Module D1

Introduction to Distribution Systems

Primary Author: Gerald B. Sheble, Iowa State University

Email Address: gsheble@iastate.edu

Prerequisite Competencies: Steady State analysis of circuits using phasors, three-phase circuit analysis and three-phase power relationships, found in Module B3.

Module Objectives:
1. Identify basic distribution substation equipment including transformers and protection equipment.
2. Perform distribution circuit voltage regulation and power efficiency calculations.
3. Perform power factor correction calculations for large industrial loads.
4. Identify basic wiring requirements for residential and commercial installations including wiring color codes and grounding requirements.

Distribution Overview

Energy is consumed at a nominal voltage of 120 or 240 volts for most residential equipment and 208, 240, 277 pr 480 volts for commercial and industrial customers. The power flows through a metering device to determine the amount of real power consumed by the equipment. The reactive power is provided without charge. The exception to this is if commercial or industrial customers operate at a low power factor, they could incur a power factor correction change.

The distribution system transports the complex power from the transmission grid to the customer. Distribution systems are typically radial because networked systems, although more reliable, are more expensive. Since three-phase power is more efficient than single phase, sets of three lines start from each distribution substation. However, close to the end of the radial feeder, it is often efficient to drop one or two of the three phases. Often only a single phase is used in very rural areas to reach the most remote customers. This less expensive design results in uneven loading of all three phases. The loading is equalized as much as possible by proper assignment of customers to each phase. It is the selection of the line capacities, number of phases over distance, the setting of transformers, voltage levels and customer geographic distribution and usage which complicates the engineering task of distribution design.

D1.1 Equipment Description / Functions

The equipment associated with the distribution system includes the substation transformers connected to the transmission grid (network), the distribution lines from the transformers to the customers and the protection and control equipment between the substation distribution transformer and the customer. The protection equipment includes lightning protectors, circuit breakers (with or without reclosers), disconnects, and fuses. The control equipment includes voltage regulators, capacitors, and demand side management (load control) equipment.

The substation transformers typically reduce the transmission grid voltage from 138 kV to 69, 34.5 or 12 kV. Older distribution systems may still use four kV as the primary voltage. The distribution lines are radially configured, due to cost considerations. However, large city distribution may be networked. Especially if the system is underground through a number of large buildings previously connected by steam pipes or coal distribution network.

Distribution lines are three-phase on leaving the substation but are often reduced to two-phases or one-phase for loads distant from the substation. The loading of each phase at the substation is as equal as possible barring unusual load usage patterns by assigning customers to each phase based on historical usage. The substation transformer reduces any final load imbalance by appropriate design of the secondary windings.

The loading of each phase is distributed equally under normal conditions and normal load patterns for the customers connected to the distribution line.

The control devices include voltage regulators and capacitors. Both devices attempt to raise the voltage to reduce the losses and to provide the proper voltage at the customer location. A voltage regulator is an autotransformer with tap positions to raise the voltage to a target value. A capacitor has a timer or a voltage-sensing device to connect the device when the voltage is below a target level. A new device used for large commercial or industrial customer is a static var controller (SVC). A SVC consists of inductors and capacitors that are switched into use by power electronic circuits to implement a desired control. A desired control may be a desired voltage or a desired harmonic content of the voltage (see module PQ1). Similar devices are considered as FACTS devices. FACT is a pneumonic for flexible AC transmission system. These devices use microprocessor technology to sense the real-time conditions of the system and respond, normally within a quarter of a cycle or less, to electronically switch components. Components are either inductors or capacitors.

The protection equipment includes lightning protectors, fuses, disconnects and breakers. Lightning protectors (arrestors) remove high voltage surges by switching the line to ground using spark gaps. Once the spark gap voltage limit has been exceeded, magnetic fields established by the flowing current by coils extend the arc to increase the voltage need to sustain the arc, thus extinguishing the arc when the voltage returns to normal. A typical resistance versus voltage graph is shown in Figure D1.1. Note that this is a nonlinear device.

