Some practice problems

Remember (also indicated in slides called G1a):

In module G1: HW Probs 1, 4, 8, 9, 10, 11. Due: 3/14 (to turn in).

The power angle

$$\bar{E}_f = \bar{V}_t + jX_s \bar{I}_a$$

Let's define the angle that $\overline{\mathrm{E}}_{\mathrm{f}}$ makes with $\overline{\mathrm{V}}_{\mathrm{t}}$ as δ

$$\bar{E}_f = E_f \angle \delta$$

For generator operation (power supplied by machine), this angle is always positive.

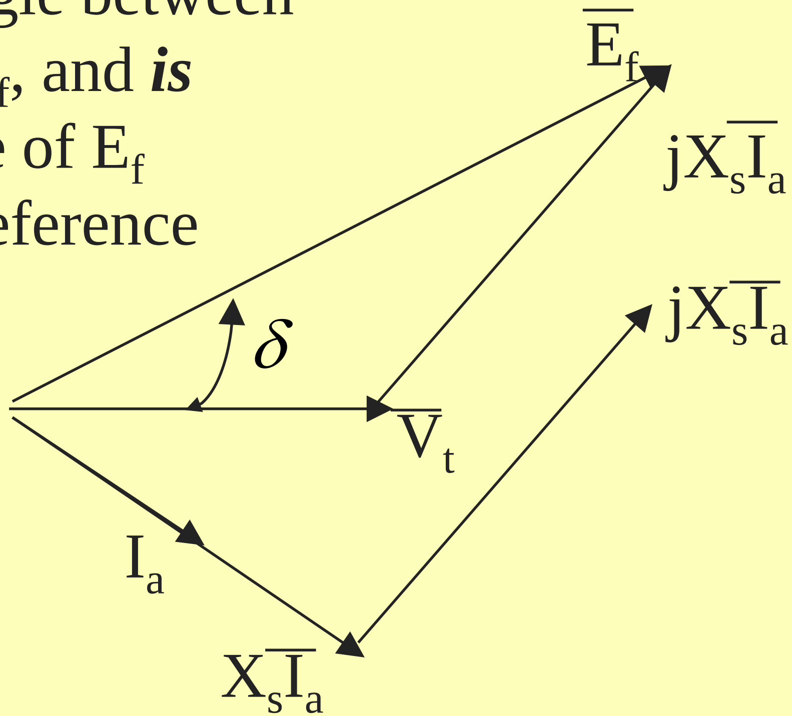
For motor operation, this angle is negative.

The power angle

$$\bar{E}_f = \bar{V}_t + jX_s\bar{I}_a$$

$$\bar{E}_f = E_f \angle \delta$$

The power angle is the angle between V_t and E_f, and ***is*** the angle of E_f if V_t is reference



This is for generator operation.

You should see if you can construct the phasor diagram for motor operation, lagging operation (just reverse the arrow on the current)

Example

A 10 MVA, 3 phase, Y-connected, two pole, 60 Hz, 13.8 kV (line to line) generator has a synchronous reactance of 20 ohms per phase. Find the excitation voltage if the generator is operating at rated terminal voltage and supplying (a) 300 Amperes at 30 degrees lagging, (b) 300 Amperes at 30 degrees leading

Example (cont'd)

Solution

$$\bar{V}_t = \frac{13.8kV}{\sqrt{3}} = 7.97kV \Rightarrow \bar{V}_t = 7.97\times10^3\angle 0°$$

$$\bar{E}_f = \bar{V}_t + jX_s\bar{I}_a$$

(a) $\bar{I}_a = 300\angle -30° \Rightarrow \bar{E}_f = 7.97\times10^3\angle 0° + (20\angle 90°)(300\angle -30°)$

$= 12.14kV\angle 25.34°$

(b) $\bar{I}_a = 300\angle 30° \Rightarrow \bar{E}_f = 7.97\times10^3\angle 0° + (20\angle 90°)(300\angle 30°)$

