how do we know which of below are correct?

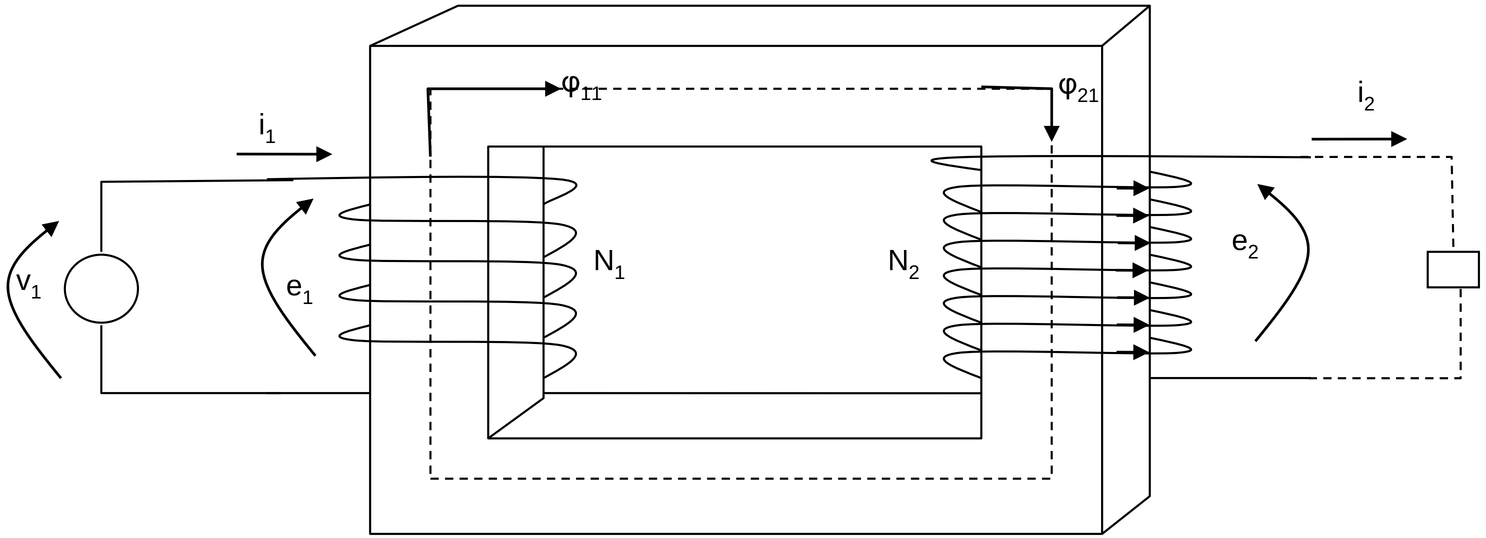
$$e_2 = +M\frac{di_1}{dt} \qquad\qquad e_2 = -M\frac{di_1}{dt}$$

Alternatively: Does assumed e_2 polarity match actual polarity of voltage induced by changing current i_1?

If yes, we choose positive sign.

If not, we choose negative sign.

Polarity & dot convention for coupled ccts



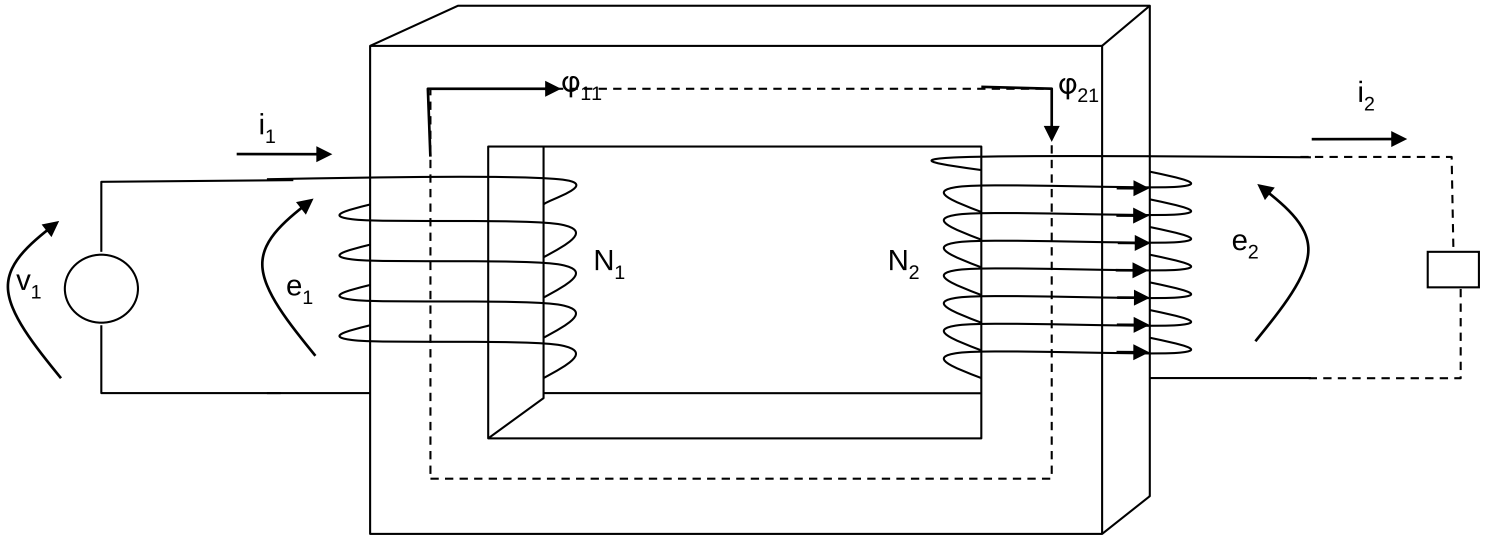
That is, how do we know which of below are correct?

$$e_2 = +M\frac{di_1}{dt} \qquad\qquad e_2 = -M\frac{di_1}{dt}$$

Use Lenz's Law: induced voltage e_2 must be in a direction so as to establish a current in a direction to produce a flux opposing the change in flux that produced e_2.

See www.khanacademy.org/science/physics/magnetic-forces-and-magnetic-fields/magnetic-flux-faradays-law/v/lenzs-law

Polarity & dot convention for coupled ccts

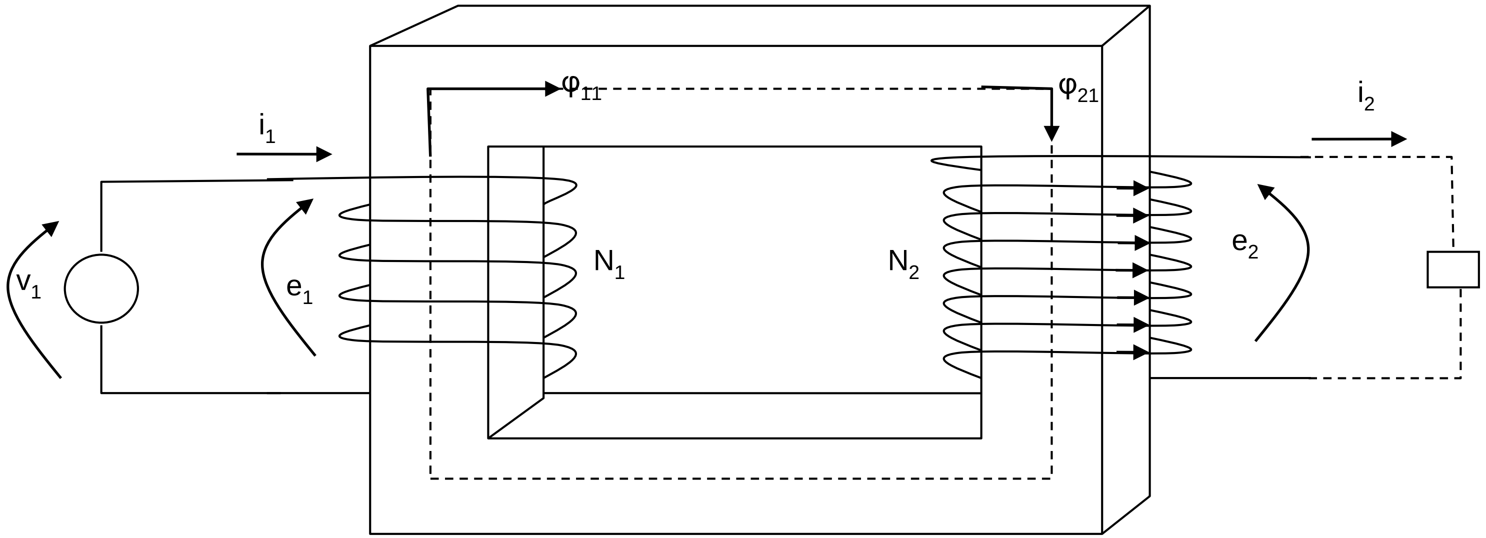


When e_1 increases, i_1 increases, and by the right-hand-rule (RHR), ϕ_{21} increases.

Assumed polarity of e_2 causes current to flow into the load in direction shown. How do we know e_2 polarity is correct?

Use Lenz's Law: induced voltage e_2 must be in a direction so as to establish a current in a direction to produce a flux opposing the change in flux that produced e_2.

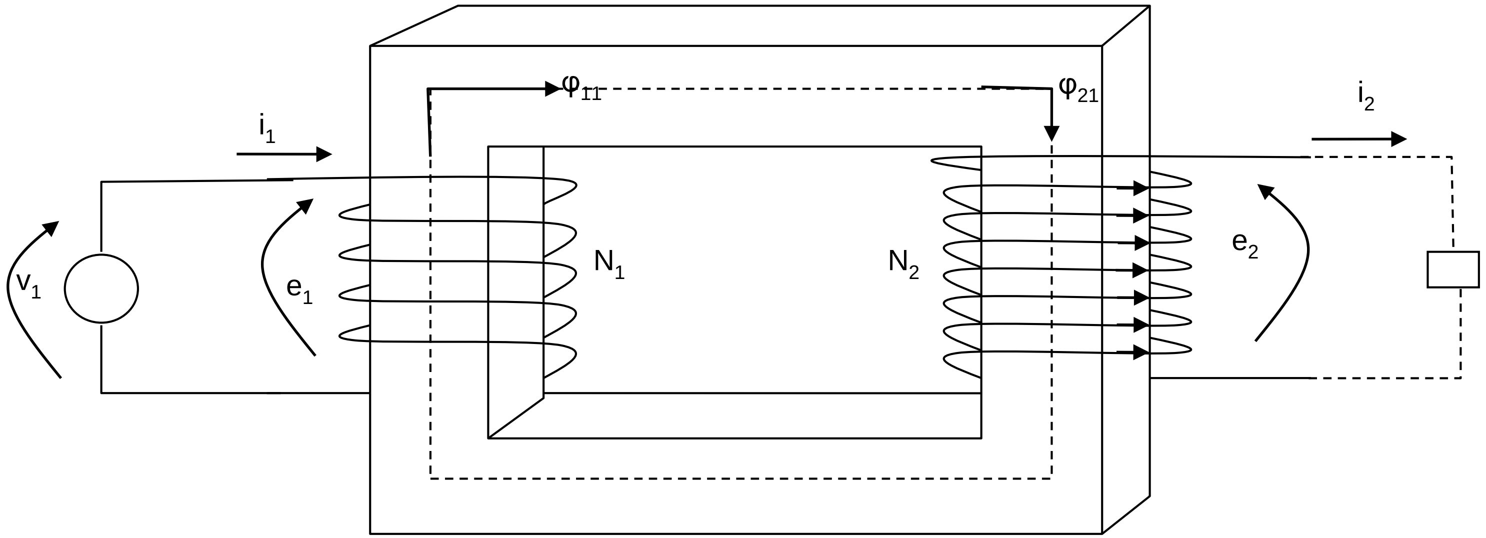
Polarity & dot convention for coupled ccts



We know e_2 polarity is correct because RHR says that a current in direction of i_2 causes flux in direction opposite to the direction of *the* ϕ_{21} *increase*.

This is "*the* ϕ_{21} *increase*," i.e., it is "the change in flux that produced e_2" and not necessarily the direction of ϕ_{21} itself (in this particular case, "*the* ϕ_{21} *increase*" is the same as the direction of ϕ_{21} itself).

Polarity & dot convention for coupled ccts

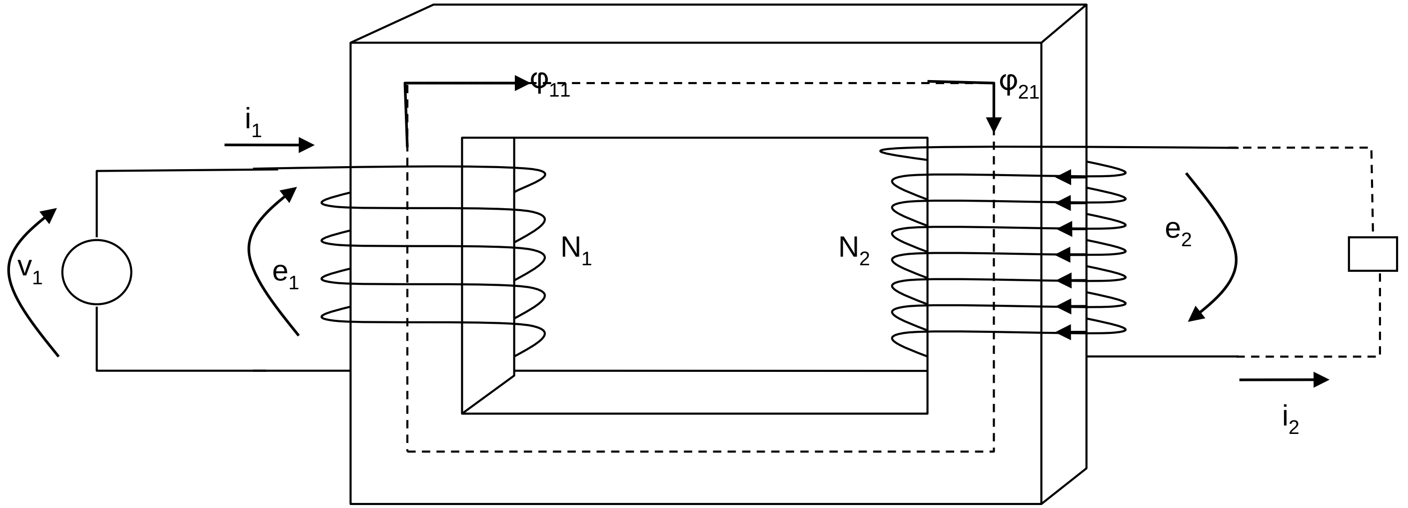


We know e_2 polarity is correct because RHR says that a current in direction of i_2 causes flux in direction opposite to the direction of *the* ϕ_{21} *increase*.

$$\Rightarrow \quad e_2 = +M\frac{di_1}{dt}$$

Question: How might we obtain a different answer?

Polarity & dot convention for coupled ccts



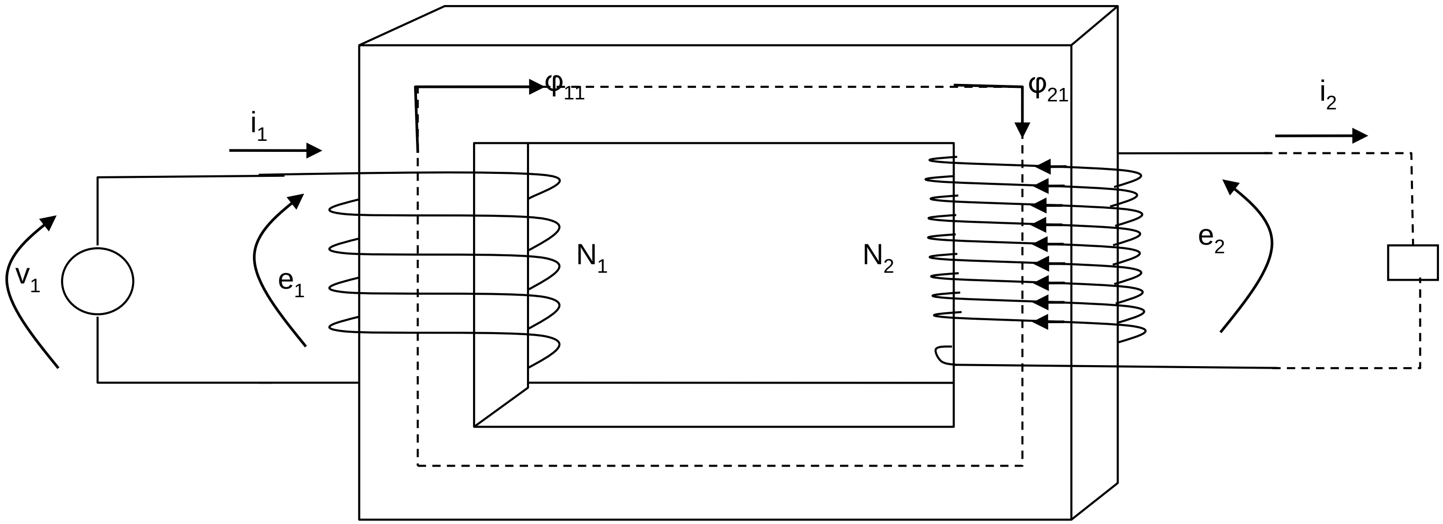
There are two ways.

First way: Switch sign of e_2, as above. Here, we also must switch current i_2 direction, because, in using Lenz's Law, the i_2 direction must be consistent with the e_2 direction.

Here, the current i_2, by RHR, produces a flux in the same direction as the ϕ_{21} increase, in violation of Lenz's Law:

$$e_2 = -M \frac{di_1}{dt}$$

Polarity & dot convention for coupled ccts



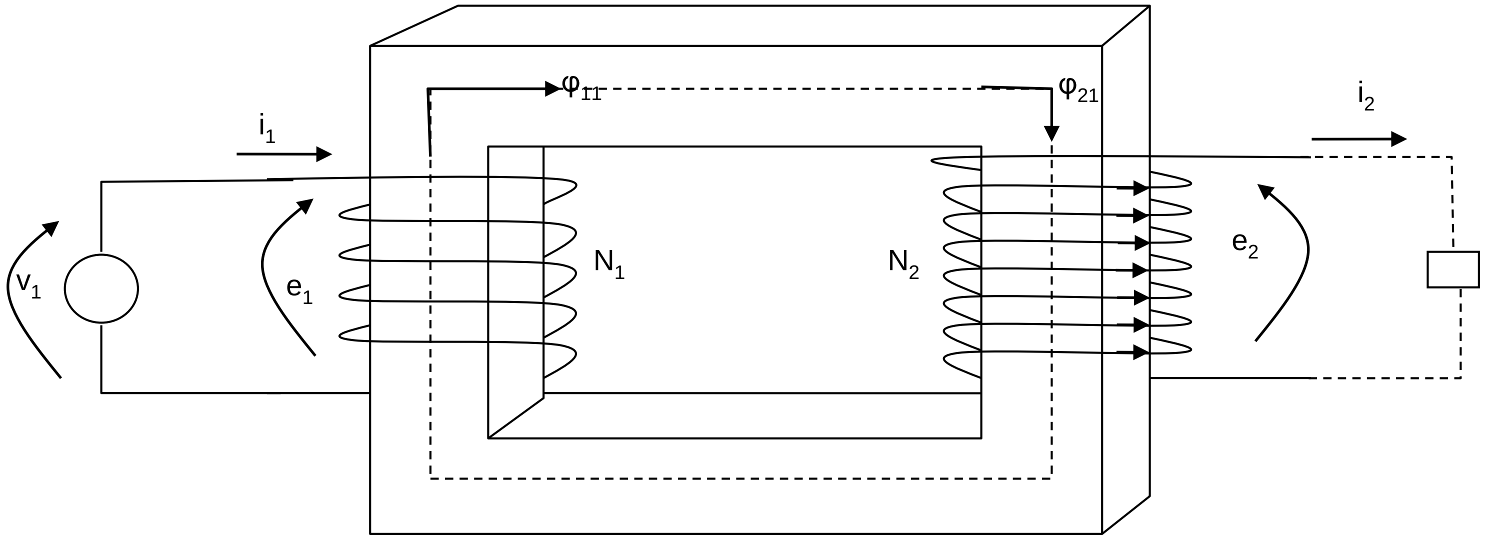
There are two ways.

<u>Second way</u>: Switch the sense of the coil 2 wrapping, while keeping the directions of e_2 and i_2 as they were originally.