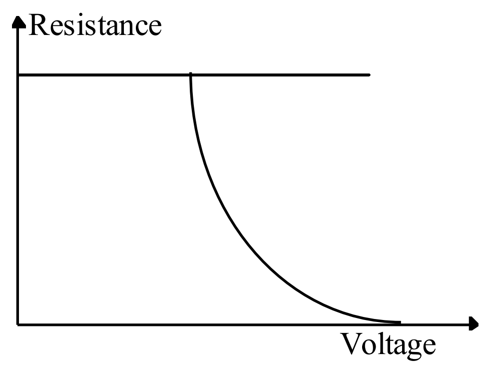


Figure D1.1 Lightning Arrestor Impedance Model

A sequence of events, shown as a graph of current versus voltage for a typical lightning arrestor is shown in Figure D1.2. As the voltage increases to the strikeover value (V_s), the device starts to conduct very quickly, resulting in a fast increase in current connected to ground. After sufficient current has passed to de-energize the excessive voltage, the current will decrease until the reclosing voltage (V_r) is reached when the device stops conducting.

Fuses and circuit breakers are used to isolate the distribution system when a fault occurs on the distribution line. A fault is simply the failure of one or more components of the system. Typical distribution faults include wire on ground after object (car, plane, etc.) hits pole, thereby breaking pole. Other faults can be temporary. One typical temporary fault is a tree limb touching the distribution wire. The tree conducts electricity to ground. Another typical temporary fault occurs when a biological object (bird, squirrel, human, etc.) connects the distribution wire to ground. A fuse is simply a piece of metal with a lower melting point than the distribution line wire. When a fault occurs on the line, a significantly higher current flows. This causes the metal to melt, an arc results clearing the path of all metal and the arc is extinguished since the normal operating voltage can not sustain the arc. The amount of overcurrent determines how quickly the fuse melts.

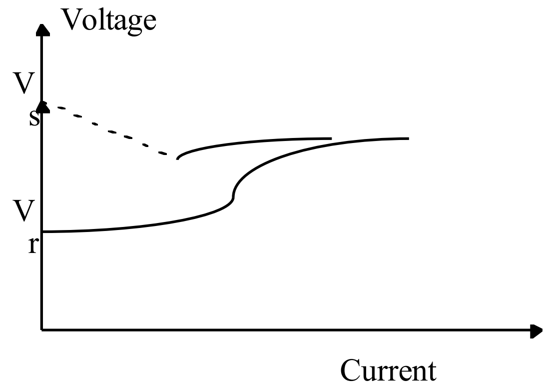


Figure D1.2 Voltage Current Characteristics of SiC Gapped Arrestor

A circuit breaker is an automatic switch, which is commanded to open when a relay senses excessive current for a defined period. The relay can be set to reclose the circuit breaker in a given sequence in the expectation that the fault is only temporary. A typical sequence is to reclose the breaker after two minutes after the initial opening. Then reclose the breaker five minutes after each subsequent opening. Normally, only three attempts are tried to close the circuit breaker. A recloser is used when the most likely cause of the fault is a tree limb or animal on the distribution equipment.

The fuse selection and the circuit breaker timing have to be coordinated such that the least amount of equipment is removed to isolate a fault.

D1.2 One-Line Diagram for Distribution System

The one-line diagram for a distribution system is the same as for the transmission grid except that the number of phases and the connections of each phase to a customer have to be recorded. Distribution maps for residential neighborhoods show routing of the distribution lines. They show where each pole or cable vault is located (numbered). Which phases are connected (continued), which wire (cable) type is used, where the protective and control equipment is located, what control and protection is present, and what settings are used to set or to control each system are also shown.