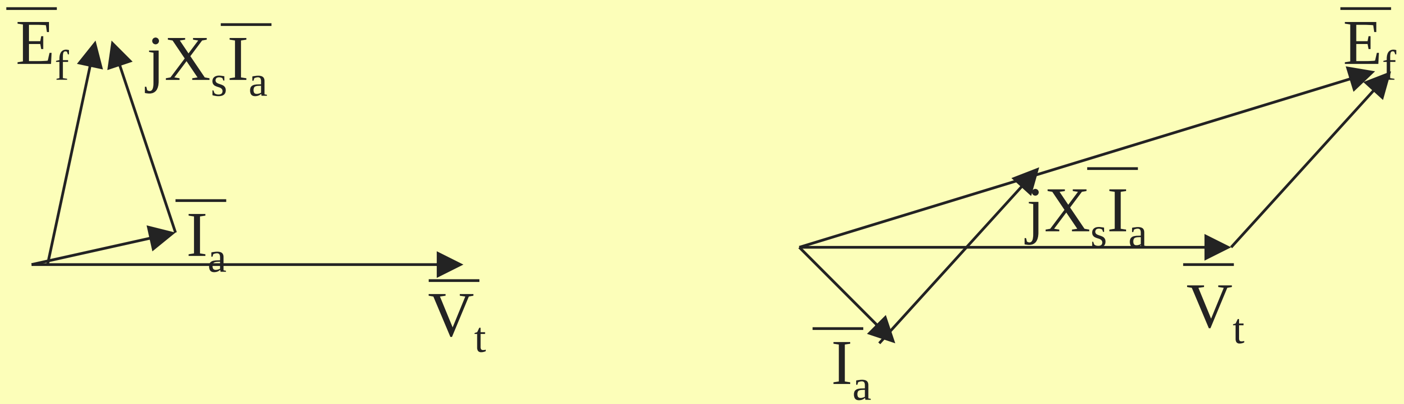
$= 7.19kV\angle 46.27°$

Note: 1) $E_{f-lag} > E_{f-lead}$

2) lag case referred as overexcited operation

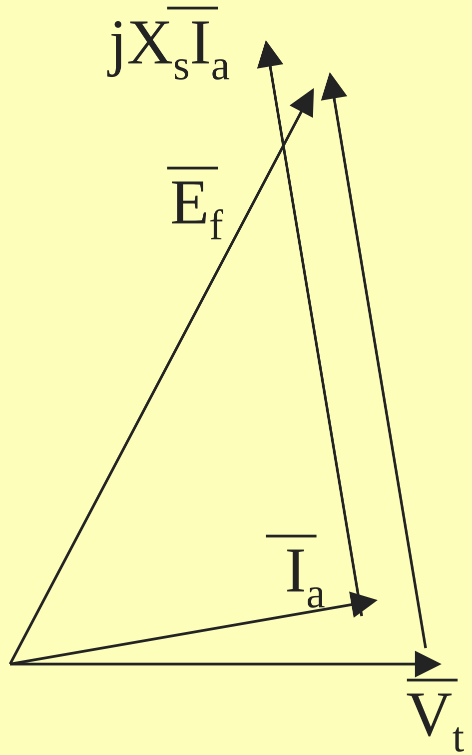
3) lead case referred as underexcited operation

It is usually the case that excitation voltage $\overline{E}_f$ is lower in magnitude for leading operation as compared to lagging operation.

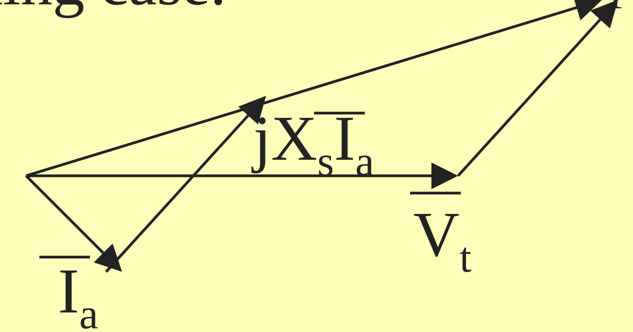


Implication: If E_f is large (small), relative to V_t, machine is <u>probably</u> lagging (leading).

It is usually the case that excitation voltage $\overline{E}_f$ is lower in magnitude for any leading operation as compared to any lagging operation, **but not always.**



Here note that E_f for leading case is larger than for lagging case. This is due to much larger I_a in the leading case.