The current i_2, by RHR, produces flux in same direction as the ϕ_{21} increase. Therefore

$$e_2 = -M\frac{di_1}{dt}$$

Polarity & dot convention for coupled ccts



Let's articulate what we are trying to do:

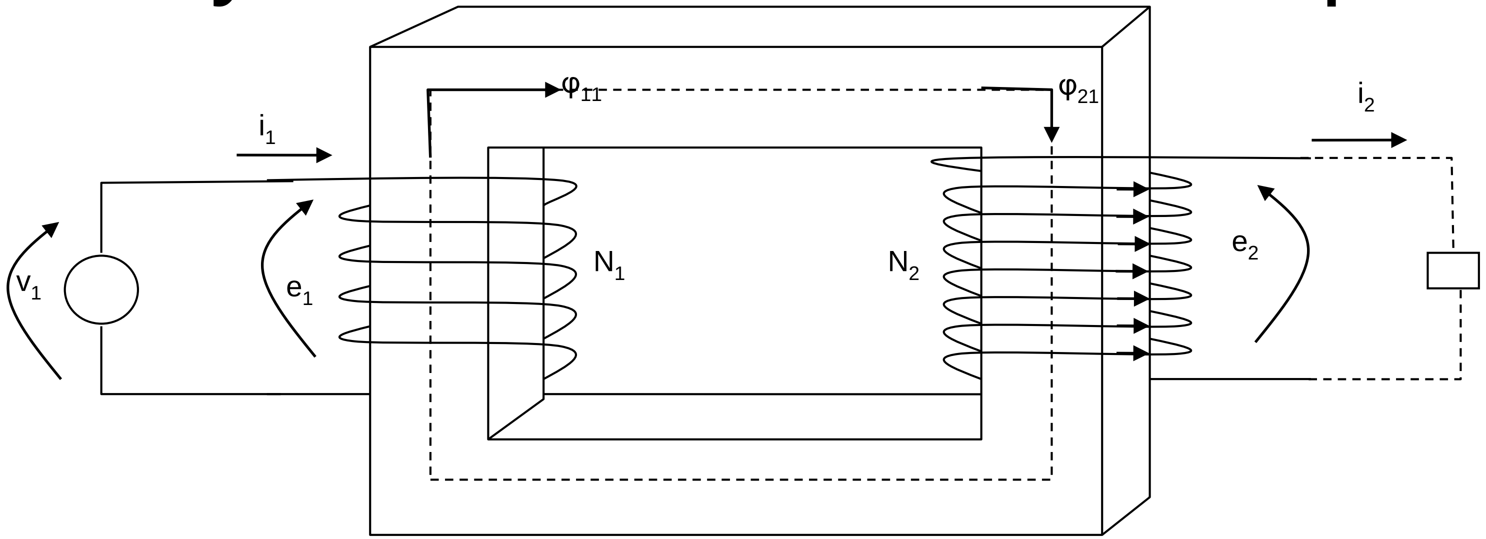
We want to know which secondary terminal, when defined with positive voltage polarity, results in using Faraday's Law with a positive sign.

$$e_2 = +M\,\frac{di_1}{dt}$$

On paper, there are 2 approaches for doing this.

1. Draw the physical winding go through Lenz's Law analysis as we have done in previous slides.

Polarity & dot convention for coupled ccts



On paper, there are 2 approaches for doing this.

1. Draw the physical winding; go through Lenz's Law analysis as we have done in previous slides.
2. Use the "dot convention."

In dot convention, we mark 1 terminal on each coil so that

- when e_2 is defined positive at dotted terminal of coil 2
- and i_1 is into the dotted terminal of coil 1, then

$$e_2 = +M\frac{di_1}{dt}$$

Example

Ex 3: Express the voltage for each pair of coils below.

From previous slide: In dot convention, we mark 1 terminal on each coil so that

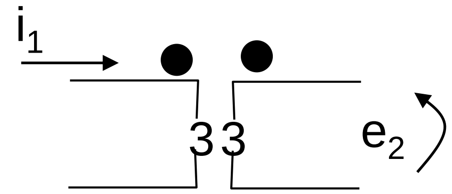
- when e_2 is defined positive at dotted terminal of coil 2, and
- i_1 is into the dotted terminal of coil 1, then

$$e_2 = +M\frac{di_1}{dt}$$

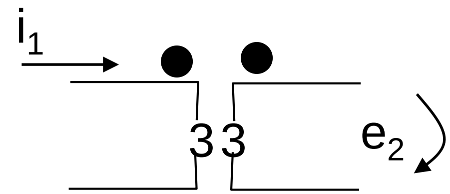
(1) Recall the sign on the RHS is determined not by direction of flux flow (or current i_1 flow) but by *<u>direction of change</u>* in flux flow (or current i_1 flow). (2) Our above dot convention seems to depend only on direction of current (i_1) flow and not on direction of change in current flow.

<u>Question</u>: How can our dot convention give correct sign if it does not account for direction of change in current i_1 flow?

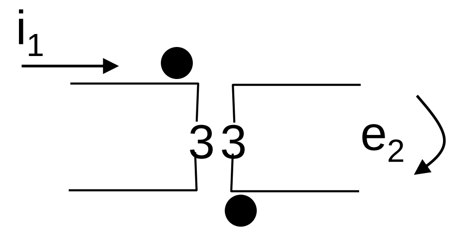
<u>Answer</u>: It does account for direction of change in current flow in that the above e_2 equation implies positive direction of change (di_1/dt is positive).



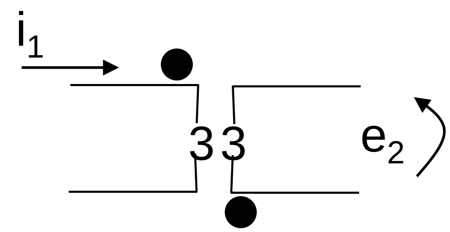
$$e_2 = +M\frac{di_1}{dt}$$



$$e_2 = -M\frac{di_1}{dt}$$



$$e_2 = +M\frac{di_1}{dt}$$



$$e_2 = -M\frac{di_1}{dt}$$

A second question

So far, we have focused on answering this question:

➔Given dotted terminals, how to determine the sign to use in Faraday's law?

A second question:

➔If you are given the physical layout, how do you obtain the dot-markings?

Approach 1: Use Lenz's Law and the right-hand-rule (RHR) to determine if a defined voltage direction at the secondary produces a current in the secondary that generates flux opposing the flux change that caused that voltage. (This is actually a conceptual summary of Approach 2 below.)

A second question

➔If you are given the physical layout, how do you obtain the dot-markings?

<u>Approach 2</u>: Do it by steps. (This is actually a step-by-step articulation of the first approach.)

1. Arbitrarily pick a terminal on one side and dot it.
2. Assign a current into the dotted terminal.
3. Use RHR to determine flux direction for current assigned in step 2.
4. Arbitrarily pick a terminal on the other side and assign a current *out of* **<u>(into)</u>** it.
5. Use RHR to determine flux direction for current assigned in Step 4.
6. Compare the direction of the two fluxes (the one from Step 3 and the one from Step 5). If the two flux directions are *opposite* **<u>(same)</u>**, then the terminal chosen in Step 4 is correct. If the two flux directions are *same* **<u>(opposite)</u>**, then the terminal chosen in Step 4 is incorrect – dot the other terminal.

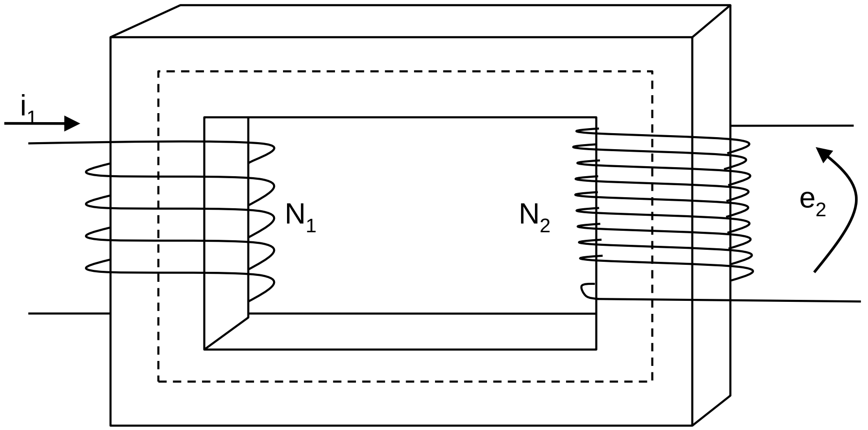
This approach depends on the following principle (consistent with words in italics in above steps): Current entering one dotted terminal and leaving the other dotted terminal should produce fluxes inside the core that are in opposite directions.

<u>An alternative statement of this principle is as follows (consistent with words in underline bold in above steps): Currents entering the dotted terminals should produce fluxes inside the core that are in the same direction.</u>

Example

Example 4: Determine the dotted terminals for the configuration below, and then write the relation between i_1 and e_2.

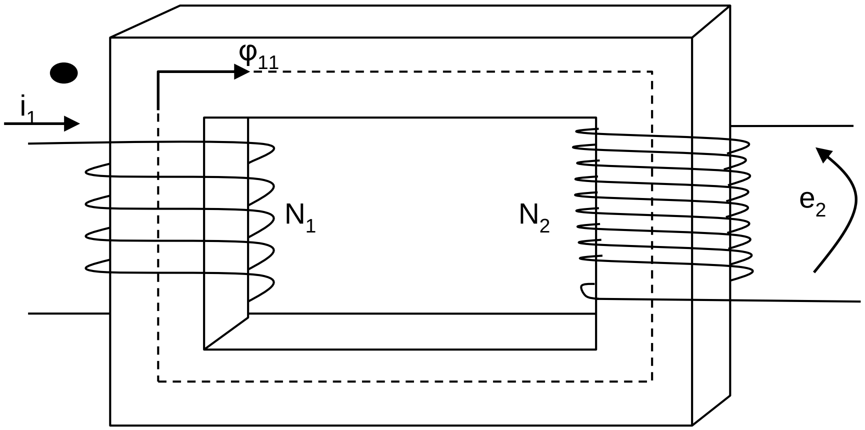
***Remember**: Current entering one dotted terminal and leaving the other dotted terminal should produce fluxes inside the core that are in opposite directions.*



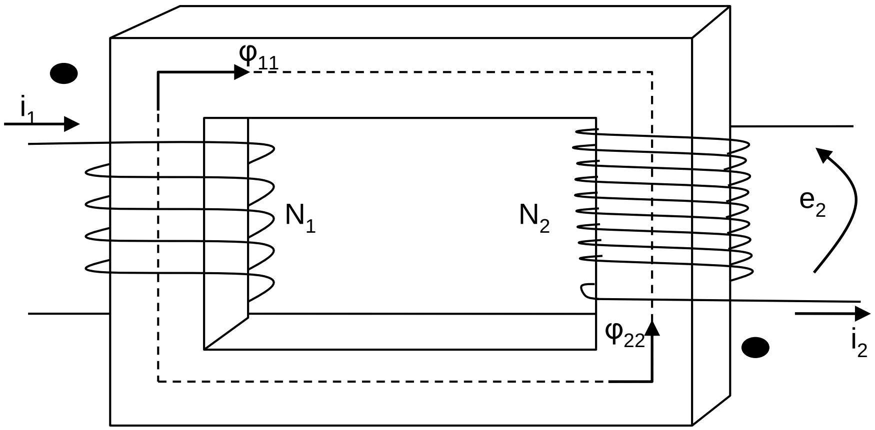
Example

Example 4: Determine the dotted terminals for the configuration below, and then write the relation between i_1 and e_2.

Solution: Steps 1-3:



Steps 4-6:



Remember: *Current entering one dotted terminal and leaving the other dotted terminal should produce fluxes inside the core that are in opposite directions.*

Example

Example 4: Determine the dotted terminals for the configuration below, and then write the relation between i_1 and e_2.

Solution:

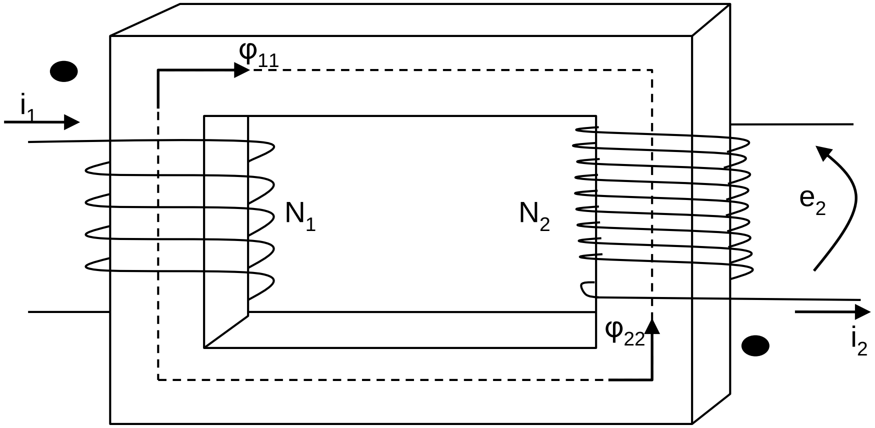
Now write equation for the coupled circuits. Recall that in dot convention, we mark 1 terminal on either side of transformer so that

- when e_2 is defined positive at the dotted terminal of coil 2 and
- i_1 is into the dotted terminal of coil 1, then

$$e_2 = +M \frac{di_1}{dt}$$

Here, however, although i_1 is into the coil 1 dotted terminal, e_2 is defined negative at the coil 2 dotted terminal. Therefore

$$e_2 = -M \frac{di_1}{dt}$$

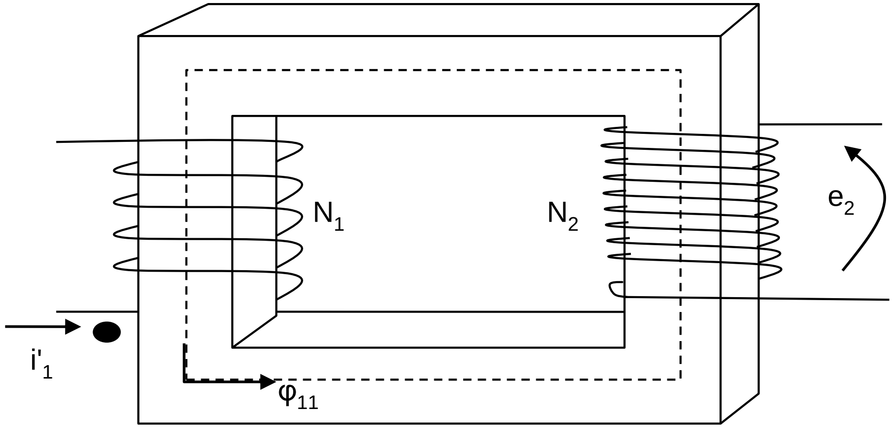


Example

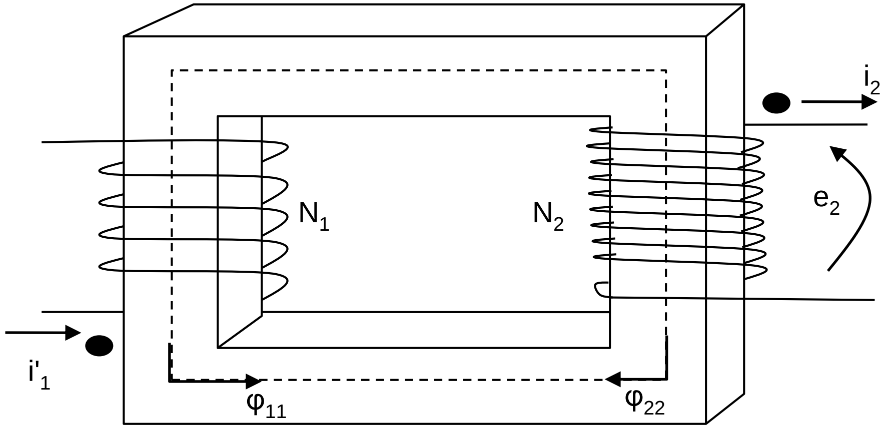
Example 4: Determine the dotted terminals for the configuration below, and then write the relation between i_1 and e_2.

Solution: there is another way we could have solved this problem, as follows

Steps 1-3:



Steps 4-6:



Example

Example 4: Determine the dotted terminals for the configuration below, and then write the relation between i_1 and e_2.

Solution: There is another way we could have solved this problem!

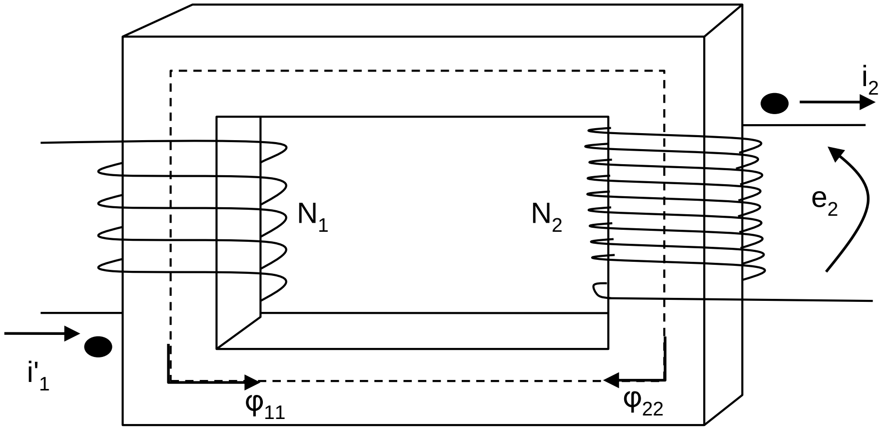
Write equation for the coupled circuits. Recall that in dot convention, we mark 1 terminal on either side of transformer so that

- when e_2 is defined positive at the dotted terminal of coil 2 and
- i_1 is into the dotted terminal of coil 1, then

$$e_2 = +M\frac{di_1}{dt}$$

Here, i'_2 is into the coil 1 dotted terminal, e_2 is defined positive at the coil 2 dotted terminal. Therefore

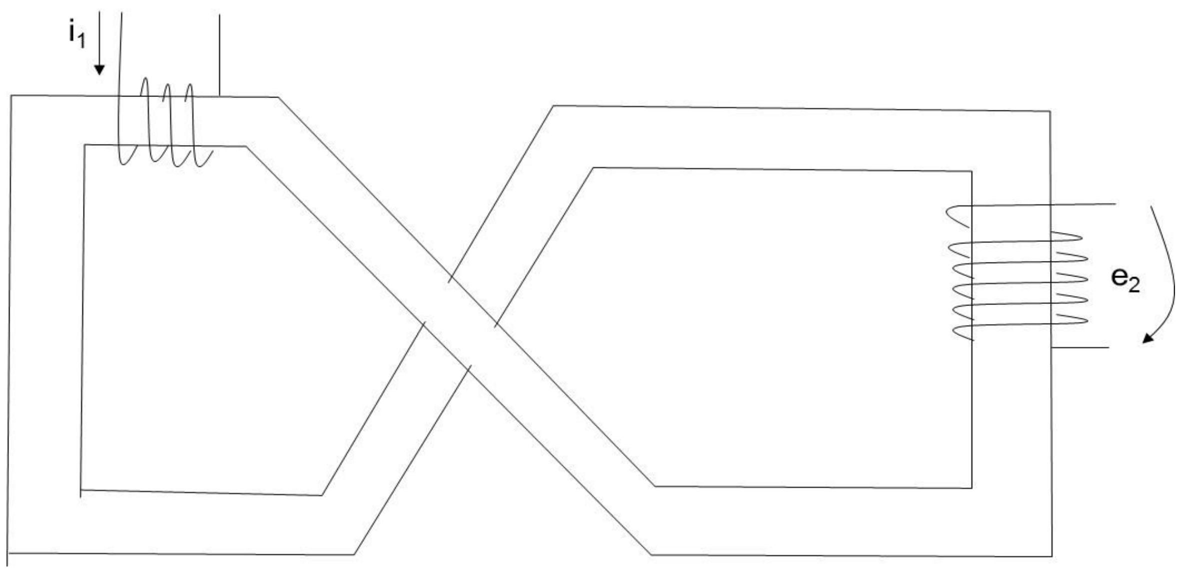
$$e_2 = +M\frac{di_1'}{dt}$$



If, however, we wanted to express e_2 as a function of i_1 (observing that i_1=-i'_1) then we would have $e_2 = -M\frac{di_1}{dt}$

Example

Example 5: For the configuration below, determine the dotted terminals and write the relation between i_1 and e_2.



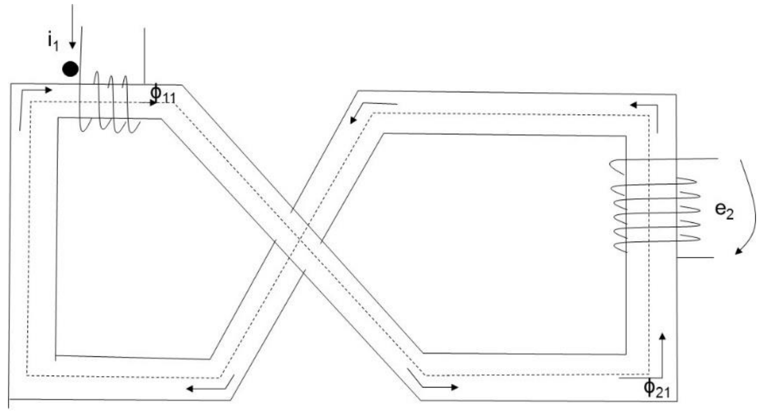
Note: Problems 1a,b,c,d are very similar to this one.

Example

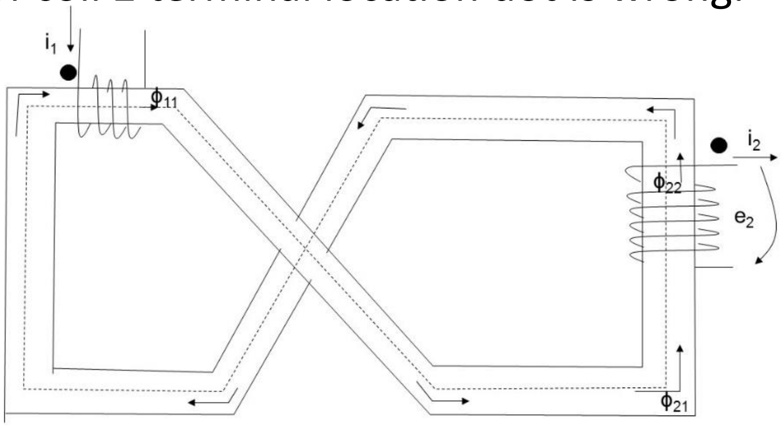
Example 5: For the configuration below, determine the dotted terminals and write the relation between i_1 and e_2.