The customer load is viewed at the low voltage transformer (220/120 volts) such that the individual service drops to each house are not represented. A simplified example is shown in Figure D1.3. Note that the location of each line is shown geographically to show the locations. Most modern diagrams would show Global Positioning Satellite (GPS) coordinates and a simplified street map. Other utility services, natural gas, telephone and cable TV might be shown. A commercial distribution system includes distribution lines (cables) and customer owned equipment within the delivery point substation (or vault).

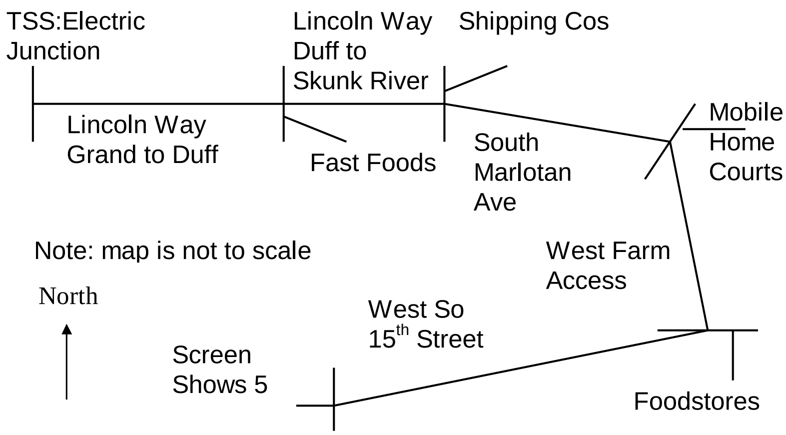


Figure D1.3 Distribution System One-Line Diagram

Light industrial systems are normally at higher voltages (e.g. 34.5 and 69 kV) and are always three phases.

Industrial distribution systems may be served at even higher voltages (e.g. 69, 138, 230 and 345 kV). Such loads are directly connected to the transmission grid (all three phases).

D1.3 Load Types

The load equipment at a customer location includes many different devices, which may be categorized into the following types: resistive, inductive, and (very infrequently) capacitive. The actual loads are primarily resistive for heating, inductive for motors, and capacitive for filtering. However, power electronics can change the actual load into an alternative characteristic by high speed switching and signal conditioning. Capacitors may also be present for power factor correction. These are control devices and not loads from the customer perspective.

The customer models are generated from surveys of home appliances based on the econometrics of each neighborhood. Additionally, load profiles are available from chart records at each substation per distribution feeder. Such charts are normally available for each month of the year. Digital recording of feeder loads is more prevalent and allows more extensive modeling of the customer use patterns.

In addition, the power factor of each feeder is recorded to determine if corrective action is required. The use of power electronic motor drives and power supplies are quickly changing the characteristics of load devices.

D1.4 Design Guidelines

The primary regulator design guide is the voltage target at the customer location. The voltage magnitude is required to be between 110 and 125 volts at the customer side of the low voltage transformer. State commissions set this limit. Additionally, voltage dips at the customer site are required to be within specified limits in most states. The voltage will dip as the inrush currents for starting machines temporarily load the system.

The state commissions also typically set reliability targets for the customer. Such considerations are beyond the scope of this text.

The power quality at the customer site is also a concern as covered in Module PQ1. Power quality now involves many aspects of delivery including frequency, harmonic content, reliability, etc. This text assumes that power quality only includes harmonic content. Other delivery specifications are discussed individually.

Any special load considerations may have to be considered by the electric utility, when such loads adversely affect the distribution system. Such considerations are beyond this text.

D1.5 Customer Tariffs

The customer pays for electricity according to the tariff approved by the state public utility commission. An example of such tariffs would contain the following details:

Residential Tariff: Price for electric usage is $0.05/kWHr. The electricity delivered is nominally 60 Hertz, 105-130 volts AC, with no more than one hour of interruption per year.

Light Industrial Tariff: Price for electric usage is $0.10/kWHr energy charge and $0.15/kW demand charge for the peak demand in any half-hour interval. The electricity delivered is nominally 60 Hertz, 105-130 volts AC, with no more than one day of interruption per year.