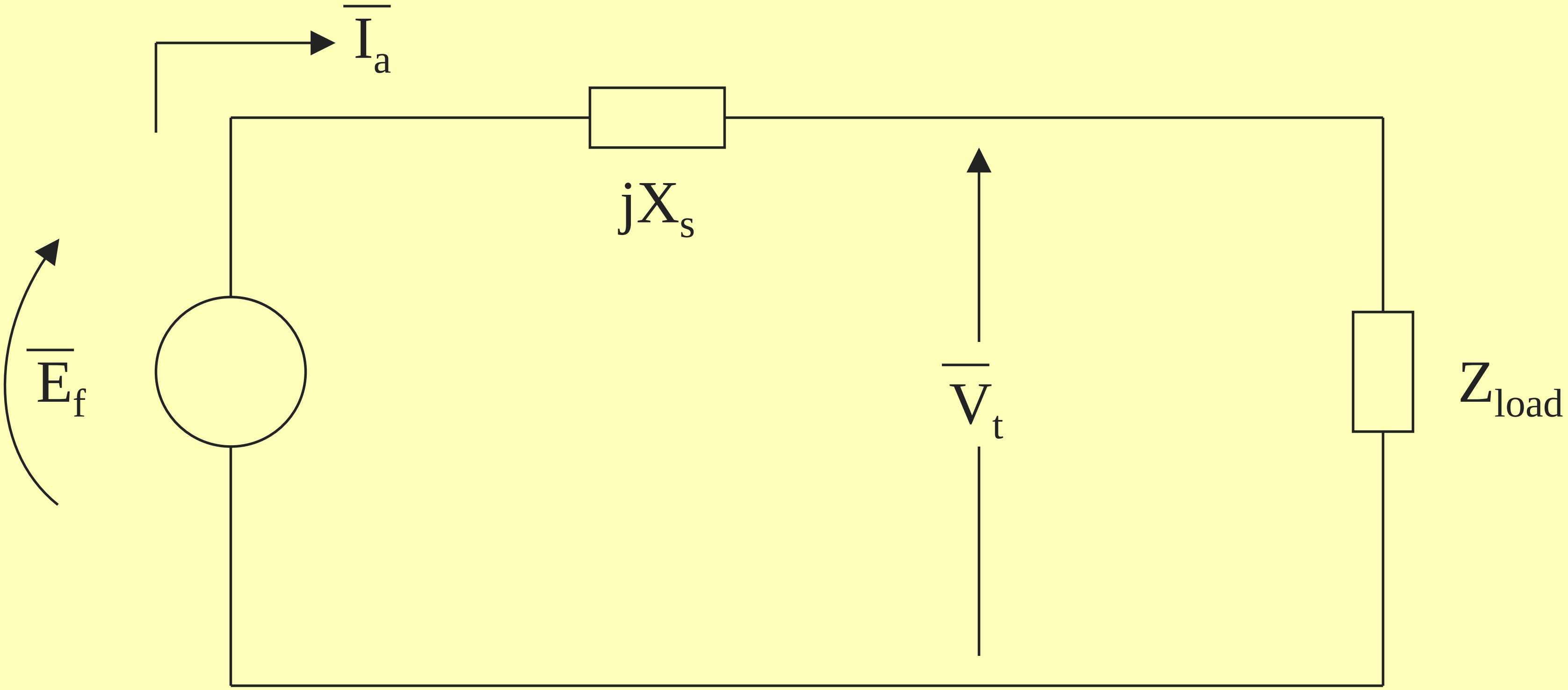
So how can we tell, using $\overline{E}_f$ and $\overline{V}_t$, whether the generator is lagging or leading?

We need to look at power relations for the answer.

We will derive power relations that do not require knowledge of the current.

Power relationships

Recall the power angle, δ, as the angle at which the excitation voltage, $\bar{E}_f = E_f \angle \delta$, leads the terminal voltage, $\bar{V}_t = V_t \angle 0^\circ$. Therefore, from the circuit….



Power relationships

$$\bar{I}_a = \frac{E_f\angle\delta - V_t\angle 0^\circ}{jX_s} = \frac{E_f\cos\delta + jE_f\sin\delta - V_t}{jX_s}$$

$$= \frac{E_f\cos\delta - V_t}{jX_s} + \frac{jE_f\sin\delta}{jX_s}$$

$$= \frac{E_f\sin\delta}{X_s} - j\left[\frac{E_f\cos\delta - V_t}{X_s}\right] \qquad (1)$$

But $\bar{I}_a = I_a\cos\theta - jI_a\sin\theta$ (2)

Power relationships (cont'd)

Equating real and imaginary parts of eqs. 1 and 2 and multiplying both sides of the equations by $3V_t$:

$$P_{out} = 3V_t I_a \cos\theta = \frac{3V_t E_f \sin\delta}{X_s} \qquad (3)$$

$$Q_{out} = 3V_t I_a \sin\theta = \frac{3V_t E_f \cos\delta}{X_s} - \frac{3V_t^2}{X_s} \qquad (4)$$

Note: reactive power is positive when the machine is operated overexcited, i.e., when it is lagging

Example

Find Pout and Qout for the conditions (a) and (b) described in the previous example.