Solution:

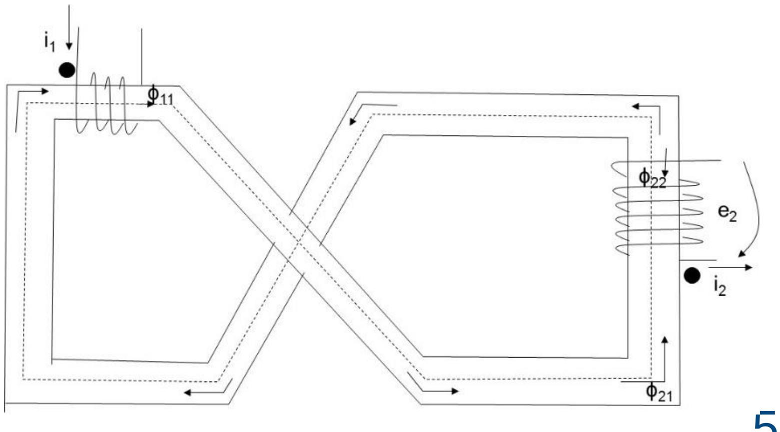
Steps 1-3:



Steps 4-6: Here we arbitrarily assign dot to upper terminal of coil 2; then, with i_2 out of this dotted terminal, we use RHR to determine flux ϕ_{22} is in same direction as coil 1 flux. This means our choice of coil 2 terminal location dot is wrong.



Therefore we know dot must be at other terminal, and the below shows clearly this is the case, since the flux from coil 2, ϕ_{22}, is opposite to the flux from coil 1, ϕ_{21}.



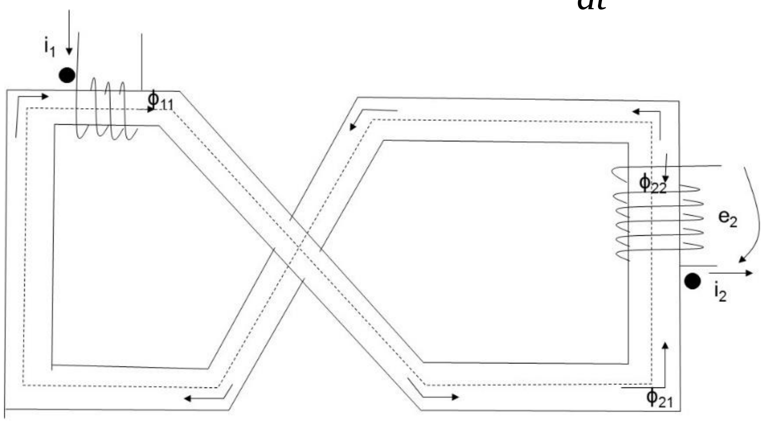
Example

Example 5: For the configuration below, determine the dotted terminals and write the relation between i_1 and e_2.

Solution:

Now we can write the equation for e_2. Recall that in the dot convention, we mark one terminal on either side so that

- when e_2 is defined positive at the dotted terminal of coil 2 and
- i_1 is into the dotted terminal of coil 1, then $e_2 = +M \dfrac{di_1}{dt}$

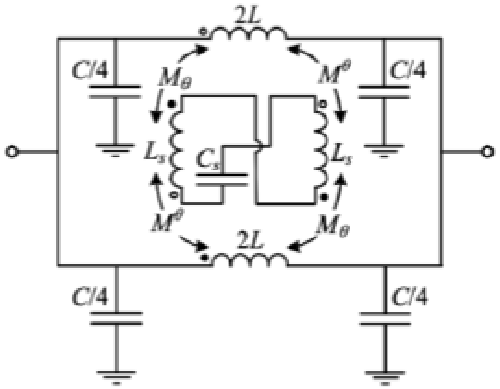


$$\Rightarrow \quad e_2 = +M \frac{di_1}{dt}$$

Writing circuit equations for coupled coils

We have so-far focused on transformers or similar circuits having magnetic coupling between coils. We may also encounter other kinds of circuits having elements that are magnetically coupled.

Example



Example: In this paper, the feasibility of resonant electrical coupling as a wireless power transfer technique is studied. Published 2015 in IEEE Transactions on Microwave Theory and…

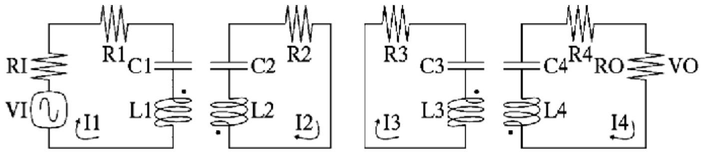


Fig. 17. Circuit model most commonly used in the analytical analysis of resonant magnetic coupling.

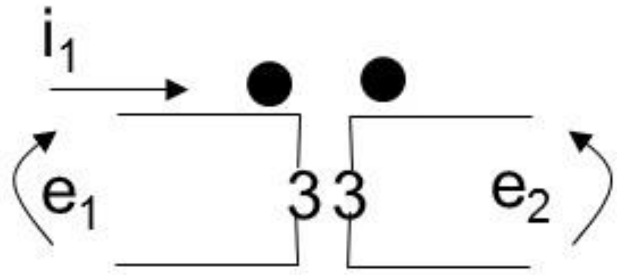
We are well-positioned to handle such circuits by combining our (new) knowledge of the dot convention with our (old) knowledge of circuit analysis.

Issue: When we write a voltage equation, we must account for

- the self-induced voltage in an inductor from its own current
- as well as any mutually-induced voltage in the inductor from a current in a coupled coil.

Writing circuit equations for coupled coils

Consider below circuit, where there may be currents in both windings:



Let's define λ_1 as the sum of

- λ_{11}, the flux from coil 1 seen by coil 1
- λ_{12}, the flux from coil 2 seen by coil 1.

Therefore:

$$\lambda_1 = \lambda_{11} + \lambda_{12}$$

What does Faraday's law say about e_1 as a function of λ_1?

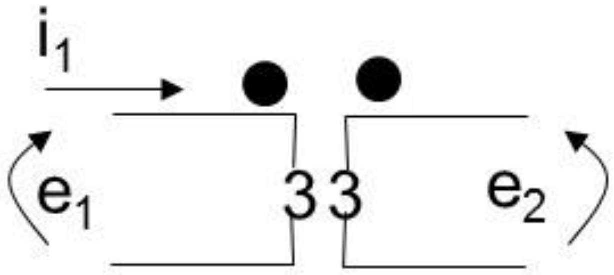
$$e_1 = \frac{d\lambda_1}{dt} = \frac{d\left(\lambda_{11} + \lambda_{12}\right)}{dt} = \frac{d\lambda_{11}}{dt} + \frac{d\lambda_{12}}{dt}$$

But from slides 12 and 25: $\lambda_{11} = L_1 i_1, \quad \lambda_{12} = M i_2$

$$\Rightarrow \quad e_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Writing circuit equations for coupled coils

Consider below circuit, where there may be currents in both windings:



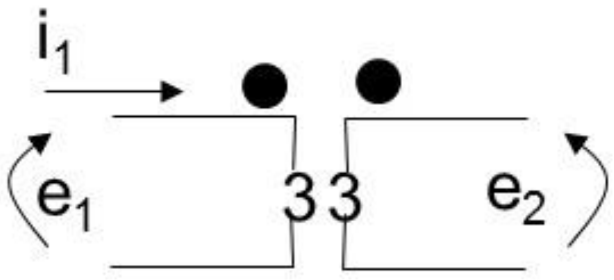
$$e_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

But wait… we have equated the sum of the derivatives to e_1, where e_1 has a certain assumed polarity. How can we be sure that the sign of both of those derivative terms is indeed positive? Until we can be sure of that, I want to write the above equation as:

$$e_1 = \pm L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

Writing circuit equations for coupled coils

Consider below circuit, where there may be currents in both windings:



$$e_1 = \pm L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

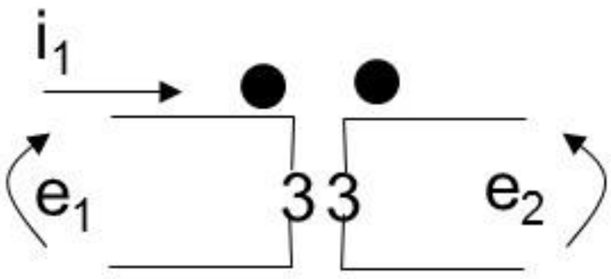
Let’s begin with the first (“self”) term (it is easiest!):

Rule for determining the sign of the self term: The polarity of the self term is determined entirely by the direction of the current i_1:

- when this current is into the positive terminal (as defined by the polarity of e_1), then the sign of the self term is positive;
- when this current is out of the positive terminal (as defined by the polarity of e_1), then the sign of the self term is negative.

Writing circuit equations for coupled coils

Consider below circuit, where there may be currents in both windings:



$$e_1 = \pm L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

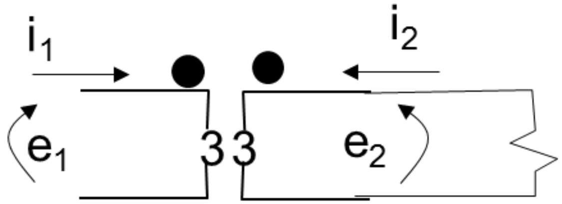
Now we need to determine how to know whether to add or subtract the mutual term from the self term. We should not be surprised to learn that we will make this determination using the dot convention.

Rule for determining the sign of the mutual term: Assume both coils correctly dotted.

1. Choose reference current directions for each coil (if not chosen for you).
2. Apply following to determine reference polarity of voltage induced by mutual effects:
   a. If reference current direction enters dotted terminal of a coil, the reference polarity of voltage that it induces in other coil is positive at its dotted terminal.
   b. If reference current direction leaves dotted terminal of a coil, the reference polarity of voltage that it induces in other coil is negative at its dotted terminal.

Writing circuit equations for coupled coils

Example 6: Express voltages e_1 and e_2 as a function of currents $i_1$and i_2 in the following circuit.



$$e_1 = \pm L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

Then, what is e_2?

$$e_2 = \pm L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$$

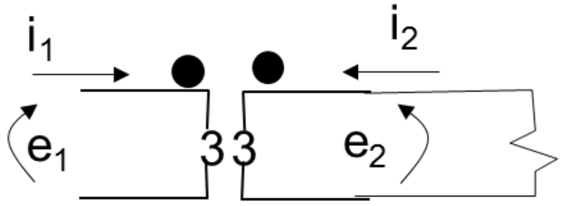
First, let's express e_1. Here, we observe two things:

1. i_1 enters the positive terminal, and therefore the self term is positive.
2. i_2 enters the dotted terminal of coil 2, therefore the reference polarity of the voltage it induces in coil 1 is positive at its dotted terminal, and its dotted terminal is the positive terminal.

$$\Rightarrow \quad e_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Writing circuit equations for coupled coils

Example 6: Express voltages e_1 and e_2 as a function of currents $i_1$and i_2 in the following circuit.



$$e_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$e_2 = \pm L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$$

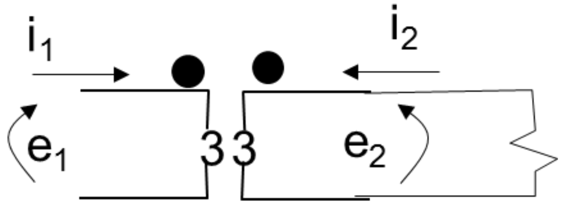
Now let's express e_2. Here, we observe two things:

1. i_2 enters the positive terminal, and therefore the self term is positive.
2. i_1 enters the dotted terminal of coil 1, therefore the reference polarity of the voltage it induces in coil 2 is positive at its dotted terminal, and its dotted terminal is the positive terminal.

$$\Rightarrow \quad e_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Writing circuit equations for coupled coils

Example 6: Express voltages e_1 and e_2 as a function of currents $i_1$and i_2 in the following circuit.

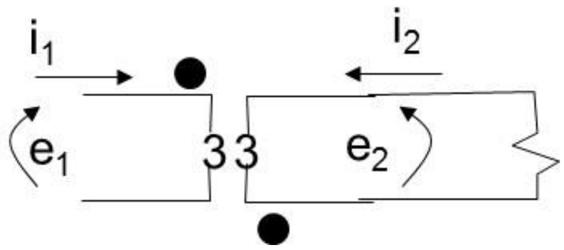


$$e_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$ ✓

$$e_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$ ✓

Writing circuit equations for coupled coils

Example 7: Express voltages e_1 and e_2 as a function of currents $i_1$and i_2 in the following circuit.



$$e_1 = \pm L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$e_2 = \pm L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$$

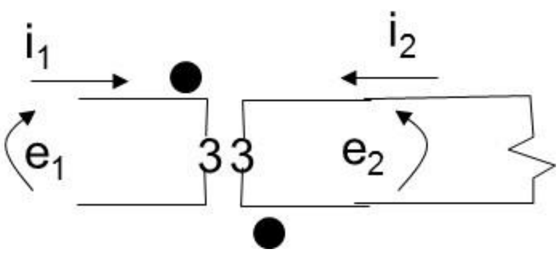
Let's express e_1. Here, we observe two things:

1. i_1 enters the positive terminal, therefore the self term is positive.
2. i_2 leaves the dotted terminal of coil 2, therefore the reference polarity of the voltage it induces in coil 1 is negative at its dotted terminal, and its undotted terminal is the positive terminal.

$$\Rightarrow \quad e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

Writing circuit equations for coupled coils

Example 7: Express voltages e_1 and e_2 as a function of currents $i_1$and i_2 in the following circuit.



$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$ ✓

$$e_2 = \pm L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$$

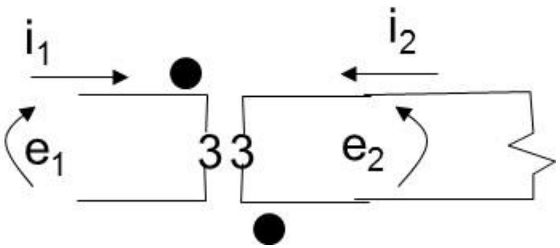
Now let's express e_2. Here, we observe two things:

1. i_2 enters the positive terminal, and therefore the self term is positive.
2. i_1 enters the dotted terminal of coil 1, therefore the reference polarity of the voltage it induces in coil 2 is positive at its dotted terminal, but its dotted terminal is the negative terminal.

$$\Rightarrow \quad e_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

Writing circuit equations for coupled coils

Example 7: Express voltages e_1 and e_2 as a function of currents i_1 and i_2 in the following circuit.

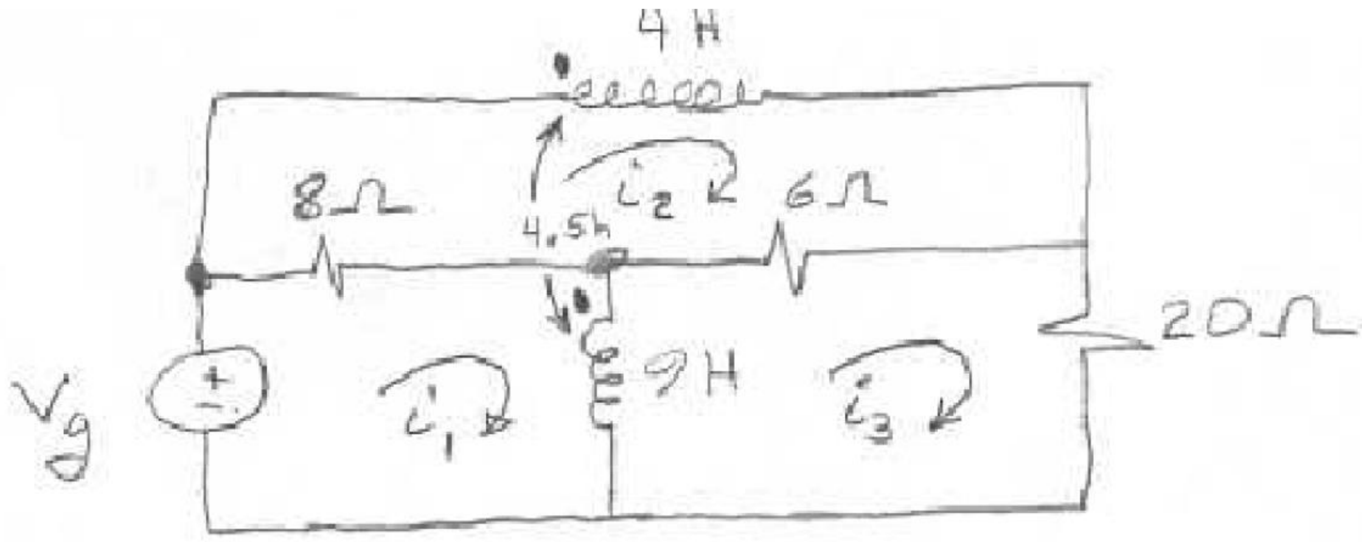


$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$ ✓

$$e_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$ ✓

Writing circuit equations for coupled coils

HW problem 4: Write a set of mesh current equations that describe the circuit below in terms of i_1, i_2, and i_3.



Number of nodes=4;

number of branches where current not known=b=6

b-(n-1)=6-3=3.

We need three mesh equations.

We write these for the three "windows" in the cct. above.

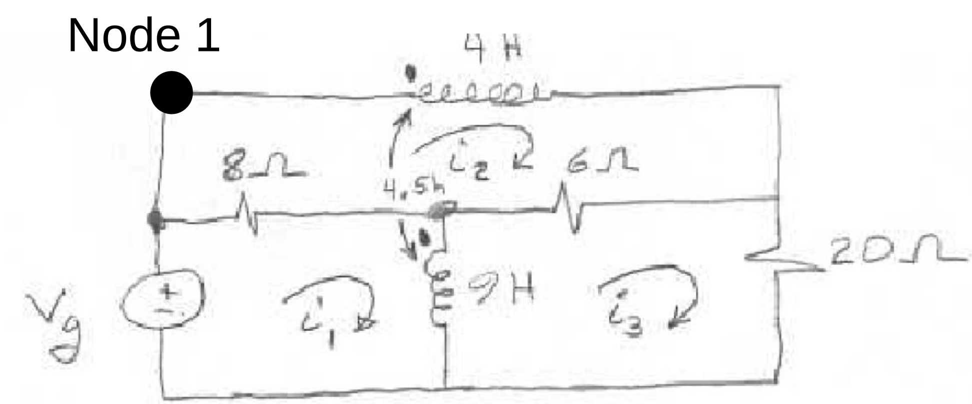
Writing circuit equations for coupled coils

HW problem 4: Write a set of mesh current equations that describe the circuit below in terms of i_1, i_2, and i_3.

Focus on top loop. Apply KVL starting from Node 1 moving clockwise. With i_2 in direction shown, we assume voltage polarity of 4H inductor is defined positive at its dotted end. With i_2 into the 4H inductor, the self term is positive. But the KVL moves across 4H inductor from positive to negative, therefore the first term in the mesh equation is negative. $-4\frac{di_2}{dt}$

The mutually induced term of 4H inductor is also negated by this KVL movement. Also, observe the coupled current i_1 is into dotted side of 9H inductor, but i_3 is out of it. So:

$$-4\frac{di_2}{dt}-4.5\left[\frac{di_1}{dt}-\frac{di_3}{dt}\right]$$



The rest of the top loop is easy.

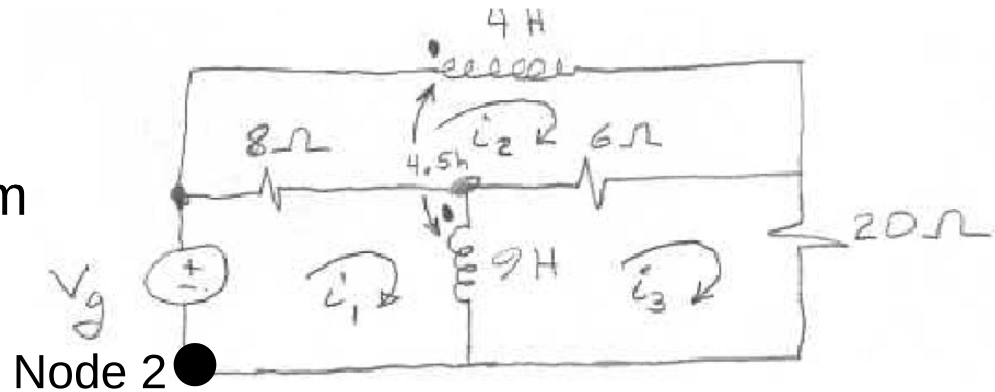
$$-4\frac{di_2}{dt}-4.5\left[\frac{di_1}{dt}-\frac{di_3}{dt}\right]+6\left[i_3-i_2\right]+8\left[i_1-i_2\right]=0$$

Writing circuit equations for coupled coils

HW problem 4: Write a set of mesh current equations that describe the circuit below in terms of i_1, i_2, and i_3.