Medium Industrial Tariff: Price for electric usage is $0.15/kWHr energy charge and $0.20/kW demand charge for the peak demand in any fifteen-minute interval. Power factor correction charge is assessed if the power factor is less than 0.95. The power factor correction charge is $0.50/kVAr for correcting demand below 0.95 and $0.70/kVAr for correcting demand below 0.90. The electricity delivered is nominally 60 Hertz, 105-130 volts AC, with no more than eight hours of interruption per year.

Heavy Industrial Tariff: Price for electric usage is $0.18/kWHr energy charge and $0.25/kW demand charge for the peak demand in any fifteen-minute interval. Power factor correction charge is assessed if the power factor is less than 0.98. The power factor correction charge is $0.50/kVAr for correcting demand below 0.98 and $0.70/kVAr for correcting demand below 0.95. The electricity delivered is nominally 60 Hertz, 105-130 volts AC, with no more than one hour of interruption per year.

The above tariffs are only examples. Practical tariffs vary greatly. However, the above detail is exemplary of the detail of most tariffs. Note that each state has a different tariff structure and price philosophy for each customer. It is common for states to give preferential treatment to companies who relocate into a state from another state or country. Such preferential treatment is termed cross subsidies since the total cost of production, transportation, distribution and financing of the utility has to come from electricity sales. Income is derived primarily from sales. Thus, other customers must pay for the preferential treatment of others. Such cross subsidies are extremely political in nature. The amount and number of cross subsidies is very large and is one of the focal points for de-regulation of electric utilities.

This module assumes that the customer will be charged an energy charge, a capacity charge, and a power factor correction charge. Table D1.1 lists the assumed charges for the examples in this chapter.

Table D1.1 Pricing Data

Tariff Factor	Qualifying factor	Rate
Energy Charge		$0.10/kWHr
Demand Charge		$0.05/kVAr
Power Factor Correction	Pf less than 0.95	$0.10/kVAr
	Pf less than 0.9	$0.30/kVAr
	Pf less than 0.85	$0.40/kVAr

This module also assumes that the cost and price estimates can be based on a single analysis for the peak period. The assumption is that the peak period is one hour of the 8,760 hours per year. The costs of this peak period can be used as an estimate of the yearly cost if a functional relationship can be found. Assume, for the examples, that the peak period can be multiplied by a utilization factor and then by the number of hours in a year to estimate the income and the costs for the year. Use a utilization factor of .6 for the examples in this chapter. The marginal cost for the peak period is an index to use for cost estimates. The marginal cost is the cost of adding one more increment of demand when all other parameters stay fixed.

D1.6 Distribution System (Voltage Drop) Calculations

We will describe voltage regulation calculations for a radial system. Single-phase calculation will be used as distribution feeders are often single phase. For three-phase distribution circuits we must use a per-phase equivalent circuit.

Power flow equations (conservation of energy) are solved by assuming a voltage at the most remote point (the receiving end) of the distribution system such that the voltage will be within the required range. The receiving end voltage is set to the target; the receiving end current is calculated from the complex power demand at the receiving end, then the sending end voltage and current are calculated for the receiving end values. This process is iteratively used for each load until the voltage at the transmission substation is calculated. The load demand could be a constant power drain (sink). The load demand could be a constant current sink. The load demand could be constant impedance. The type of load demand may even be a composition of all three load types.

The recursive steps in the process are broken down into the following. First, assume a voltage at the farthest end (last customer). Use this voltage to find the current at the customer bus. Find the sending end voltage and current for this distribution segment. Use the voltage at this bus to find the current required for the customer at this bus. Add to this current the current needed for any distribution lines leaving this bus. Now repeat the process until the voltage at the sending end bus (transmission substation) is found.

The procedure for distribution calculations can be depicted by the following pseudo-code in Table D1.2. This procedure is implemented in MATLAB code for the corresponding simulator.