Solution:

(a) $\delta = 25.34°, V_t = 7.97kV, E_f = 12.14kV$

$$\Rightarrow P_{out} = \frac{3(7.97 \times 10^3)(12.14 \times 10^3)\sin 25.34°}{20} = 6.21 \; MW$$

$$\Rightarrow Q_{out} = \frac{3(7.97 \times 10^3)(12.14 \times 10^3)\cos 25.34°}{20} - \frac{3(7.97 \times 10^3)^2}{20} = 3.59 \; MVARS$$

Example (cont'd)

(b) $\delta = 46.27°, V_t = 7.97kV, |E_f| = 7.19kV$

$$\Rightarrow P_{out} = \frac{3(7.97\times10^3)(7.19\times10^3)\sin 46.27°}{20} = 6.21\ MW$$

$$\Rightarrow Q_{out} = \frac{3(7.97\times10^3)(7.19\times10^3)\cos 46.27°}{20} - \frac{3(7.97\times10^3)^2}{20} = -3.59\ MVARS$$

Example (cont'd)

Consider:

- Why is real power the same under the two conditions?
- When the generator is operating lagging, is it absorbing vars from or supplying vars to the network? What about when the generator is operating leading?
- For a particular angle θ, how are the terms "lagging" and "leading" meaningful with respect to real power? With respect to reactive power?

1.0 power factor condition in terms of Ef, Vt, and delta

We have seen that gen lagging operation results in +Qout and leading operation results in -Qout. So 1.0 power factor implies Qout=0. Applying this to reactive power equation:

1.0 power factor condition

$$Q_{out} = 0 = 3V_t I_a \sin\theta = \frac{3V_t E_f \cos\delta}{X_s} - \frac{3V_t^2}{X_s}$$

$$\Rightarrow \frac{3V_t E_f \cos\delta}{X_s} = \frac{3V_t^2}{X_s}$$

$$\Rightarrow E_f \cos\delta = V_t$$

This is the condition for which the generator is operating at 1.0 power factor.

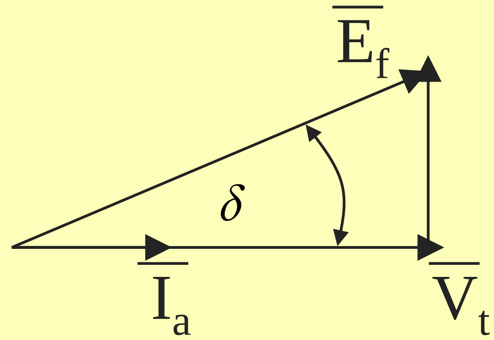
What about leading/lagging operation?

In the final equation on the previous page, just replace the equals sign by > (for lagging) or < (for leading).

$$\Rightarrow E_f \cos\delta > V_t \quad \text{(for lagging or Qout} > 0)$$

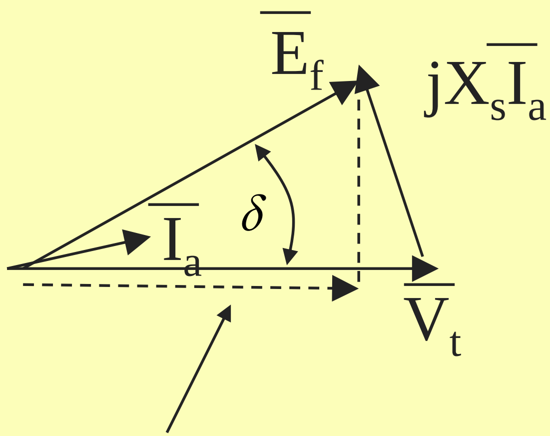
$$\Rightarrow E_f \cos\delta < V_t \quad \text{(for leading or Qout} < 0)$$

Let's look at the phasor diagrams.



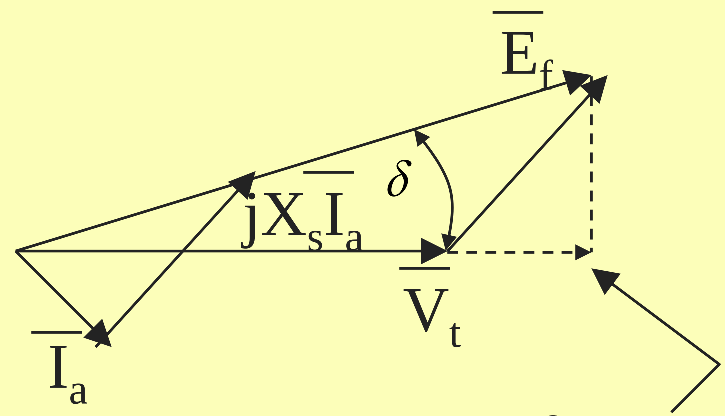
$$\Rightarrow E_f \cos\delta = V_t$$

Unity power factor condition (0 vars)



$$\Rightarrow E_f \cos\delta < V_t$$

Leading Condition (absorbing vars)



$$\Rightarrow E_f \cos\delta > V_t$$

Lagging Condition (suppling vars)