Focus on left loop. Apply KVL starting from Node 2 moving clockwise. The first two parts are easy.

$$v_g - 8[i_1 - i_2]$$



The next part is the self-induced term of the 9H inductor. With i_1 in direction shown, we assume voltage polarity of 9H inductor is defined positive at its dotted end. With i_1 into the 9H inductor, the self term is positive. But the KVL moves across 9H inductor from positive to negative, therefore the self term in the mesh equation is negative. Note it is comprised of current i_1 into the dot (positive) and i_3 out of the dot (negative)

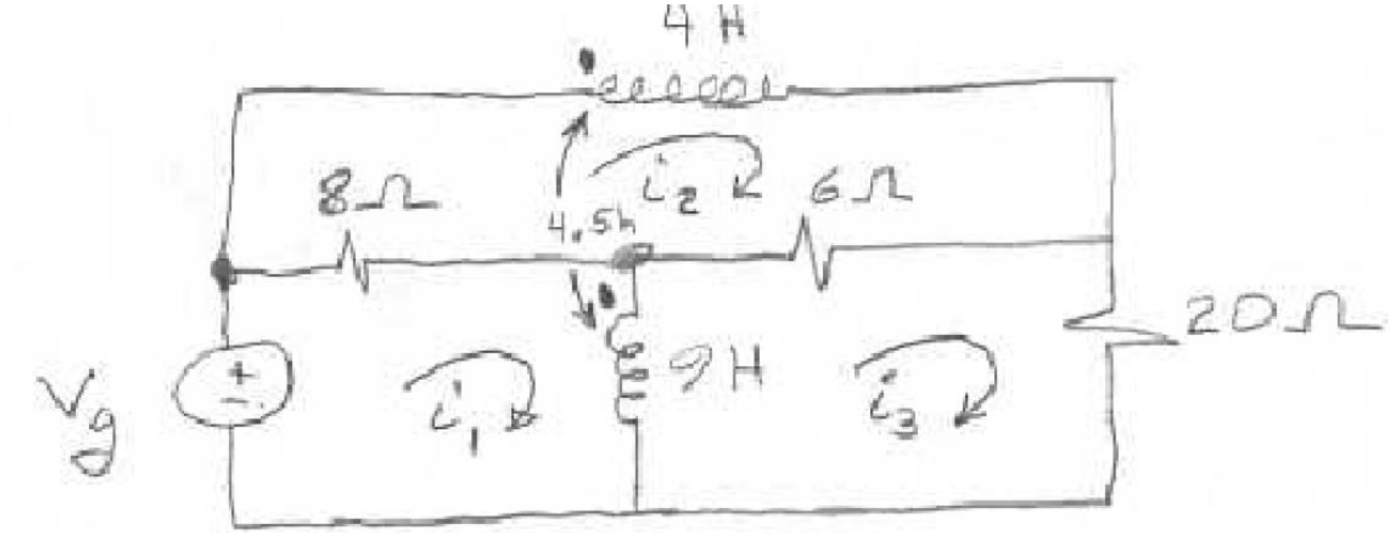
$$v_g - 8[i_1 - i_2] - 9\left[\frac{di_1}{dt} - \frac{di_3}{dt}\right]$$

We still need the mutual term from the 4H inductor. It would be positive since i_2 is into the dot of the 4 H inductor and out voltage is defined positive at the dot of the 9H inductor, but it is also influenced by the KVL direction of movement.

$$v_g - 8[i_1 - i_2] - 9\left[\frac{di_1}{dt} - \frac{di_3}{dt}\right] - 4.5\frac{di_2}{dt} = 0$$

Writing circuit equations for coupled coils

HW problem 4: Write a set of mesh current equations that describe the circuit below in terms of i_1, i_2, and i_3.



Top Loop: $$-4\frac{di_2}{dt} - 4.5\left[\frac{di_1}{dt} - \frac{di_3}{dt}\right] + 6\left[i_3 - i_2\right] + 8\left[i_1 - i_2\right] = 0$$

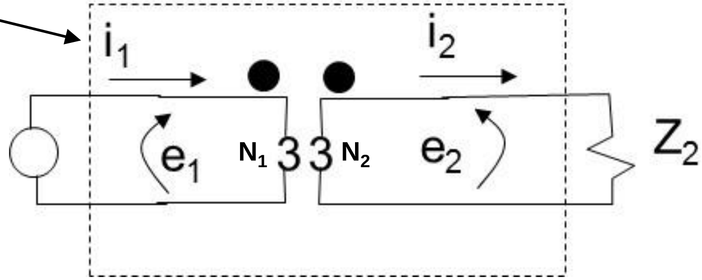
Left Loop: $$v_g - 8\left[i_1 - i_2\right] - 9\left[\frac{di_1}{dt} - \frac{di_3}{dt}\right] - 4.5\frac{di_2}{dt} = 0$$

Right Loop: $$9\left[\frac{di_1}{dt} - \frac{di_3}{dt}\right] + 4.5\frac{di_2}{dt} - 6\left[i_3 - i_2\right] - 20i_3 = 0$$

Development of turns ratio relations for ideal xfmr

Ideal xfmr: No Losses, infinite permeability.

Dashed box:
The “ideal” xfmr



Objective: See how
- i_1 and i_2 are related
- e_1 and e_2 are related

in the steady-state.

Remember:
- Self term: when same-side current enters its positive terminal, self term is positive.
- Mutual term: when opposite-side current enters its dotted terminal, mutual term is positive at its dotted terminal.

Voltage equation for left-hand-loop:

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad (*)$$

Voltage equation for right-hand-loop:

$$e_2 = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = i_2 Z_2$$

$$\Rightarrow M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + i_2 Z_2 \qquad (**)$$

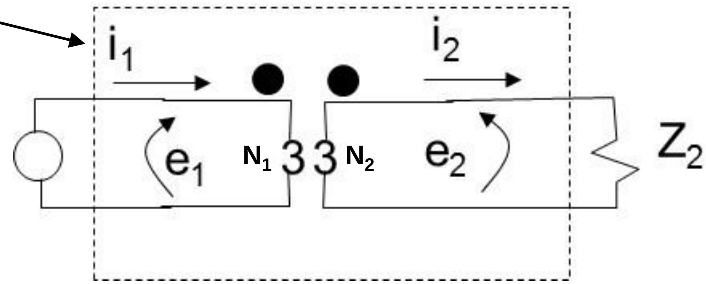
So we have equations (*) and (**):

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad (*) \qquad M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + i_2 Z_2 \qquad (**)$$

Development of turns ratio relations for ideal xfmr

Ideal xfmr: No Losses, infinite permeability.

Dashed box:
The "ideal" xfmr



Objective: See how
- i_1 and i_2 are related
- e_1 and e_2 are related

in the steady-state.

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad (*)$$

$$M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + i_2 Z_2 \qquad (**)$$

Important concept (see xfmr HW prob #6):

- *Differential equations*: characterize electrical relationships for
  - any time periods
  - under any type of excitation.
- *Phasor equations*: characterize electrical relationships for
  - time periods where conditions are in a steady-state
  - under sinusoidal excitation

We may convert differential equations to phasor equations.

How?...

Development of turns ratio relations for ideal xfmr

Ideal xfmr: No Losses, infinite permeability.

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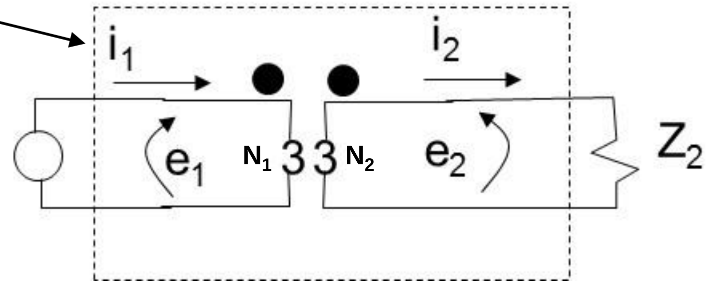
The "ideal" xfmr



Objective: See how
- i_1 and i_2 are related
- e_1 and e_2 are related

in the steady-state.

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad (*)$$

$$M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + i_2 Z_2 \qquad (**)$$

We may convert differential equations to phasor equations.
How?... Observe what we get when we differentiate a sinusoid:

Time Domain	Phasor Domain
$i(t)=\|I\|\sin\omega t$	$\mathbf{I}=\|I\|\angle 0$
$di/dt=\|I\|\omega\cos\omega t$	$\omega\|I\|\angle 90$
$di/dt=\|I\|\omega\sin(\omega t+90)$	$=\omega\|I\|\angle 0\angle 90=j\omega\mathbf{I}$

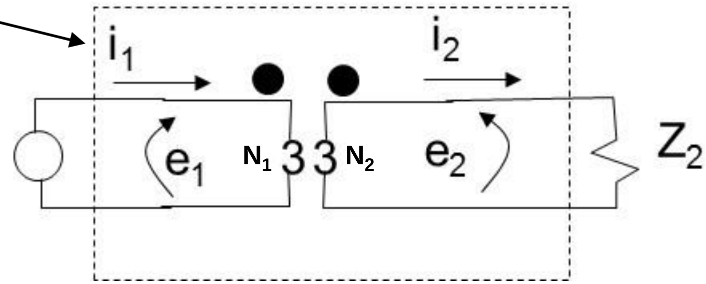
Original function scaled by ω; rotated forward by 90°.

Differentiation in time domain is multiplication by jω in phasor (Fourier) domain! Let's use this to transform (*) and (**)…

Development of turns ratio relations for ideal xfmr

Ideal xfmr: No Losses, infinite permeability.

Dashed box:
The "ideal" xfmr



Objective: See how
- i_1 and i_2 are related
- e_1 and e_2 are related

in the steady-state.

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad (*)$$

$$M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + i_2 Z_2 \qquad (**)$$

$$\boldsymbol{E}_1 = j\omega L_1 \boldsymbol{I}_1 - j\omega M \boldsymbol{I}_2 \qquad (\&)$$

$$j\omega M \boldsymbol{I}_1 = j\omega L_2 \boldsymbol{I}_2 + Z_2 \boldsymbol{I}_2$$

$$\Rightarrow \boldsymbol{I}_2 = \frac{j\omega M}{j\omega L_2 + Z_2} \boldsymbol{I}_1 \qquad (\#)$$

Recall:

$$L_2 = \frac{\mu A N_2^2}{l} = \frac{N_2^2}{\mathscr{R}}$$

$$\mathscr{R} = \frac{l}{\mu A}$$

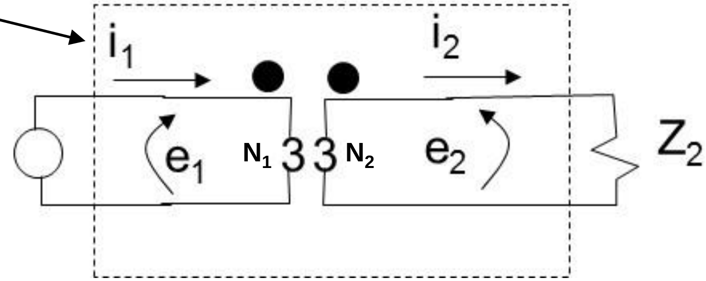
Assume: μ is very large (infinite permeability).
Then $\mathscr{R}$ is very small.
Then L_2 is very large.
Then $|j\omega L_2| >> |Z_2|$.

So (#) becomes $\boldsymbol{I}_2 \approx \frac{j\omega M}{j\omega L_2} \boldsymbol{I}_1 = \frac{M}{L_2} \boldsymbol{I}_1$

Development of turns ratio relations for ideal xfmr

Ideal xfmr: No Losses, infinite permeability.

Dashed box:
The "ideal" xfmr



Objective: See how
- i_1 and i_2 are related
- e_1 and e_2 are related

in the steady-state.

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad (*)$$

$$M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + i_2 Z_2 \qquad (**)$$

$$\boldsymbol{E}_1 = j\omega L_1 \boldsymbol{I}_1 - j\omega M \boldsymbol{I}_2 \qquad (\&)$$

$$\boldsymbol{I}_2 = \frac{M}{L_2} \boldsymbol{I}_1 \qquad (!!)$$

Recall, slide 25: $M = \dfrac{N_1 N_2}{\mathscr{R}}$ Recall, slide 14: $L_{22} \triangleq L_2 = \dfrac{N_2^2}{\mathscr{R}}$

Substitution into (!!)

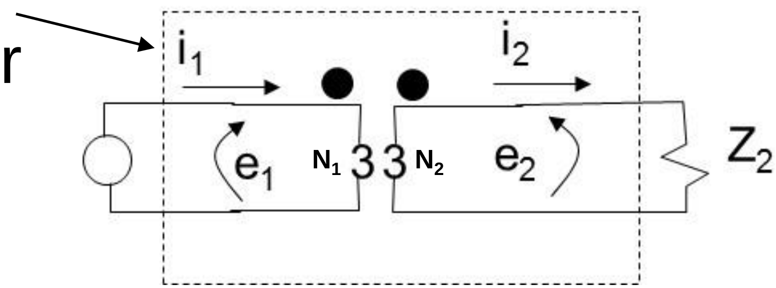
$$\boldsymbol{I}_2 = \frac{M}{L_2}\boldsymbol{I}_1 = \frac{\frac{N_1 N_2}{\mathscr{R}}}{\frac{N_2^2}{\mathscr{R}}}\boldsymbol{I}_1 = \frac{N_1 N_2}{N_2^2}\boldsymbol{I}_1 = \frac{N_1}{N_2}\boldsymbol{I}_1 \Rightarrow \boldsymbol{I}_2 = \frac{N_1}{N_2}\boldsymbol{I}_1$$

ratio of currents in coils on either side of an ideal transformer is in inverse proportion to the ratio of the coils' turns

Development of turns ratio relations for ideal xfmr

Ideal xfmr: No Losses, infinite permeability.

Dashed box:
The "ideal" xfmr



Objective: See how
- i_1 and i_2 are related
- e_1 and e_2 are related

in the steady-state.

$$e_1 = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt} \qquad (*)$$

$$M\frac{di_1}{dt} = L_2\frac{di_2}{dt} + i_2 Z_2 \qquad (**)$$

$$\boldsymbol{E}_1 = j\omega L_1 \boldsymbol{I}_1 - j\omega M \boldsymbol{I}_2 \qquad (\&)$$

$$\boldsymbol{I}_2 = \frac{j\omega M}{j\omega L_2 + Z_2}\boldsymbol{I}_1 \qquad (\#)$$

$$\boldsymbol{I}_2 = \frac{N_1}{N_2}\boldsymbol{I}_1 \qquad (\#*)$$

Substitute (#) into (&); obtain common denominator; simplify:

$$\boldsymbol{E}_1 = \left[\frac{\omega^2 M^2 - \omega^2 L_1 L_2 + j\omega L_1 Z_2}{j\omega L_2 + Z_2}\right]\boldsymbol{I}_1$$

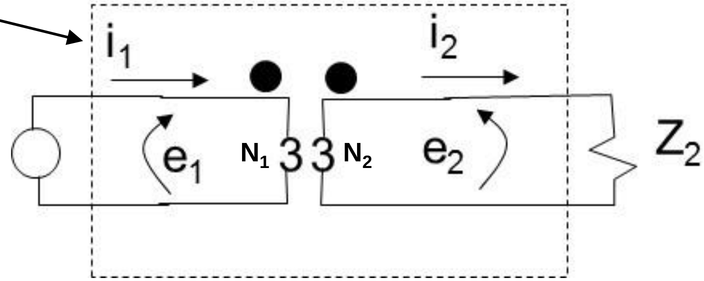
Use $M = \sqrt{L_1 L_2} \Rightarrow$

$$\boldsymbol{E}_1 = \left[\frac{j\omega L_1 Z_2}{j\omega L_2 + Z_2}\right]\boldsymbol{I}_1$$

Development of turns ratio relations for ideal xfmr

Ideal xfmr: No Losses, infinite permeability.

Dashed box:
The "ideal" xfmr



Objective: See how
- i_1 and i_2 are related
- e_1 and e_2 are related

in the steady-state.

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad (*)$$

$$M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + i_2 Z_2 \qquad (**)$$

$$\boldsymbol{E}_1 = j\omega L_1 \boldsymbol{I}_1 - j\omega M \boldsymbol{I}_2 \qquad (\&)$$

$$\boldsymbol{I}_2 = \frac{j\omega M}{j\omega L_2 + Z_2} \boldsymbol{I}_1 \qquad (\#)$$

$$\boldsymbol{I}_2 = \frac{N_1}{N_2} \boldsymbol{I}_1 \qquad (\#*)$$

$$\boldsymbol{E}_1 = \left[\frac{j\omega L_1 Z_2}{j\omega L_2 + Z_2} \right] \boldsymbol{I}_1$$

Use $|j\omega L_2| >> |Z_2|$. $\Rightarrow \boldsymbol{E}_1 = \frac{L_1 Z_2}{L_2} \boldsymbol{I}_1$

Recall, slide 14: $L_1 = \frac{N_1^2}{\mathscr{R}} \quad L_2 = \frac{N_2^2}{\mathscr{R}}$

$$\Rightarrow \boldsymbol{E}_1 = \frac{L_1 Z_2}{L_2} \boldsymbol{I}_1 = \frac{\frac{N_1^2}{\mathscr{R}} Z_2}{\frac{N_2^2}{\mathscr{R}}} \boldsymbol{I}_1 = \frac{N_1^2}{N_2^2} Z_2 \boldsymbol{I}_1$$

From (#*) $\boldsymbol{I}_1 = \frac{N_2}{N_1} \boldsymbol{I}_2$

Substitute: $\boldsymbol{E}_1 = \frac{N_1^2}{N_2^2} Z_2 \frac{N_2}{N_1} \boldsymbol{I}_2 = \frac{N_1}{N_2} Z_2 \boldsymbol{I}_2$

Use $Z_2 I_2 = E_2$:

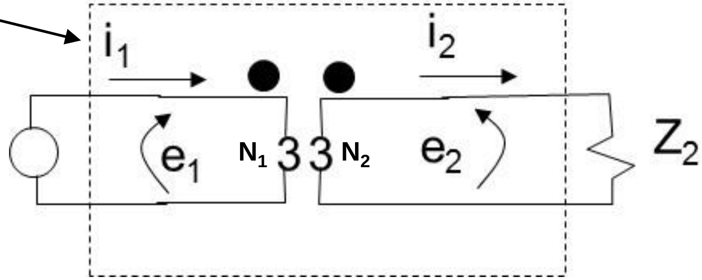
$$\boldsymbol{E}_1 = \frac{N_1}{N_2} \boldsymbol{E}_2$$

ratio of voltage across coils on either side of an ideal transformer is in proportion to the ratio of the coils' turns

Development of turns ratio relations for ideal xfmr

Ideal xfmr: No Losses, infinite permeability.

Dashed box:
The "ideal" xfmr



Objective: See how
- i_1 and i_2 are related
- e_1 and e_2 are related

in the steady-state.

$$e_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad (*)$$

$$M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} + i_2 Z_2 \qquad (**)$$

$$\frac{\boldsymbol{E}_1}{\boldsymbol{E}_2} = \frac{N_1}{N_2}$$

ratio of voltage across coils on either side of an ideal transformer is in proportion to the ratio of the coils' turns

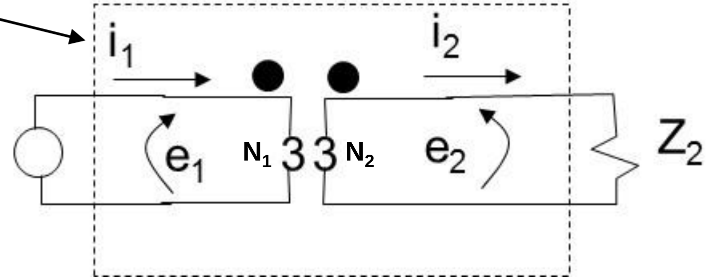
$$\frac{\boldsymbol{I}_2}{\boldsymbol{I}_1} = \frac{N_1}{N_2}$$

ratio of currents in coils on either side of an ideal transformer is in inverse proportion to the ratio of the coils' turns

Power for ideal xfmr

Ideal xfmr: No Losses, infinite permeability.

Dashed box:
The "ideal" xfmr



Objective: Does ideal "power transformer" transform power?

$$\frac{\boldsymbol{E}_1}{\boldsymbol{E}_2} = \frac{N_1}{N_2} \Rightarrow \boldsymbol{E}_2 = \frac{N_2}{N_1}\boldsymbol{E}_1 \qquad \frac{\boldsymbol{I}_2}{\boldsymbol{I}_1} = \frac{N_1}{N_2} \Rightarrow \boldsymbol{I}_2 = \frac{N_1}{N_2}\boldsymbol{I}_1$$

Preliminary comment: This discussion has nothing to do with losses in a real xfmr. We are still considering an ideal xfmr (no losses).