Table D1.2 Voltage Regulation Calculations

1.		For the last customer calculate the current for the required voltage:
	a.	If impedance load, then use ohm's law,
	b.	If power load, then use $S=VI^*$.
2.		Calculate the sending end voltage and current using current from 1.
3.		For the next customer calculate the current for the required load (sending end voltage from 2.):
	a.	If impedance load, then use ohm's law,
	b.	If power load, then use $S=VI^*$.
4.		Calculate the total current, using KCL to find the current required from the next line toward the source.
5.		Calculate the sending end voltage and current matrix using current from 4.
6.		Repeat steps 3 through 5 for all customers and lines until the source is reached.
7.		Calculate the sending end power from $S=VI^*$.
10.		Calculate the voltage regulation $(V_s-V_r)/V_r$.

The voltage regulation is calculated as shown in (D1.11). The voltage regulation is a measure of the stiffness of the distribution system. Ideally, the voltage regulation at all load buses should be small. However, a recurring problem is voltage flicker due to motor start-up inrush currents. The presence of such flicker is an indication that the distribution system is not stiff and that significant voltage correction is needed.

$$VR=\frac{|V_s|-|V_R|}{|V_s|}*100\% \tag{D1.11}$$

Once all line currents are known, then the losses can be completed as the sum of I^2R over all lines. Then efficiency is

$$\eta=\frac{P_{IN}-P_{LOSS}}{P_{IN}}$$

where P_{IN} is the power from the source.

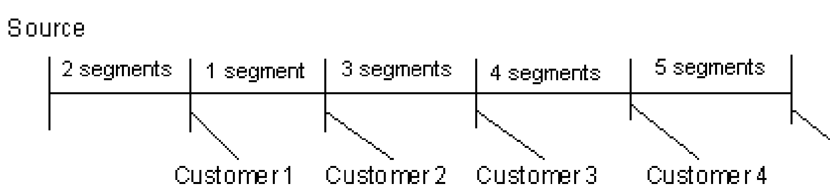
One line segment is 1000 feet. Note that the equivalent pi model for distribution lines often does not include the impact of mutual coupling between the phases since the length of a such lines are "electrically short." This is equivalent to saying that the inclusion of such effects is negligible.

Table D1.3 Line Data

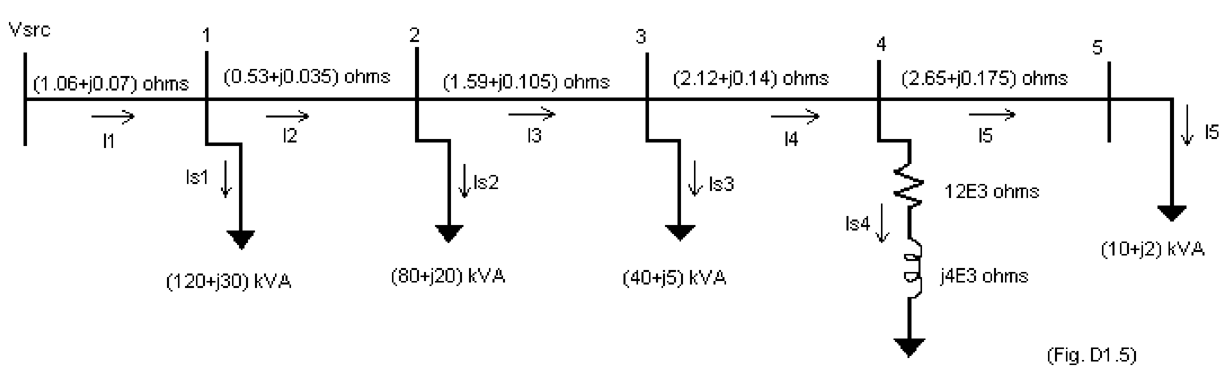
Aluminum Conductor Size	Copper Neutral	Resistance (90c) (Ω/1000 ft)	Reactance (Ω/1000 ft)	Ampacity Direct Burial	Ampacity In Duct (A)	Cost ($k/1000 ft)
4 AWG	6-#14	.53	.035	130	90	1.5
2 AWG	10-#14	.33	.030	170	120	1.7
1 AWG	13-#14	.26	.029	190	140	1.9
1/0 AWG	16-#14	.21	.028	210	160	2.1
2/0 AWG	13-#12	.16	.026	250	180	2.3
3/0 AWG	16-#12	.13	.024	280	200	2.5
4/0 AWG	20-#12	.11	.023	320	230	2.7
250 kcml	25-#12	.09	.022	360	260	2.9
300 kcml	18-#10	.07	.021	400	290	3.2
350 kcml	20-#10	.06	.020	440	320	3.4
400 kcml	22-#10	.04	.019	470	330	3.6