Problem 12 in Module G1

A synchronous generator having synchronous reactance of X_S=2 ohms is operating with an 18.0 line-to-line terminal voltage. The power out of the machine terminals is P_{out}=140 MW, Q_{out}=0.

a. Compute the magnitude E_f and angle $\angle\delta$ of the internal (excitation) voltage.

$$V_t = \frac{18\times10^3}{\sqrt{3}} = 10{,}392 volts,$$

$$P_{out} = \frac{3V_t\,|E_f|\sin\delta}{X_s} \Rightarrow \quad |E_f|\sin\delta = \frac{P_{out}X_s}{3V_t} = \frac{(140\times10^6)(2)}{3(10392)} = 8961.2$$

$$Q_{out} = 0 = \frac{3V_t\,|E_f|\cos\delta}{X_s} - \frac{3V_t^2}{X_s} \Rightarrow |E_f|\cos\delta = V_t = 10{,}392$$

$$\frac{|E_f|\sin\delta}{|E_f|\cos\delta} = \tan\delta = \frac{8961.2}{10392} = 0.8642 \Rightarrow \delta = 40.84^\circ$$

Another way to work this problem:

$$P_{out} = 3V_tI_a\cos\theta \Rightarrow 140E6 = 3(10.392)I_a(1) \Rightarrow I_a = 4490.6$$

Because we know operation is at unity power factor ➔ $\bar{I}_a = 4490.6\angle 0^\circ$

$$\bar{E}_f = \bar{V}_t + jX_s\bar{I}_a = 10392\angle0^\circ + j2(4490.6\angle0^\circ)$$

$$= 10392 + j8981.2 = 13735\angle40.83^\circ \Rightarrow \delta = 40.83^\circ$$

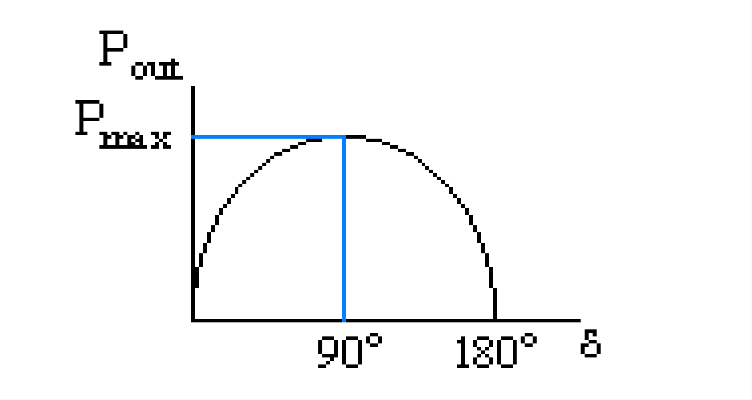
b. A large reactor (e.g., inductive element) is suddenly connected in parallel with the load R_L, and the field current is adjusted. Indicate what would happen to each of the below by checking the appropriate space:

- Field current: increase__√__ decrease_____
- Reactive power out of the generator: increase__√__ decrease_____ no change_____
- Current: lead_________ lag___√____ neither________

Generation Operation: Generator pull-out power

From equ. 3, the plot of the electric power P_{out} against the power angle δ

$$P_{\max} = \frac{3V_t E_f}{X_s}$$



Generation Operation: Generator pull-out power (cont'd)

- For simplicity neglect all real power losses associated with:
  - windage and heat loss in turbine
  - friction in turbine
  - friction in generator bearings
  - winding resistances
- $P_{mech} = P_{out}$ when operating in steady-state
- if $\delta > 90$ then P_{out} decreases while P_{mech} remains unchanged $\Rightarrow P_{mech} > P_{out}$ which causes machine to accelerate beyond synchronous speed-i.e. the machine has pulled out or lost synchronism

Example

Compute the pull-out power for the two conditions described in Example 2

<u>Solution</u>

(a) Overexcited case (lagging):

$$P_{\max} = \frac{3(7.97 \times 10^3)(12.14 \times 10^3)}{20} = 14.51 \; MW$$

(b) Underexcited case (leading):

$$P_{\max} = \frac{3(7.97 \times 10^3)(7.19 \times 10^3)}{20} = 8.6 \; MW$$

Note: The limit is lower when the generator is underexcited because E_f is lower