Express power on both sides of transformer:

$$S_1 = \boldsymbol{E}_1\boldsymbol{I}_1^* \qquad S_2 = \boldsymbol{E}_2\boldsymbol{I}_2^*$$

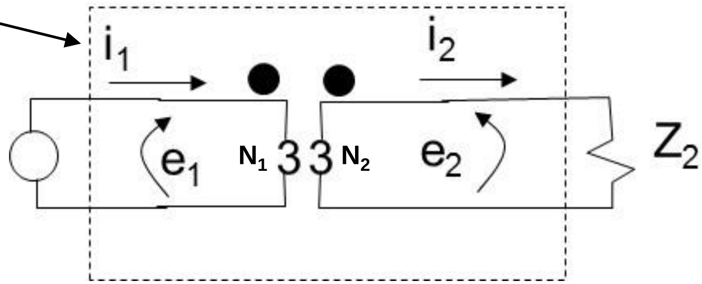
Substitute expressions for $\mathbf{E}_2$ and $\mathbf{I}_2$ into expression for S_2:

$$S_2 = \boldsymbol{E}_2\boldsymbol{I}_2^* = \frac{N_2}{N_1}\boldsymbol{E}_1\frac{N_1}{N_2}\boldsymbol{I}_1^* = \boldsymbol{E}_1\boldsymbol{I}_1^* \quad \Rightarrow S_2 = \boldsymbol{E}_1\boldsymbol{I}_1^* = S_1$$

Power for ideal xfmr

Ideal xfmr: No Losses, infinite permeability.

Dashed box:
The "ideal" xfmr



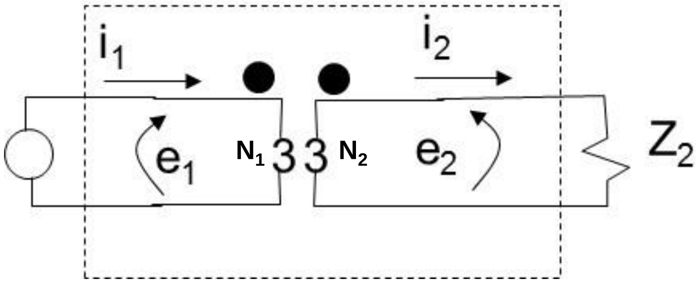
Objective: Does ideal "power transformer" transform power?

We have just proved that, for an ideal transformer, $S_1=S_2$, enabling us to conclude that "power transformers" do not transform power. It is a good thing, because doing so would result in a violation of the conservation of energy (otherwise known as the first law of thermodynamics), since "power transformation" would imply that we could provide one side with a certain amount of power P_1 and get out a greater amount of power P_2 on the other side. If we allowed, then, such a device to operate for an amount of time T, the output energy P_2T would be greater than the input energy P_1T, thus, the violation.

IDEAL TRANSFORMER

$$\frac{\boldsymbol{E}_1}{\boldsymbol{E}_2} = \frac{N_1}{N_2} \qquad S_2 = S_1 \qquad \frac{\boldsymbol{I}_2}{\boldsymbol{I}_1} = \frac{N_1}{N_2}$$

Referring Quantities



Objective: Can we somehow "move" Z_2 to the primary side of the ideal xfmr and then do all analysis there, so that our analysis need not have to account for the effect of the ideal xfmr?

Closely-related question: What impedance is "seen" looking into the primary terminals of the ideal xfmr? In other words, what is $Z_1=\mathbf{E}_1/\mathbf{I}_1$?

From our turns ratio relations:

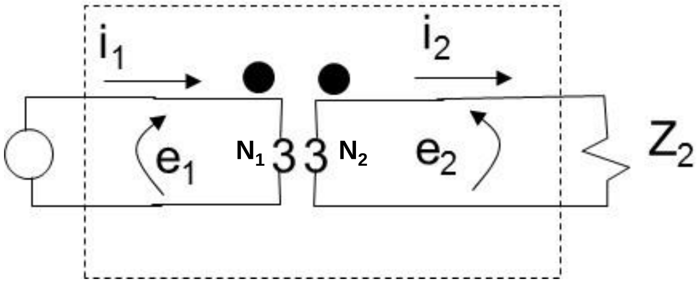
$$\boldsymbol{E}_1 = \frac{N_1}{N_2}\boldsymbol{E}_2 \qquad \boldsymbol{I}_1 = \frac{N_2}{N_1}\boldsymbol{I}_2$$

Substitution into the expression for Z_1:

$$Z_1 = \frac{\frac{N_1}{N_2}\boldsymbol{E}_2}{\frac{N_2}{N_1}\boldsymbol{I}_2} = \frac{N_1^2}{N_2^2}\frac{\boldsymbol{E}_2}{\boldsymbol{I}_2} = \frac{N_1^2}{N_2^2}Z_2 \qquad \Rightarrow Z_1 = \frac{N_1^2}{N_2^2}Z_2$$

Answer to closely-related question: Looking into the primary terminals of an ideal xfmr supplying Z_2 across its secondary terminals, we "see" the impedance Z_2 scaled by the turns ratio squared.

Referring Quantities



Primary side

Secondary side

Objective: Can we somehow "move" Z_2 to the primary side of the ideal xfmr and then do all analysis there, so that our analysis need not have to account for the effect of the ideal xfmr?

Closely-related question: What impedance is "seen" looking into the primary terminals of the ideal xfmr? In other words, what is $Z_1=\mathbf{E}_1/\mathbf{I}_1$?

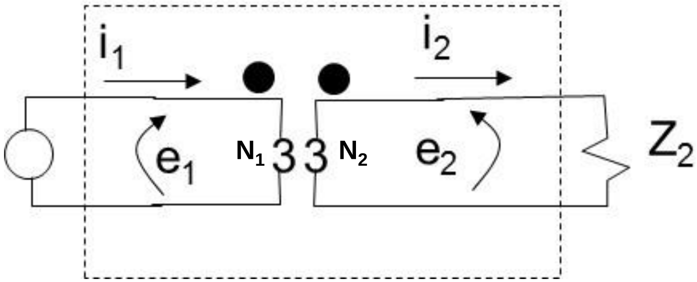
IDEAL TRANSFORMER

$$\frac{\boldsymbol{E}_1}{\boldsymbol{E}_2}=\frac{N_1}{N_2} \qquad S_2=S_1 \qquad \frac{Z_1}{Z_2}=\left(\frac{N_1}{N_2}\right)^2 \qquad \frac{\boldsymbol{I}_2}{\boldsymbol{I}_1}=\frac{N_1}{N_2}$$

These equations relate currents, voltages, impedances, and powers that

- exist on one side of the transformer, i.e., the secondary (primary), to
- corresponding currents, voltages, impedances, and powers, that exist on the other side of the transformer, i.e., the primary (secondary).

Referring Quantities



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Secondary side

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IDEAL TRANSFORMER

$$\frac{\boldsymbol{E}_1}{\boldsymbol{E}_2}=\frac{N_1}{N_2}$$

$$S_2=S_1$$

$$\frac{Z_1}{Z_2}=\left(\frac{N_1}{N_2}\right)^2$$

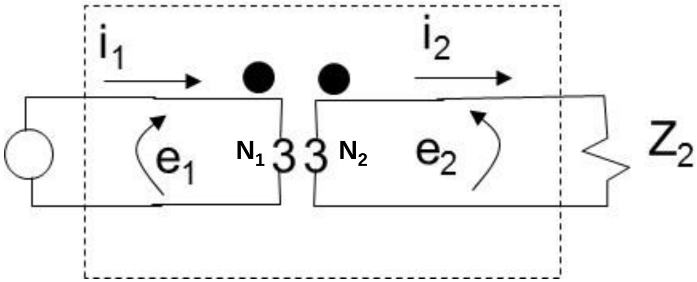
$$\frac{\boldsymbol{I}_2}{\boldsymbol{I}_1}=\frac{N_1}{N_2}$$

Relating quantities on one side of the xfmr to quantities on the other side is fine, but does that accomplish our above main objective?

Can we "move" Z_2 to the primary side of the ideal xfmr?

I think we can, based on our answer to the "closely-related question" if we move it scaled by the turns ratio squared. Let's see how this looks…

Referring Quantities



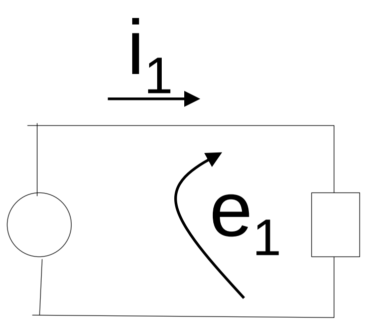
Primary side

Secondary side

<u>Objective</u>: Can we somehow "move" Z_2 to the primary side of the ideal xfmr and then do all analysis there, so that our analysis need not have to account for the effect of the ideal xfmr?

<u>Closely-related question</u>: What impedance is "seen" looking into the primary terminals of the ideal xfmr? In other words, what is $Z_1=\mathbf{E}_1/\mathbf{I}_1$?

Equivalent ccts



$$Z_1 = \frac{N_1^2}{N_2^2} Z_2$$

Write Ohm's law on primary side:

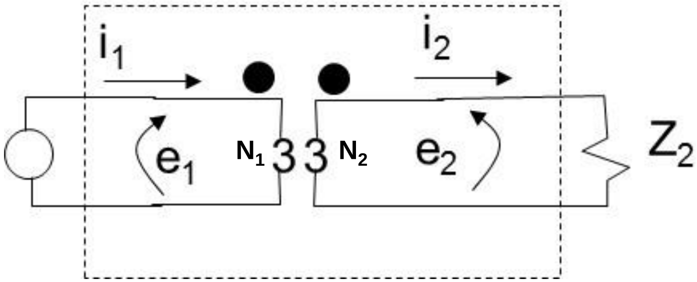
$$i_1 = \frac{e_1}{Z_1} = \frac{e_1}{\frac{N_1^2}{N_2^2} Z_2}$$

$$\Rightarrow \quad \frac{N_1}{N_2} i_1 = \frac{\frac{N_2}{N_1} e_1}{Z_2}$$

$$\Rightarrow \quad i_2 = \frac{e_2}{Z_2} \Rightarrow Z_2 = \frac{e_2}{i_2}$$

What is this telling us? It is taking the answer to the "closely related question" (which is that we "see" $Z_1=(N_1/N_2)^2 Z_2$ from the primary side) and showing that it is equivalent to "seeing" Z_2 looking into the load terminals from the secondary.

Referring Quantities



Primary side

Secondary side

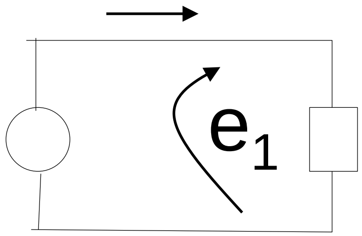
Objective: Can we somehow "move" Z_2 to the primary side of the ideal xfmr and then do all analysis there, so that our analysis need not have to account for the effect of the ideal xfmr?

Closely-related question: What impedance is "seen" looking into the primary terminals of the ideal xfmr? In other words, what is $Z_1=\mathbf{E}_1/\mathbf{I}_1$?

What is this telling us? It is taking the answer to the "closely related question" (which is that we "see" $Z_1=(N_1/N_2)^2 Z_2$ from the primary side) and showing that it is equivalent to "seeing" Z_2 looking into the load terminals from the secondary.

The point is that we can "see" Z_2 on the primary side, but it looks like $Z_1=(N_1/N_2)^2 Z_2$.

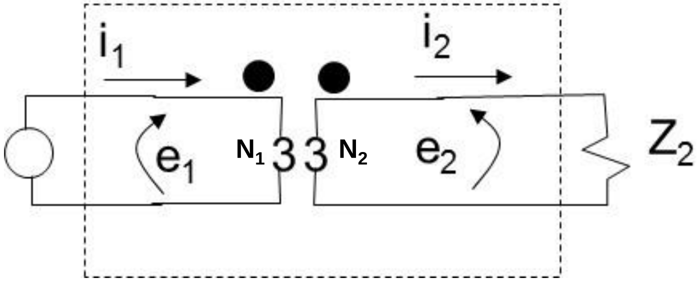
This means that the below circuit gives us everything we need to know about primary-side quantities when the secondary side is loaded with Z_2.



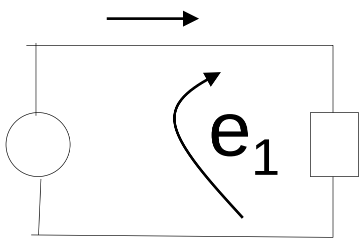
$$Z_1 = \frac{N_1^2}{N_2^2} Z_2$$

This is actually what we set out to achieve in our objective! The circuit to the left is the circuit we need! We have achieved our objective ☺.

Referring Quantities



Primary side Secondary side



$$Z_1 = \frac{N_1^2}{N_2^2} Z_2$$

This is actually what we set out to achieve in our objective! The circuit to the left is the circuit we need! We have achieved our objective ☺.

In fact, in addition to the impedances Z_1 and Z_2, we can say similar things about the voltages and currents.

- $Z_1=(N_1/N_2)^2Z_2$ is primary side equivalent of Z_2.
- $e_1=(N_1/N_2)e_2$ is the primary side equivalent of e_2.
- $i_1=(N_2/N_1)i_2$ is the primary side equivalent of i_2.

You can move quantities from secondary side to primary side!

And it works the other way too…

- $Z_2=(N_2/N_1)^2Z_1$ is secondary side equivalent of Z_1.
- $e_2=(N_2/N_1)e_1$ is the secondary side equivalent of e_1.
- $i_2=(N_1/N_2)i_1$ is the secondary side equivalent of i_1.

You can move quantities from primary side to secondary side!

Referring Quantities

Notation:

- First, observe that subscripts "1" and "2" (for turns, currents, voltages, impedances) tell us whether the quantity actually exists (physically) on the primary side (having subscript of "1") or the secondary side (having subscript of "2").

- Second, we use **<u>unprimed notation</u>**, i.e., I_1, E_1, Z_1, and I_2, E_2, Z_2, to denote the quantity represented on the side on which it actually exists. Thus, we say
  - I_1, E_1, Z_1 are the current, voltage, and impedance of *primary side quantities referred to the primary side*, and
  - I_2, E_2, Z_2 are the current, voltage, and impedance of secondary side quantities referred to the secondary side.

- Third, we use **<u>primed notation</u>**, i.e., I''_1, E''_1, Z''_1, and I'_2, E'_2, Z'_2, to denote the quantity represented on the opposite side from where it actually exists. Thus, we say
  - I''_1, E''_1, Z''_1 are the current, voltage, and impedance of *primary side quantities referred to the secondary side*, and
  - I'_2, E'_2, Z'_2 are the current, voltage, and impedance of secondary side quantities referred to the primary side.

Referring Quantities

Referring quantities from secondary to primary

$$\frac{\boldsymbol{I}_2}{\boldsymbol{I}_2'} = \frac{N_1}{N_2} \Rightarrow \underbrace{\boldsymbol{I}_2' = \frac{N_2}{N_1}\boldsymbol{I}_2}$$

Referring secondary current to primary

$$\frac{\boldsymbol{E}_2'}{\boldsymbol{E}_2} = \frac{N_1}{N_2} \Rightarrow \underbrace{\boldsymbol{E}_2' = \frac{N_1}{N_2}\boldsymbol{E}_2}$$

Referring secondary voltage to primary

$$\frac{Z_2'}{Z_2} = \left(\frac{N_1}{N_2}\right)^2 \Rightarrow \underbrace{Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2}$$

Referring secondary impedance to primary

Referring quantities from primary to secondary

$$\frac{\boldsymbol{I}_1''}{\boldsymbol{I}_1} = \frac{N_1}{N_2} \Rightarrow \underbrace{\boldsymbol{I}_1'' = \frac{N_1}{N_2}\boldsymbol{I}_1}$$

Referring secondary current to primary

$$\frac{\boldsymbol{E}_1}{\boldsymbol{E}_1''} = \frac{N_1}{N_2} \Rightarrow \underbrace{\boldsymbol{E}_1'' = \frac{N_2}{N_1}\boldsymbol{E}_1}$$

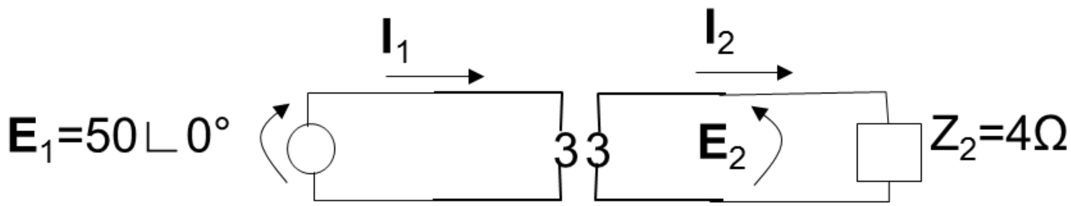
Referring secondary voltage to primary

$$\frac{Z_1}{Z_1''} = \left(\frac{N_1}{N_2}\right)^2 \Rightarrow \underbrace{Z_1'' = \left(\frac{N_2}{N_1}\right)^2 Z_1}$$

Referring secondary impedance to primary

Referring Quantities

Example 8: Find the current $\mathbf{I}_1$ in the primary side of the below circuit.



N1/N2=5/10

Solution: We can solve this problem in one of two ways.

Approach 1: Refer all quantities to secondary, solve for $\mathbf{I}_2$; then refer this current to the primary.

Approach 2: Refer all quantities to primary, solve for $\mathbf{I}_1$.

Approach 2:

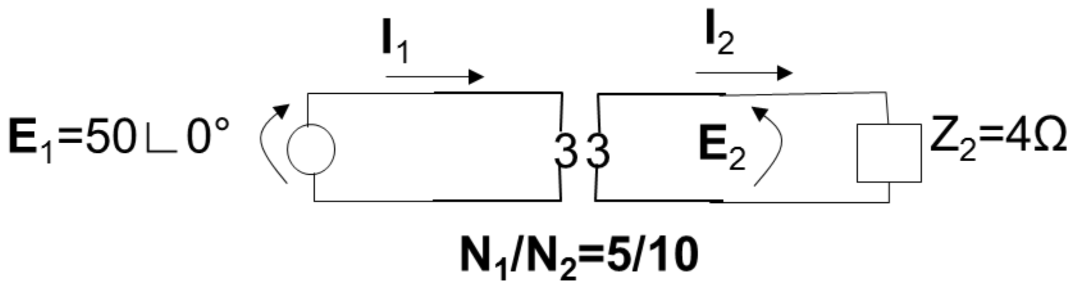
We begin by referring Z_2 to the primary side via:

$$Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2 = \left(\frac{5}{10}\right)^2 4 = \frac{25}{100} 4 = 1$$

So we know what Z'_2 is… it is the secondary impedance Z_2 as seen from the primary side. Let's redraw the circuit accordingly.

Referring Quantities

Example 8: Find the current I_1 in the primary side of the below circuit.

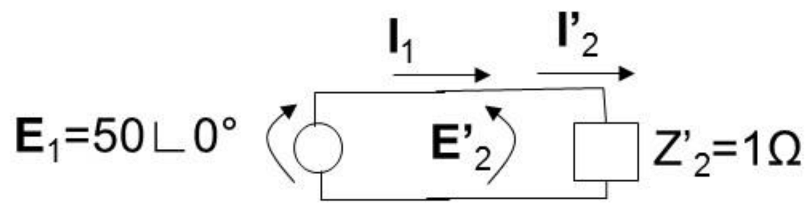


Solution: We can solve this problem in one of two ways.

Approach 1: Refer all quantities to secondary, solve for I_2; then refer this current to the primary.

Approach 2: Refer all quantities to primary, solve for I_1.

Approach 2:



Use of Ohm's Law results in

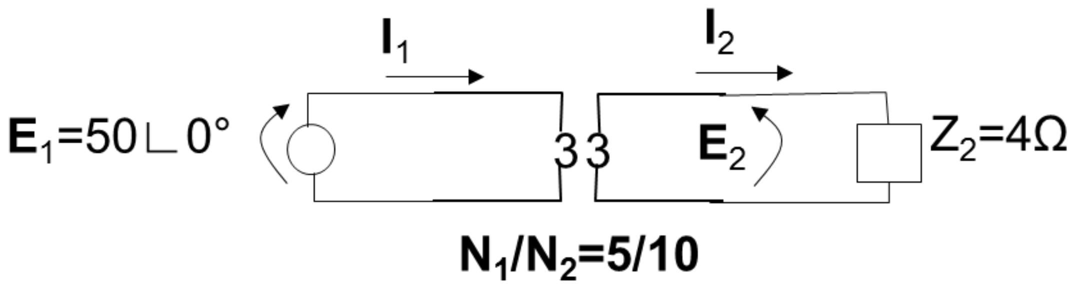
$$I_1 = I_2' = \frac{50\angle 0°}{1} = 50\angle 0°$$

And we are done ☺

But wait…what if we wanted to obtain secondary quantities, such as I_2 and E_2?

Referring Quantities

Example 8: Find the current $\mathbf{I}_1$ in the primary side of the below circuit.

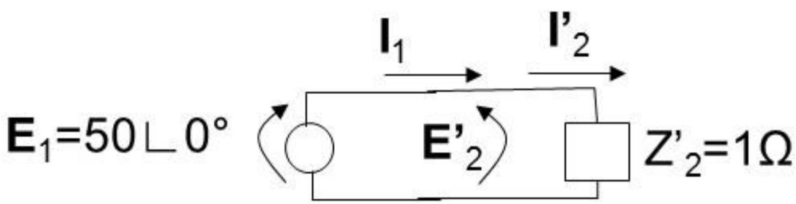


Solution: We can solve this problem in one of two ways.

Approach 1: Refer all quantities to secondary, solve for $\mathbf{I}_2$; then refer this current to the primary.

Approach 2: Refer all quantities to primary, solve for $\mathbf{I}_1$.