Example D 1.1 (Bus Distribution System Analysis)

Consider the following three-phase distribution system. The busses are numbered: source (src), 1,2,3,4, and 5. The number of segments is given in each line (2,1,3,4,5 respectively). The conductor used is 4AWG. Also, load levels (3-phase power) are given under each bus in kW and kVARs. Buses 1,2,3, and 5 have constant power loads. The load at bus 4 is constant impedance and is given as (12000+j4000) ohms per phase (Wye connected). Determine the source voltage required to hold bus 5 at 11kV (line-to-line). Also, determine the source current, the source power, the total system losses, the efficiency, and the percent-regulation. Note again compensator that all kVA quantities given are three-phase.



(Fig. D1.4)



(Fig. D1.5)

$$V_5 = \frac{11000 \angle -30^\circ}{\sqrt{3}} = 6350 \angle -30^\circ$$

$$I_5 = \left(\frac{S}{V}\right)^* = \left(\frac{\left(\frac{10-j2}{3}\right)\cdot 10^3}{6350 \angle 30^\circ}\right) = 0.4021 - j0.3533\,A$$

$$V_4 = V_5 + I_5 \cdot Z_{45} = (6350 \angle -30^\circ) + (0.4021 - j.3533)(2.65 + j0.175) = 6352.3 \angle -30.002^\circ$$

$$\Rightarrow \quad V_{4LL} = 11002.44 \angle -.002^\circ$$

$$I_{S4} = \frac{V_4}{Z_4} = \frac{6352.3 \angle -30.002^\circ}{12000 + j4000} = 0.33318 - j0.37575$$

$$I_4 = I_5 + I_{S4} = (0.4021 - j0.3533) + (0.33318 - j0.37575) = (0.7353 - j0.7291)$$

$$V_3 = V_4 + I_4 \cdot Z_{34} = (6352.3 \angle -30.002^\circ) + (0.7353 - j0.7291)(2.12 + j0.14) = 6354.46 \angle -30.006^\circ$$

$$\Rightarrow \quad V_{3LL} = 11006.25 \angle -.006^\circ$$

$$I_{S3} = \frac{\left(\frac{40-j5}{3}\right)\cdot 10^3}{6354.46 \angle 30.006^\circ} = 1.6859 - j1.2765$$

$$I_3 = I_4 + I_{S3} = (0.7353 - j0.7291) + (1.6859 - j1.2765) = (2.4212 - j2.0055)$$

$$V_2 = V_3 + I_3 \cdot Z_{23} = (6354.46 \angle -30.006^\circ) + (2.4212 - j2.0055)(1.59 + j0.105) = 6359.44 \angle -30.0106^\circ$$

$$\Rightarrow \quad V_{2LL} = 11014.88 \angle -.0106^\circ$$

$$I_{S2} = \frac{\left(\frac{80-j20}{3}\right)\cdot 10^3}{6359.44 \angle 30.0106} = 3.10674 - j3.00506$$

$$I_2 = I_3 + I_{S2} = (2.4212 - j2.0055) + (3.10674 - j3.00506) = (5.5299 - j5.0106)$$