Approach 2:



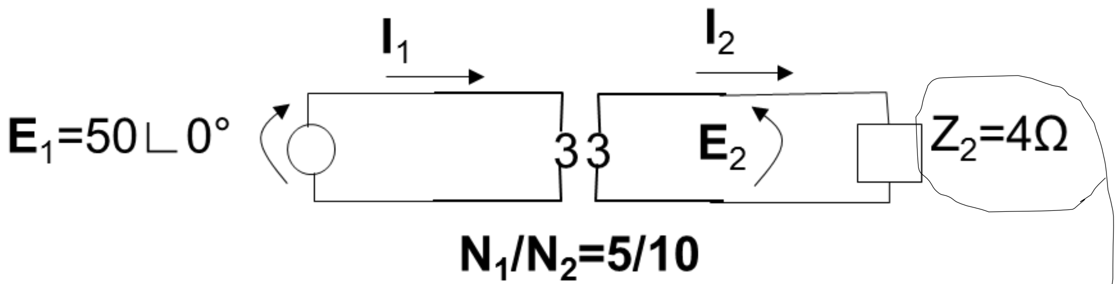
$$\boldsymbol{I}_1 = \boldsymbol{I}_2' = \frac{50\angle 0^\circ}{1} = 50\angle 0^\circ$$

But wait…what if we wanted to obtain secondary quantities, such as $\mathbf{I}_2$ and $\mathbf{E}_2$?

Then we refer $\mathbf{I'}_2$ and $\mathbf{E'}_2$ (which are quantities we obtain on the primary side) back to the secondary side. We already know $\mathbf{I'}_2$; we obtain $\mathbf{E'}_2$ by inspection of the above circuit, observing that it is the same as $\mathbf{E}_1$, i.e., $\mathbf{E'}_2 = 50\angle 0^\circ$.

Referring Quantities

Example 8: Find the current $\mathbf{I}_1$ in the primary side of the below circuit.

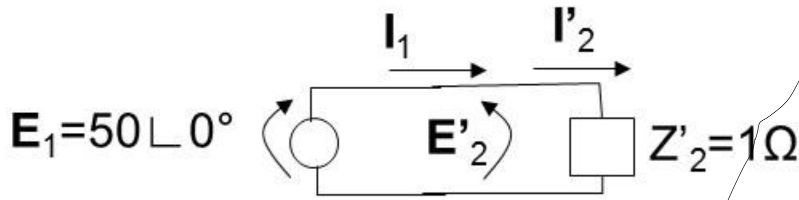


Solution: We can solve this problem in one of two ways.

Approach 1: Refer all quantities to secondary, solve for $\mathbf{I}_2$; then refer this current to the primary.

Approach 2: Refer all quantities to primary, solve for $\mathbf{I}_1$.

Approach 2:



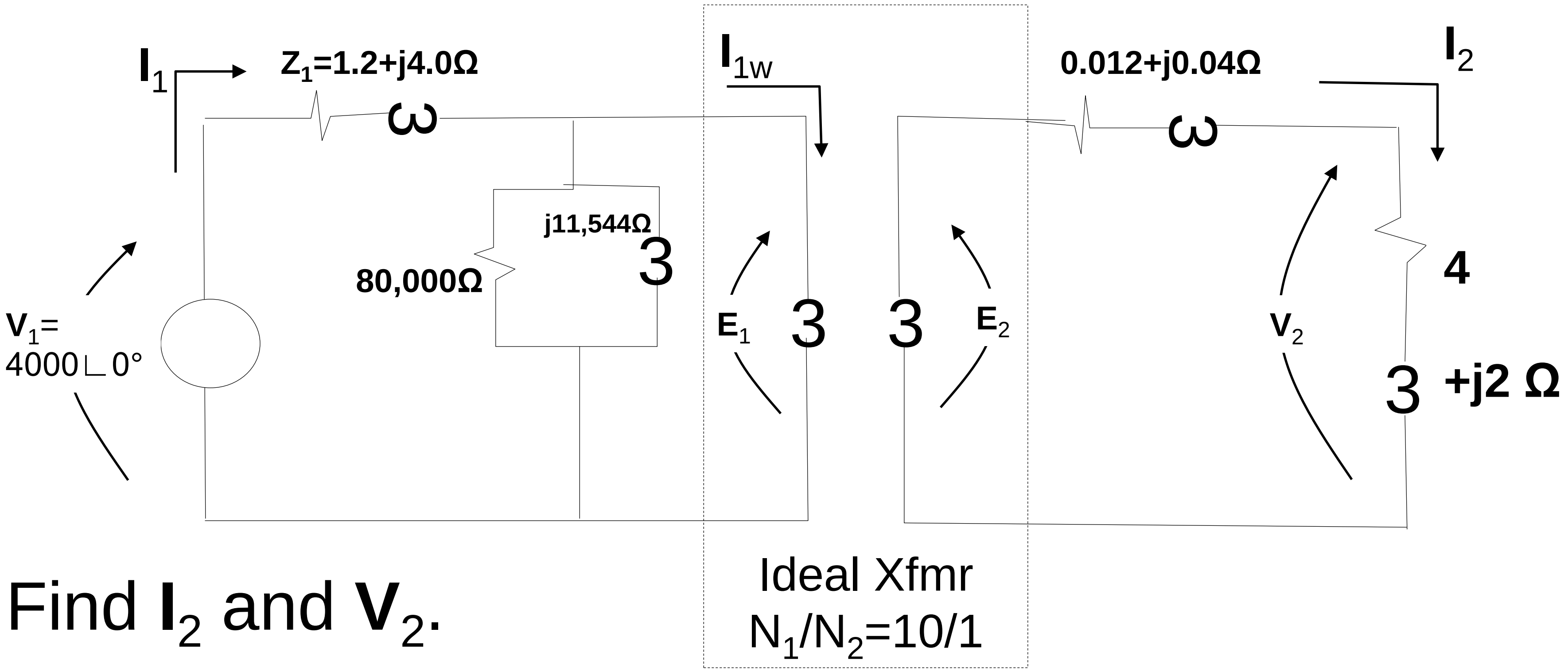
$$\boldsymbol{I}_1 = \boldsymbol{I}_2' = \frac{50\angle 0^\circ}{1} = 50\angle 0^\circ$$

$$\frac{\boldsymbol{I}_2}{\boldsymbol{I}_2'} = \frac{N_1}{N_2} \Rightarrow \boldsymbol{I}_2 = \frac{N_1}{N_2}\boldsymbol{I}_2' = \frac{5}{10}50\angle 0^\circ = 25\angle 0^\circ$$

$$\frac{\boldsymbol{E}_2'}{\boldsymbol{E}_2} = \frac{N_1}{N_2} \Rightarrow \boldsymbol{E}_2 = \frac{N_2}{N_1}\boldsymbol{E}_2' = \frac{10}{5}50\angle 0^\circ = 100\angle 0^\circ$$

Check: $$Z_2 = \frac{\boldsymbol{E}_2}{\boldsymbol{I}_2} = \frac{100\angle 0^\circ}{25\angle 0^\circ} = 4\Omega$$

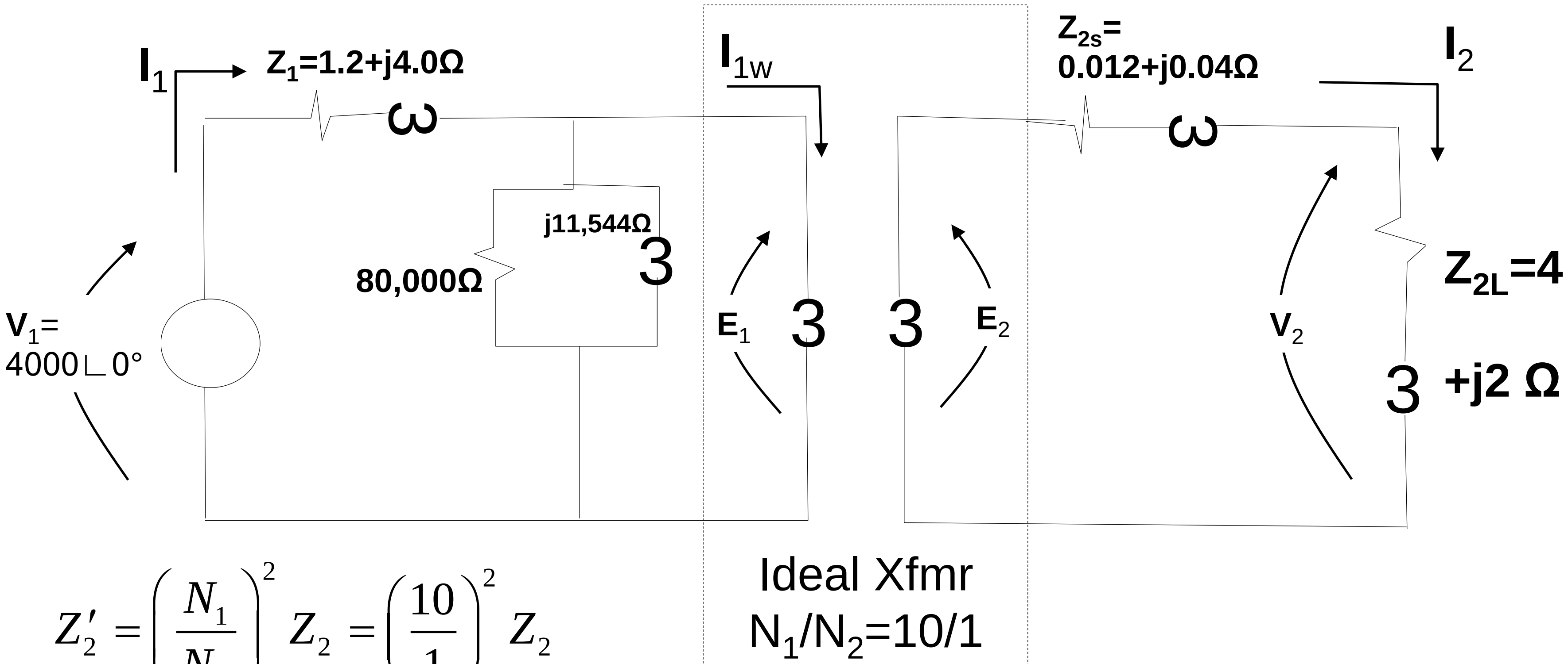
Referring Quantities: Example 9



Find $\mathbf{I}_2$ and $\mathbf{V}_2$.

First, we need to refer all quantities to one side or the other. Because there are more elements on the primary side, it is easier to refer secondary quantities to primary quantities.

Referring Quantities: Example 9



$$Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2 = \left(\frac{10}{1}\right)^2 Z_2$$

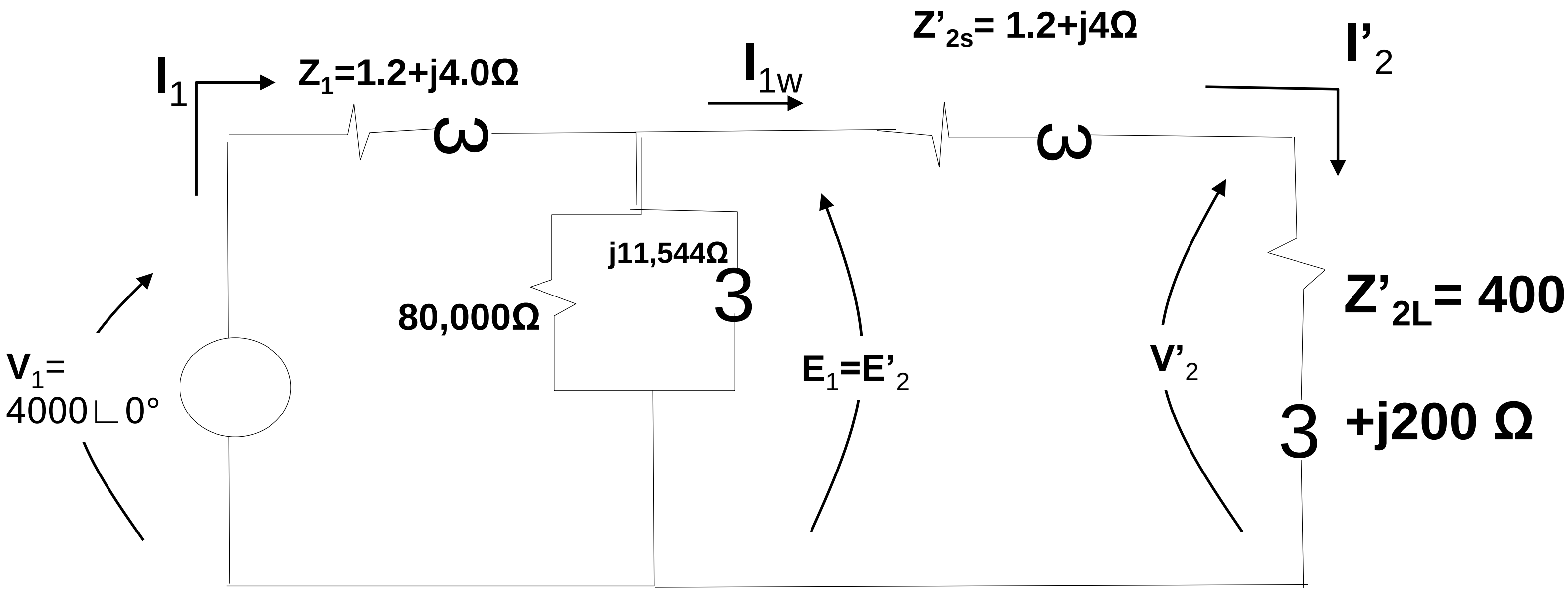
Apply to first impedance (call it Z_{2s}):

$$Z_{2s}' = \left(\frac{N_1}{N_2}\right)^2 Z_{2s} = \left(\frac{10}{1}\right)^2 (0.012 + j0.04) = 1.2 + j4\Omega$$

Apply to second impedance (call it Z_{2L}):

$$Z_{2L}' = \left(\frac{N_1}{N_2}\right)^2 Z_{2L} = \left(\frac{10}{1}\right)^2 (4 + j2) = 400 + j200\Omega$$

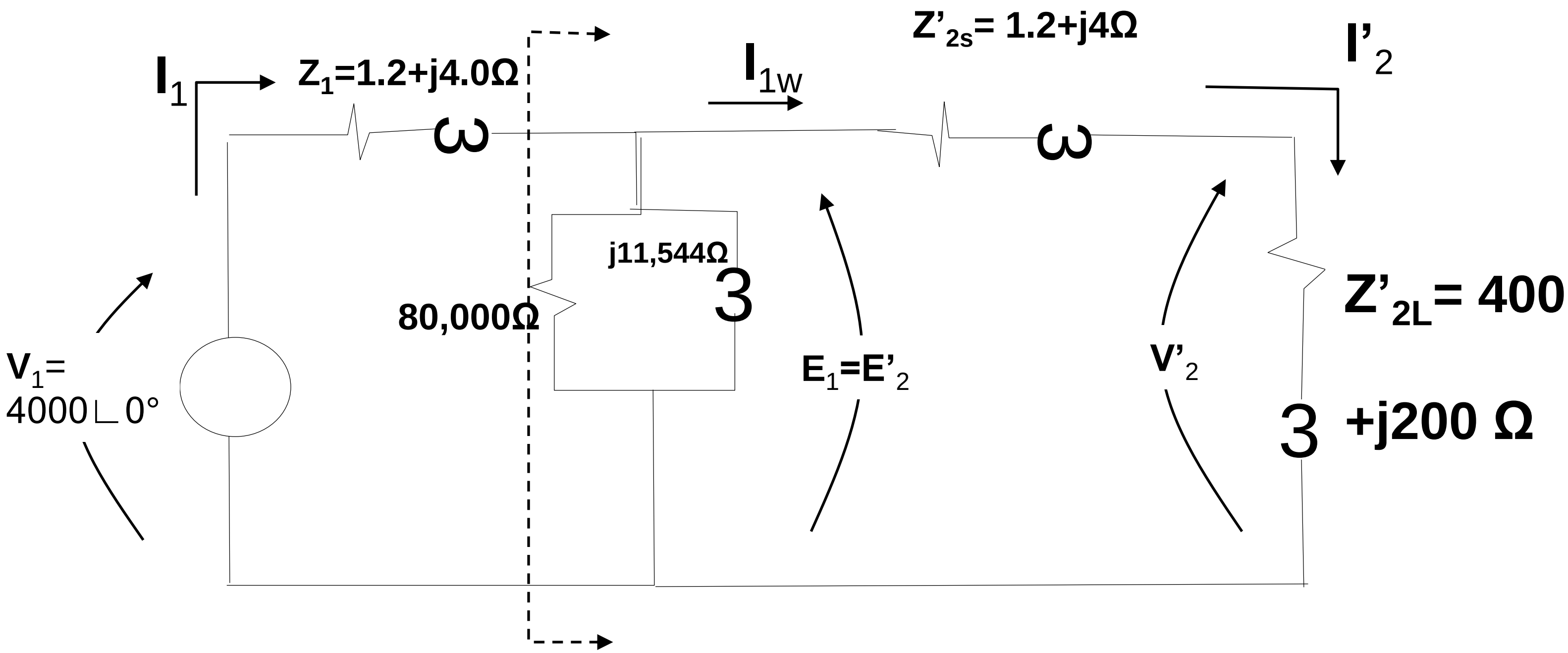
Referring Quantities: Example 9



Now we have a circuit to solve. Recall our goal is to find $\mathbf{I}_2$ & $\mathbf{V}_2$.
I am going to do this as follows:

1. Get Z_{eq} =80,000//j11,544//(401.2+j204) (parallel combination)
2. Get $\mathbf{I}_1=\mathbf{V}_1/[Z_1+Z_{eq}]$.(Ohm's law)
3. Use KVL or voltage division to get $\mathbf{E}'_2$
4. Get $\mathbf{I}_{1w}=\mathbf{I}'_2=\mathbf{E}'_2/[Z'_{2s}+Z'_{2L}]$ (Ohm's law)
5. Get $\mathbf{V}'_2=\mathbf{I}'_2[Z'_{2L}]$ (Ohm's law)
6. Refer $\mathbf{I}'_2$ and $\mathbf{V}'_2$ back to the secondary (turns ratio)

Referring Quantities: Example 9

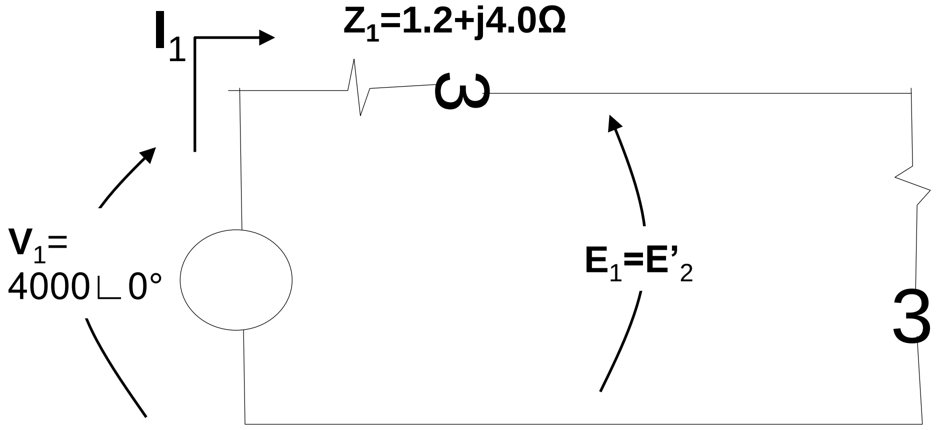


1. Get Z_{eq} =80,000//j11,544//(401.2+j204). This is a parallel combination of the 80,000Ω resistor, the j11,544Ω inductor, and the series combination $Z'_{2a}+Z'_{2L}$, as indicated by the dotted arrows above. Recalling the parallel combination of three impedances is given by

$$Z_{eq} = \frac{Z_a Z_b Z_c}{Z_a Z_b + Z_b Z_c + Z_a Z_c}$$

$$Z_{eq} = \frac{(80,000)(j11,544)(401.2 + j204)}{(80,000)(j11,544) + (j11,544)(401.2 + j204) + (80,000)(401.2 + j204)} = 385 + j218$$

Referring Quantities: Example 9



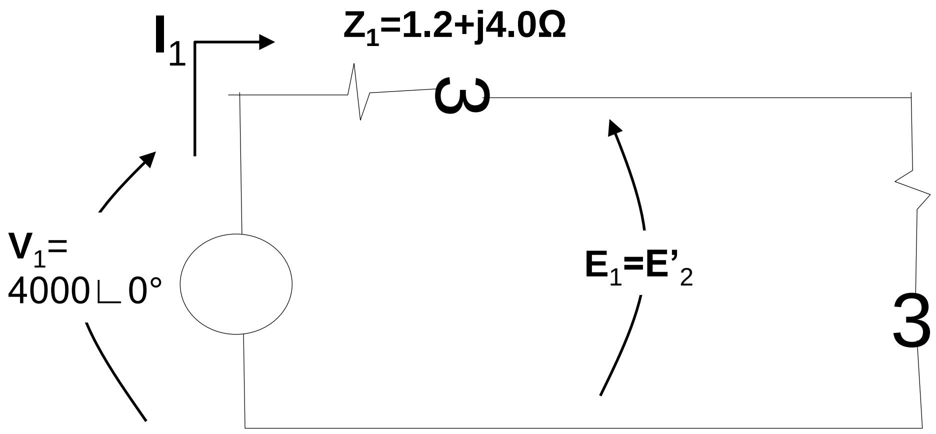
$$Z_{eq} = 385 + j218$$

2. Get $\mathbf{I}_1=\mathbf{V}_1/[Z_1+Z_{eq}]$. This is just an application of Ohm's law

$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_1}{Z_1 + Z_{eq}} = \frac{4000\angle 0°}{(1.2 + j4) + (385 + j218)} = 8.98\angle -29.9°$$

<u>*Comment*</u>: We really don't need $\mathbf{I}_1$, because we can get $\mathbf{E}_1=\mathbf{E}'_2$ (step 3) by voltage division. But if you want to perform step 3 by KVL, we do need $\mathbf{I}_1$.

Referring Quantities: Example 9



$$Z_{eq} = 385 + j218$$

3. Use KVL or voltage division to get **E'**$_2$

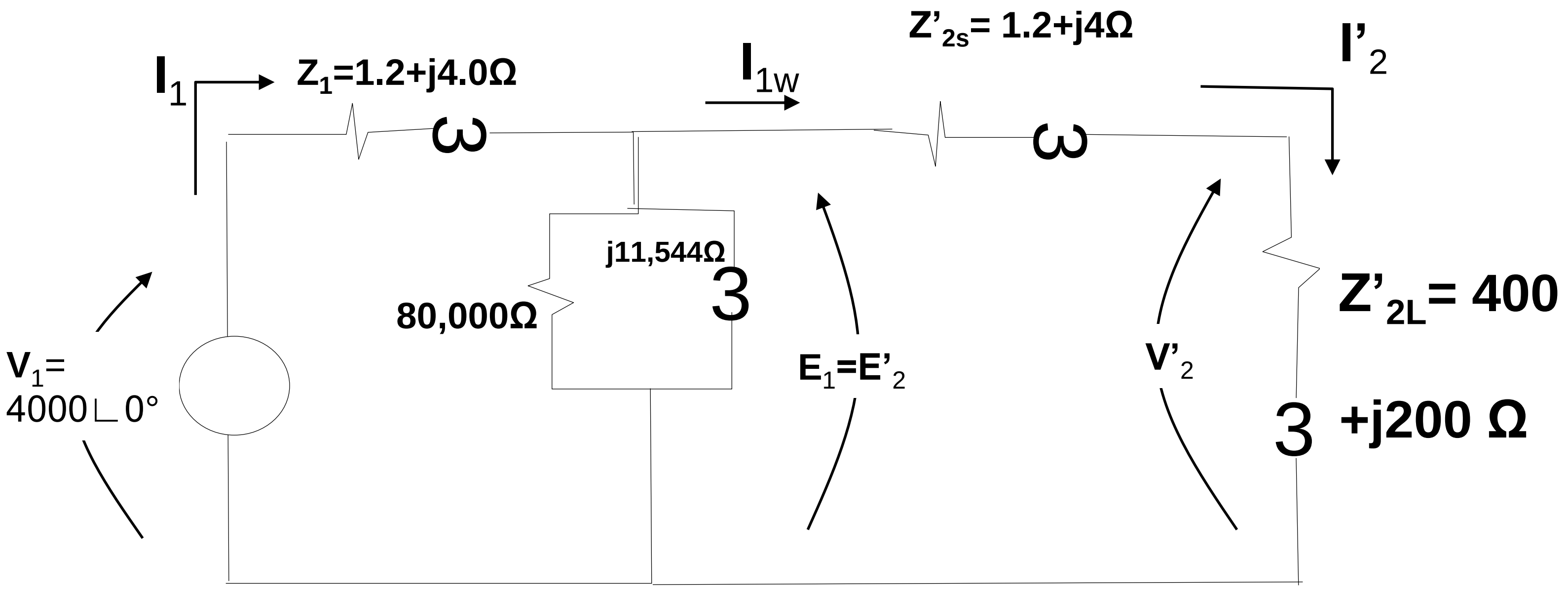
I will first do it by KVL:

$$\boldsymbol{E}_1 = \boldsymbol{E}_2' = \boldsymbol{V}_1 - \boldsymbol{I}_1 Z_1$$
$$= 4000\angle 0° - (8.98\angle -29.9°)(1.2 + j4)$$
$$= 3973\angle -0.4°$$

Now do it by voltage division:

$$\boldsymbol{E}_1 = \boldsymbol{E}_2' = \boldsymbol{V}_1 \left[\frac{Z_{eq}}{Z_1 + Z_{eq}} \right] =$$
$$= 4000\angle 0° \left[\frac{385 + j218}{(1.2 + j4) + (385 + j218)} \right]$$
$$= 3973\angle -0.4°$$

Referring Quantities: Example 9



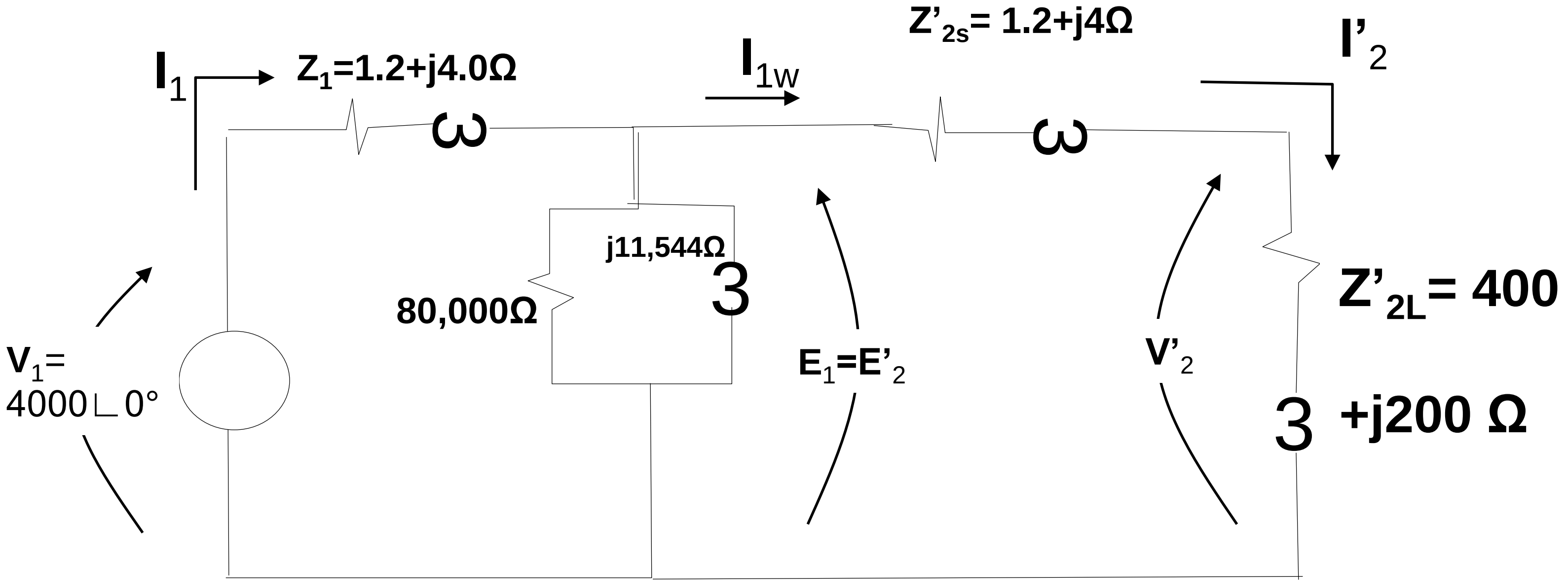
4. Get $\mathbf{I}_{1w}=\mathbf{I}'_2=\mathbf{E}'_2/[Z'_{2s}+Z'_{2L}]$ (Ohm's law).

$$\boldsymbol{I}_{1w} = \boldsymbol{I}'_2 = \frac{\boldsymbol{E}_2'}{Z'_{2s} + Z_{2L}'}$$

$$= \frac{3973\angle -0.4^\circ}{(1.2 + j4) + (400 + j200)}$$

$$= 8.83\angle -27.4^\circ$$

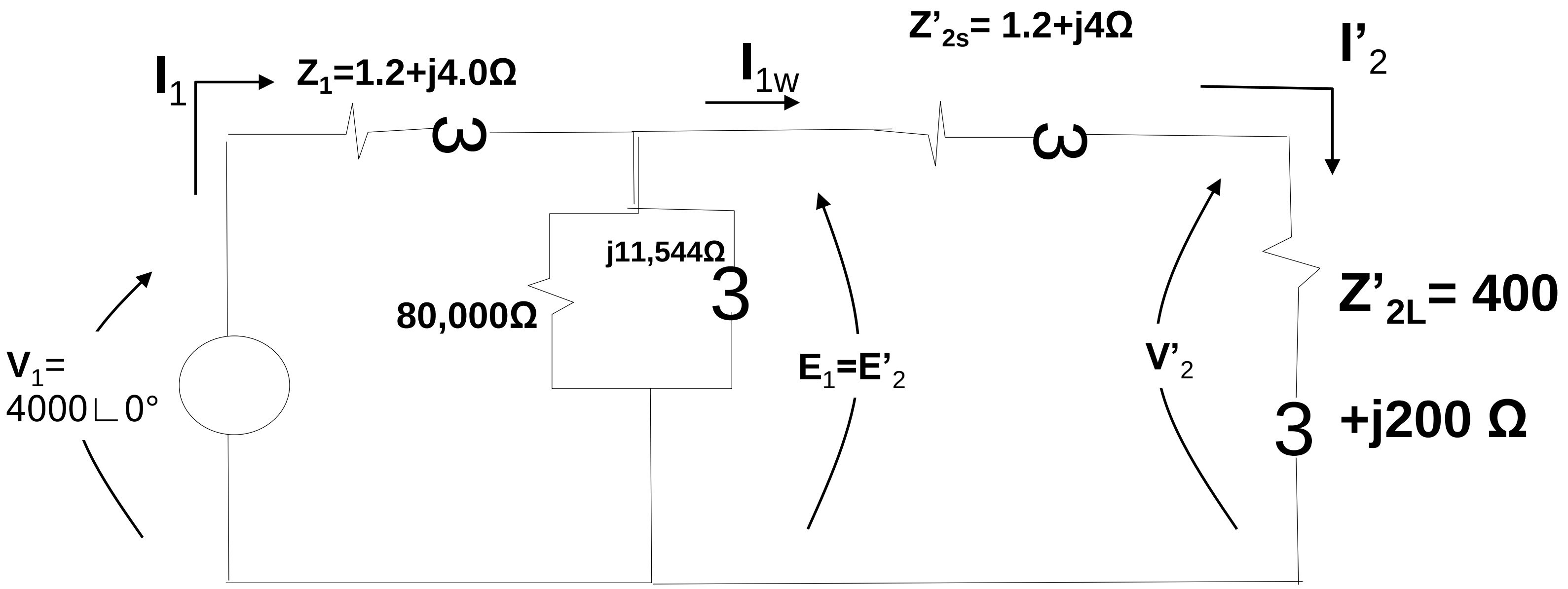
Referring Quantities: Example 9



5. Get **V'**$_2$=**I'**$_2$[Z'$_{2L}$] (Ohm's law)

$$\boldsymbol{V_2'} = \boldsymbol{I'}_2\boldsymbol{Z'}_{2L} = (8.83\angle -27.4°)(400 + j200)$$
$$= 3950\angle -0.8°$$

Referring Quantities: Example 9



6. Refer **I'**$_2$ and **V'**$_2$ back to the secondary (turns ratio)

$$\boldsymbol{V}_2' = 3950\angle -0.8^\circ$$

$$\Rightarrow \boldsymbol{V}_2 = \boldsymbol{V}_2' \frac{N_2}{N_1} = 3950\angle -0.8^\circ \frac{1}{10} = 395\angle -0.8^\circ$$

$$\boldsymbol{I'}_2 = 8.83\angle -27.4^\circ$$

$$\Rightarrow \boldsymbol{I}_2 = \boldsymbol{I'}_2 \frac{N_1}{N_2} = 8.83\angle -27.4^\circ \frac{10}{1} = 88.3\angle -27.4^\circ$$

Comment

I strongly encourage you to read chapters 5 and 6 in Kirtley's text.

- Chapter 5: Magnetic circuits
- Chapter 6: Transformers

Don't get stuck. Read through whole thing.

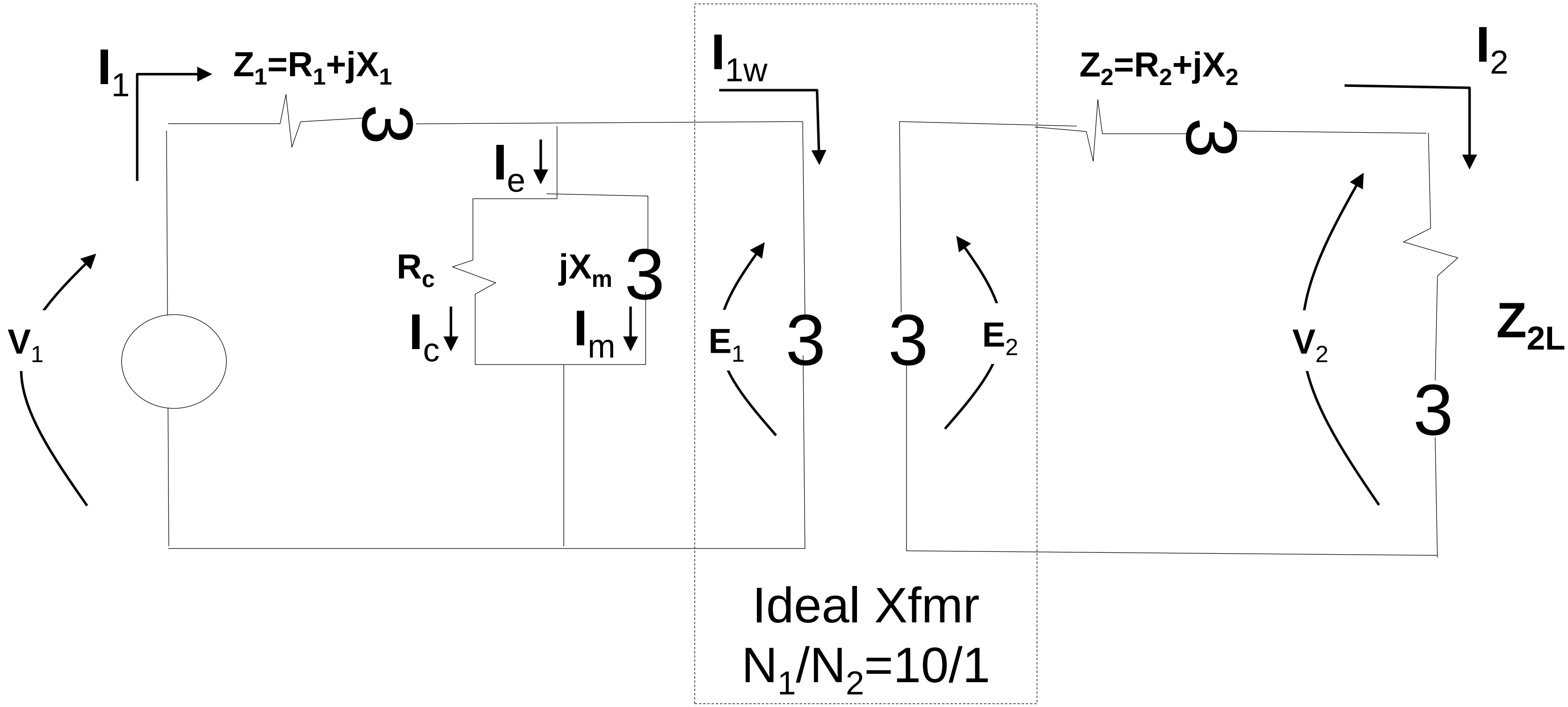
Find where he addresses the same thing I address.

- Identify what he emphasizes that is different.
- Identify what he emphasizes that is similar.

Be motivated to learn/understand so as to do well in this and other classes.

Be motivated to learn/understand engineering to help you become a competent engineer in the workplace.

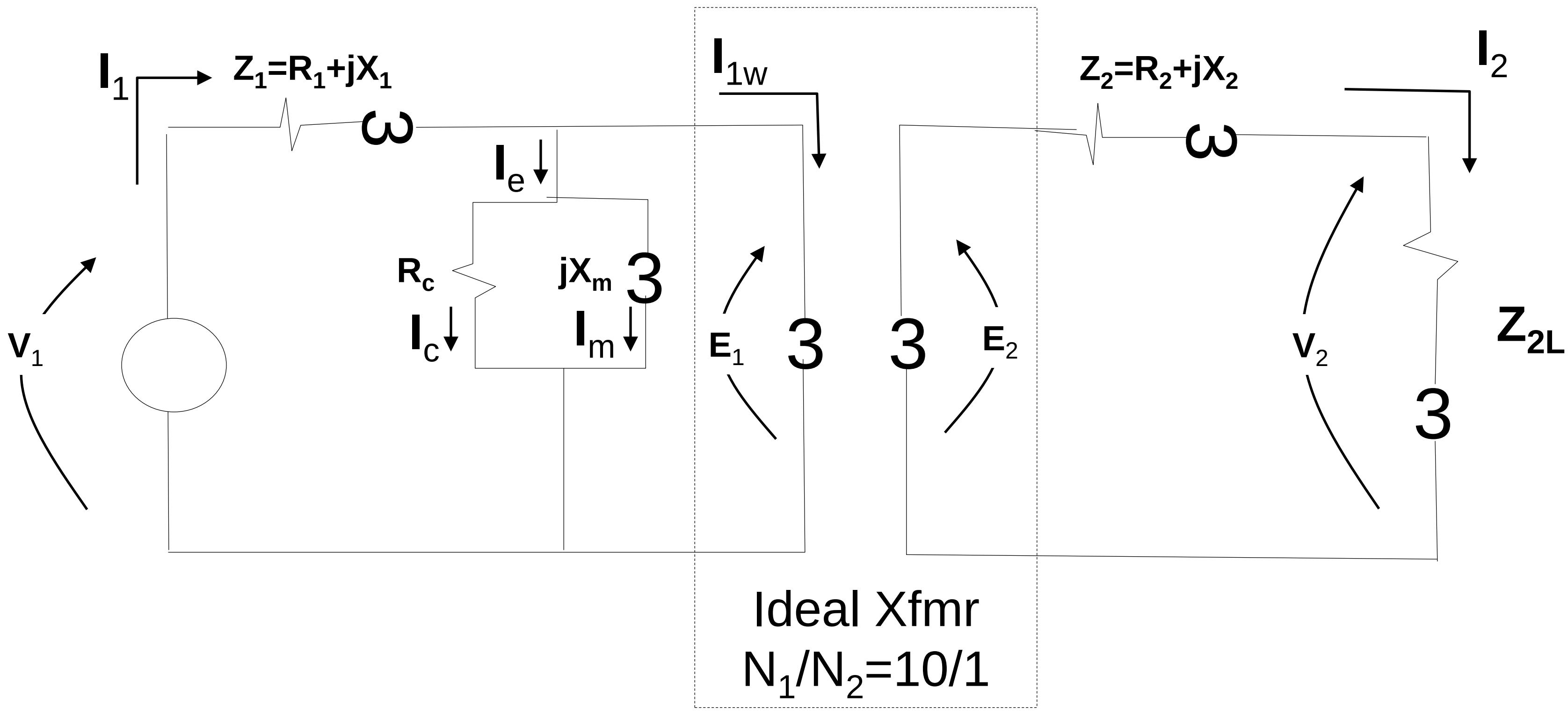
Exact & approximate transformer models



The circuit we have been using in the last example is actually the "exact equivalent model" of the xfmr. (Aside: "Exact" and "model" is an oxymoron.)

Note the nomenclature given to the voltages, currents, and impedances above. Let's define them.