$$V_1 = V_2 + I_2 \cdot Z_{12} = (6359.44 \angle -30.0106^\circ) + (5.5299 - j5.0106)(0.53 + j0.035) = 6363.36 \angle -30.0158^\circ$$

$$\Rightarrow \quad V_{1LL} = 11021.66 \angle -.0158^\circ$$

$$I_{S1} = \frac{\left(\frac{120-j30}{3}\right)\cdot 10^3}{6363.36 \angle 30.0158} = 4.6568 - j4.50523$$

$$I_1 = I_2 + I_{S1} = (5.5279 - j5.0106) + (4.6568 - j4.50523) = (10.1847 - j9.5158)$$

$$V_{SRC} = V_1 + I_1 \cdot Z_{S1} = (6363.36 \angle -30.0158^\circ) + (10.1847 - j9.5158)(1.06 + j0.07) = 6377.97 \angle -30.0372^\circ$$

$$\Rightarrow \quad V_{SRC,\,LL} = 11046.98 \angle -.0372^\circ$$

$$I_{SRC} = I_1$$

$$P_{SRC} = P_{IN} = 3\,\mathrm{Re}\left\{(V_{SRC} \cdot I_{SRC})^*\right\} = 3\,\mathrm{Re}\left\{((6377.97 \angle -30.0372^\circ)(10.1847 - j9.5158))^*\right\}$$

$$= 3\,\mathrm{Re}\left\{(25854.066 - j85056.029)^*\right\} = 3\,\mathrm{Re}\left\{(25854.066 + j85056.029)^*\right\} = 77562.198\,W$$

$Losses:$

$I_5^2 \cdot R_{45} = .7592$

$I_4^2 \cdot R_{34} = 2.273$

$I_3^2 \cdot R_{23} = 15.716$

$I_2^2 \cdot R_{12} = 29.502$

$I_1^2 \cdot R_{S1} = 205.935$

$TOTAL = 3 \cdot (254.19) = P_{LOSS}$

$regulation\%reg:$

$$\%reg = \frac{|V_{LL,\,SRC}| - |V_{LL,5}|}{|V_{LL,\,SRC}|} \times 100\%$$

$$\%reg = \frac{11046.98 - 11000}{11046.98} \times 100\% = 0.425\%$$

$Efficiency\ (\eta):$

$$\eta = \frac{P_{IN} - P_{LOSS}}{P_{IN}} \times 100\%$$

$$\eta = \frac{77562.198 - 762.57}{77562.198} \times 100\% = 99.02\%$$

D1.7 Split Distribution Line Analysis

The student should determine how to solve the equations if the distribution line splits into two sections as shown in Figure D1.6. Note that the calculations are a problem when the two distribution lines connect at bus 2. The solution is to invent an iterative procedure to adjust the customer voltage at bus 6 until the voltages at the connection point are (nearly) the same.

An example procedure is to update the ending voltage (customer 5) assuming that the last three solutions can approximate a quadratic curve to find the same solution as found for the first branch (Customers 2 and 3).

Other power flow techniques, such as Gauss-Seidel, are typically used. Such techniques are the subject of senior elective or graduate courses.