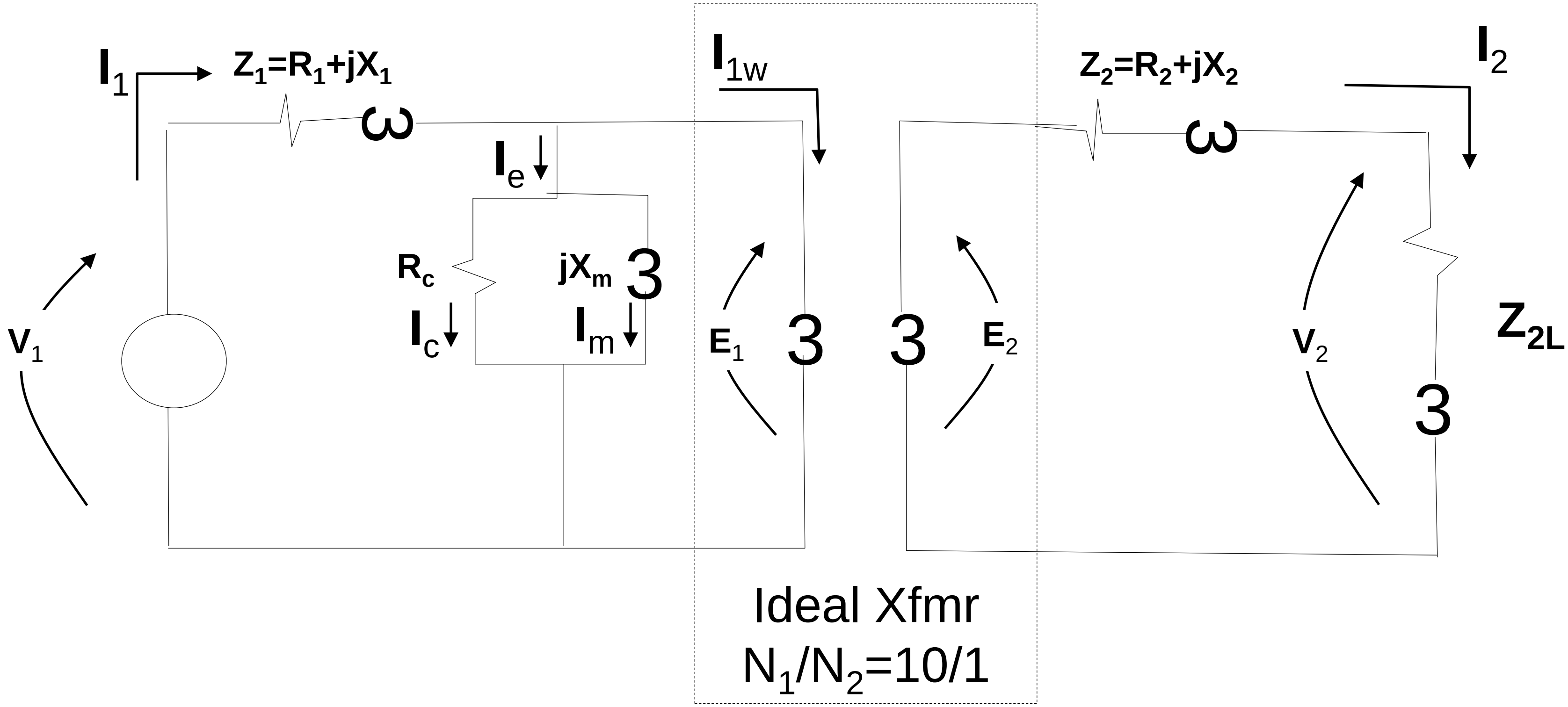
Exact & approximate transformer models



- Z_1, Z_2: series impedances of each side
- R_1, R_2: winding resistances of each side
- X_1, X_2: leakage reactance of each side
- R_c: core loss resistance; represents losses due to eddy currents and hysteresis.
- X_m: magnetizing inductance; represents current necessary to overcome the core reluctance in setting up flux.

- $\mathbf{V}_1$, $\mathbf{V}_2$: Source, load voltages, respectively
- $\mathbf{E}_1$, $\mathbf{E}_2$: Voltages across internally-modeled prim & sec coils, respectively.
- $\mathbf{I}_1$, $\mathbf{I}_2$: Currents into and out of xfmr prim & sec terminals, respectively.
- $\mathbf{I}_{1w}$, $\mathbf{I}_2$: Currents in internally-modeled prim & sec coils, respectively.
- I_e: Exciting current.
- I_c, I_m: Core loss &magnetizing currents

Exact & approximate transformer models

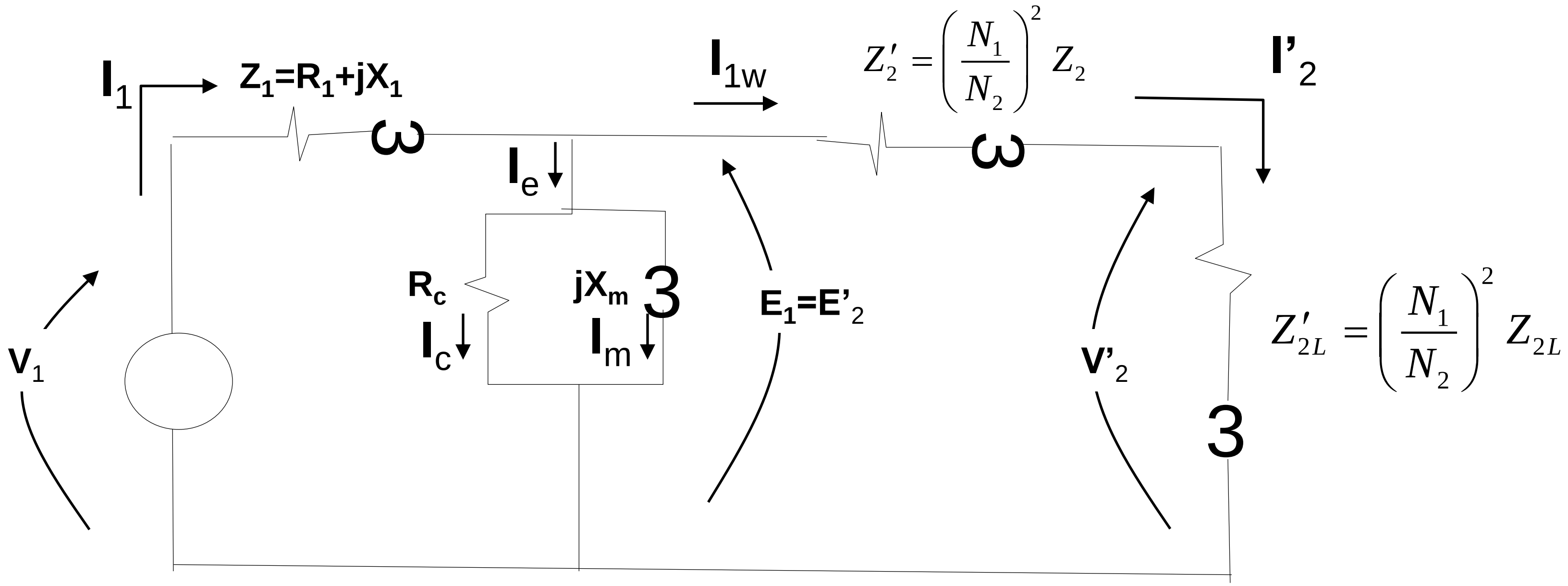


Let's illustrate how to refer secondary-side quantities to primary side. There are only two of them: Z_2, Z_{2L}.

$$Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2 \qquad Z_{2L}' = \left(\frac{N_1}{N_2}\right)^2 Z_{2L}$$

Now draw the circuit…

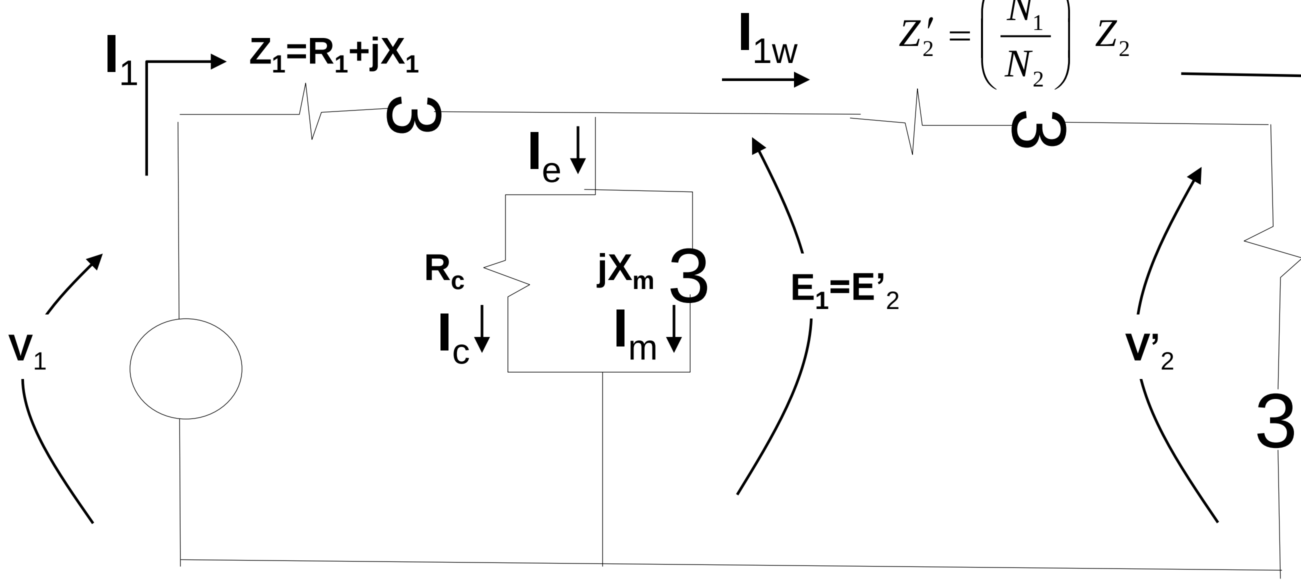
Exact & approximate transformer models



Observe that the ideal transformer is no longer used.

This model is called the "exact equivalent model referred to the primary."

Exact & approximate transformer models



$$Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2$$
$$Z_{2L}' = \left(\frac{N_1}{N_2}\right)^2 Z_{2L}$$

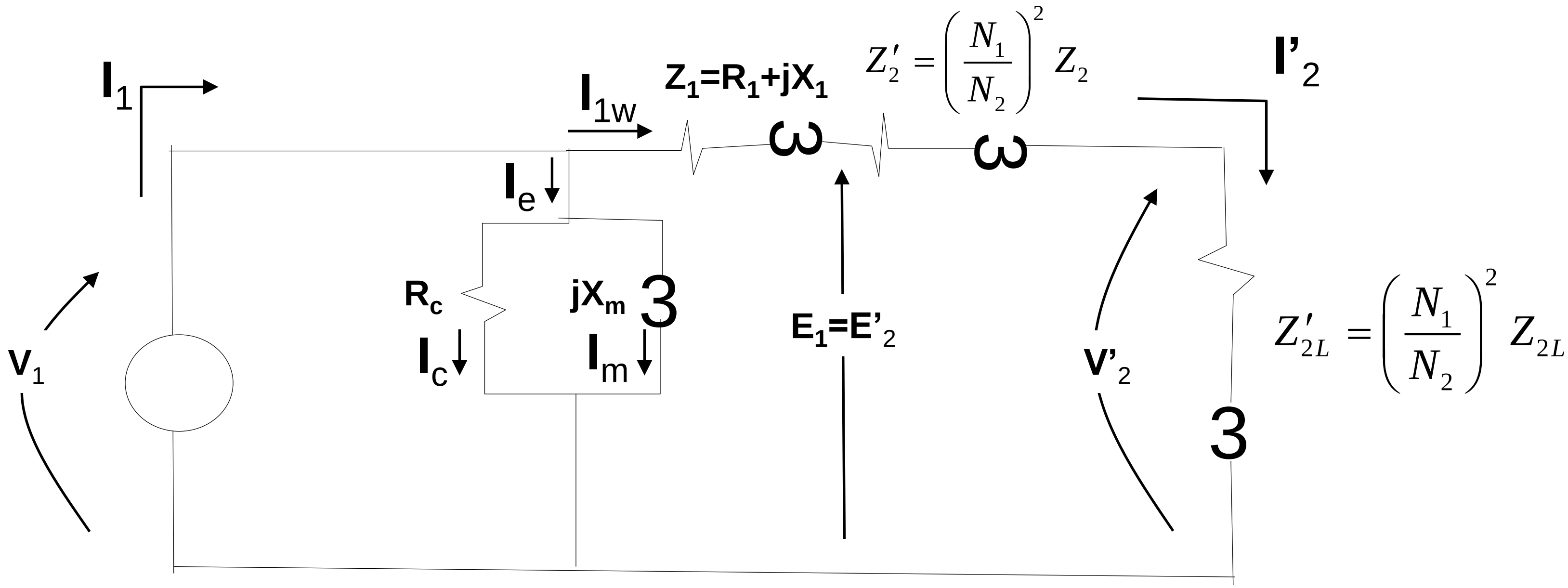
Fact: The shunt element, $R_C//jX_m$, is actually quite large, in comparison to the series impedances Z_1, Z'_2.

$$\left|R_c // jX_m\right| >> \left|R_1 + jX_1\right|$$

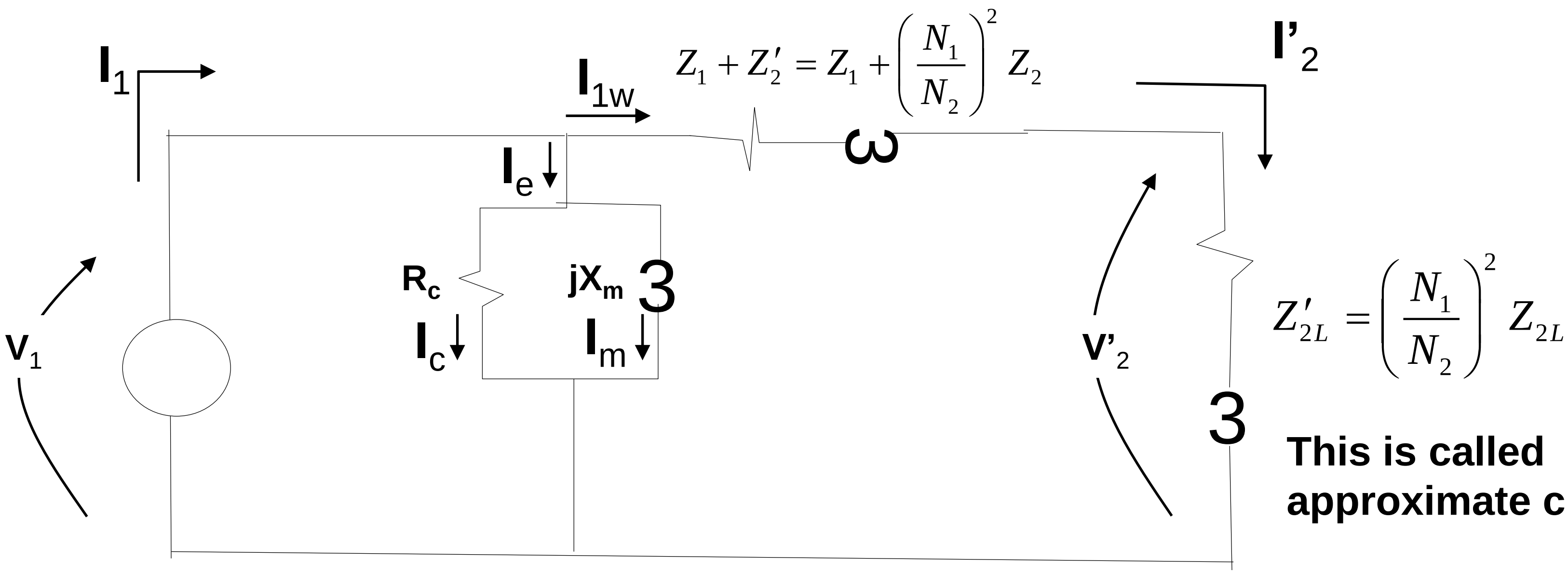
This implies

1. Voltage drop across $Z_1$is small; most of $\mathbf{V}_1$ appears across shunt $R_C//jX_m$.
2. I_e very small; current thru Z_1 same as current thru Z_2.

Exact & approximate transformer models



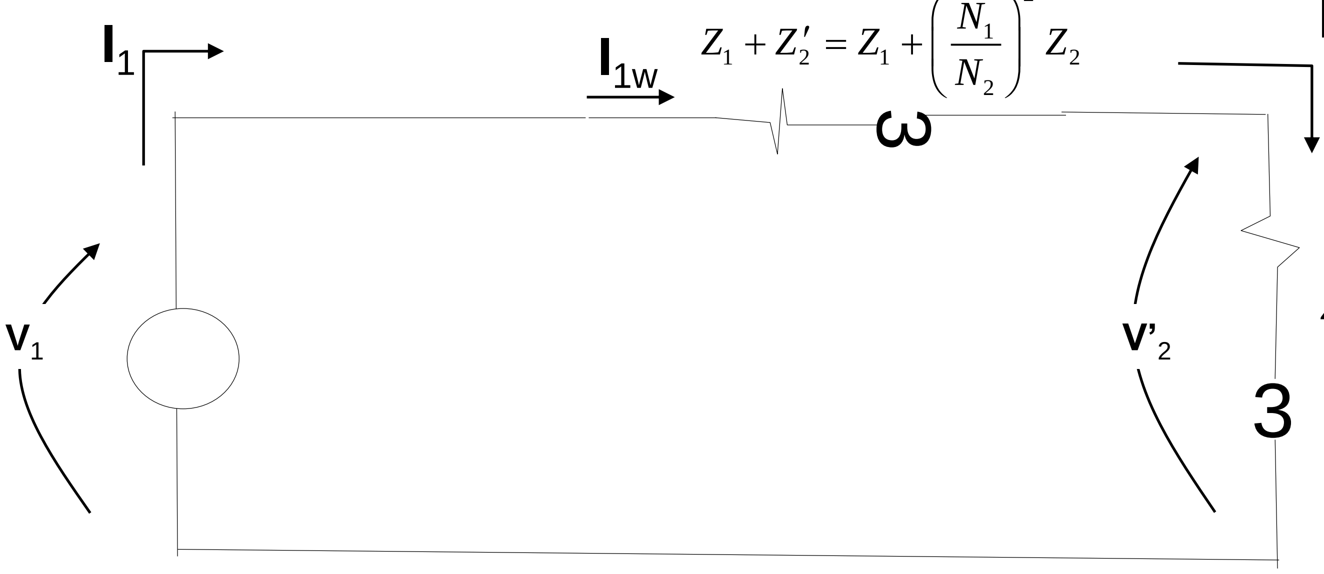
Combining Z_1 and Z_2, the above becomes:



This is called approximate circuit #1.

Exact & approximate transformer models

But note that with approximate circuit #1, the voltage seem by the portion of the circuit with $Z_1+Z'_{2L}$ is $\mathbf{V}_1$. This implies that the shunt $R_c//jX_m$ does not affect $\mathbf{I'}_2$. Thus, if I am not interested in loss analysis (and therefore don't care about $I_c^2R_c$), meaning I am mainly interested in voltage drop across the transformer, then the below is a good model.



$$Z_1 + Z_2' = Z_1 + \left(\frac{N_1}{N_2}\right)^2 Z_2$$

$$Z_{2L}' = \left(\frac{N_1}{N_2}\right)^2 Z_{2L}$$

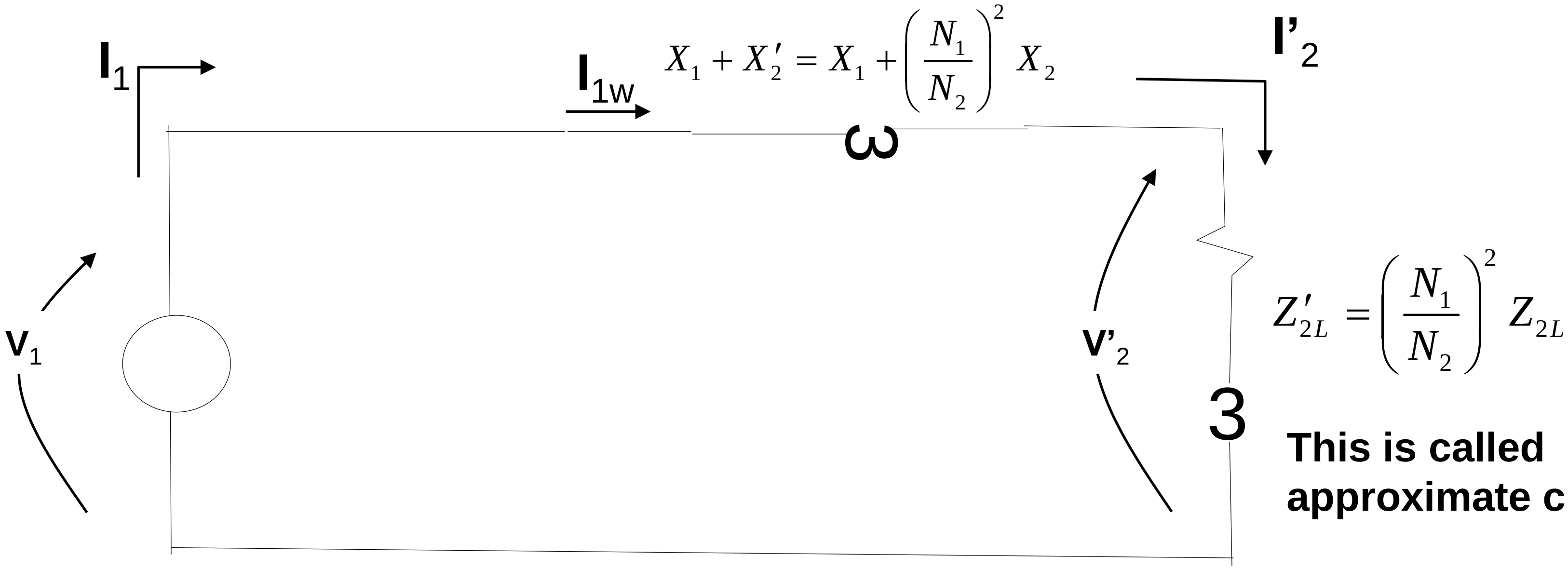
This is called approximate circuit #2.

Exact & approximate transformer models

A final model results from the fact that, for a transformer, the series reactance is significantly larger than the series impedance, i.e., with $Z_1=R_1+jX_1$, $Z'_2=R'_2+jX'_2$:

$$X_1 + X_2' >> R_1 + R_2'$$

Then the following VERY simple model becomes quite reasonable. Indeed, this model, consisting of a single reactance, is often used in analysis of large power systems.



This is called approximate circuit #3.