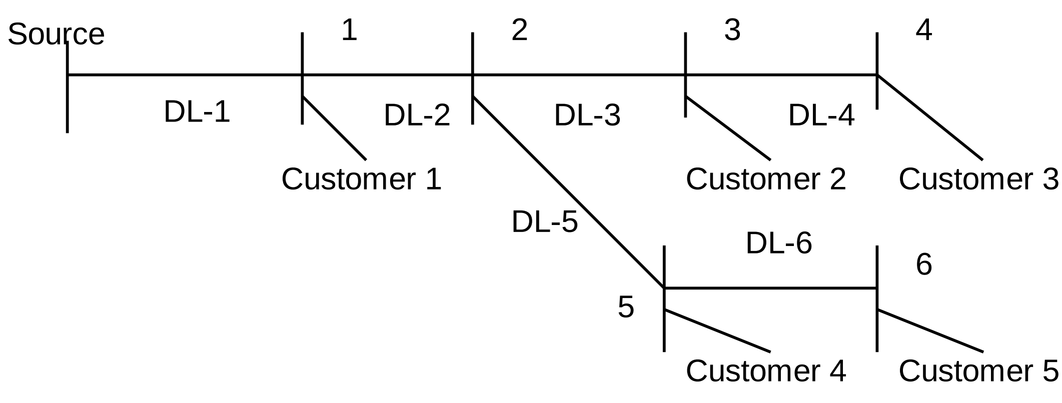


Figure D1.6 Multiple Branch Distribution System

D1.8 Power Factor Correction Compensation

The customer is required to maintain the power factor within limits since reactive power is not charged to the customer at this time. The standard correction configuration is to place a capacitor in parallel to the load as shown in Figure D1.7. The value of the capacitance is to reduce the impedance as seen by the electric utility to a real component only. Alternatively, this implies that the reactive power component is zero.

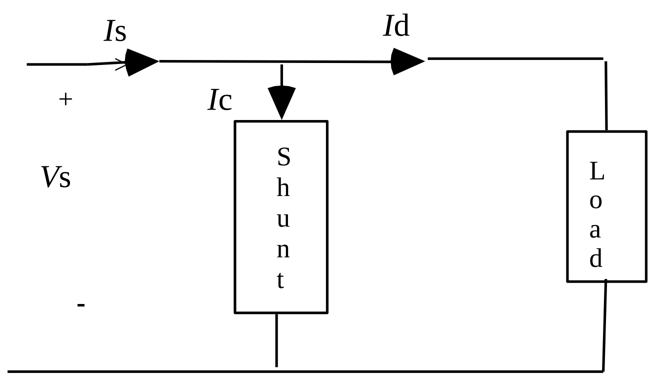


Figure D1.7 Shunt Compensation

The calculations are shown in (D1.11) through (D1.16). The demand requirement calculation (D1.11) determines the amount of reactive power required by the load equipment. The complex power required (provided) by the compensation equipment (D1.38) determines how much reactive demand requirement can be provided. The configuration of the compensation determines how the requirements are satisfied (D1.39 or D1.40). The one-port (shunt device/equipment) determines the component size of the compensation.

$$S_d = V_d\left(I_d \cos(\theta) - jI_d \sin(\theta)\right) \tag{D1.11}$$

$$\bar{S}_d = \bar{V}_d \bar{I}_d^* = P_d + jQ_d \tag{D1.12}$$

$$\bar{S}_c = \bar{V}_c \bar{I}_c^* = P_c + jQ_{cd} \tag{D1.13}$$

$$\bar{I}_s = \bar{I}_c + \bar{I}_d \tag{D1.14}$$

$$\bar{S}_s = \bar{S}_c + \bar{S}_d \tag{D1.15}$$

$$\begin{aligned} \bar{S}_c &= \bar{V}_c \bar{I}_c^* = 0 + jQ_c \\ &= \bar{V}_c\,(j\omega C \bar{V}_c)^* \\ &= -j\omega C \left|V_c\right|^2 \end{aligned} \tag{D1.16}$$

Example D 1.2 (Power Factor Correction)

Consider a one ϕ demand operating at 500V at 60 Hz with P_1 = 48kW @ pf = 0.60 lagging and P_2 = 24 kW at pf = 0.96 leading. Add a capacitor in parallel to these and find the capacitance value to zero the reactive demand.

$$S_1 = P_1 + jQ_1 = (48 + j64)kVA$$
$$S_2 = P_2 + jQ_2 = (24 - j7)kVA$$
$$S_{net} = (P_1 + P_2) + j(Q_1 + Q_2) = 72 + j57\,kVA = 91.8\,kVA \quad @ \quad pf = 0.784\,lagging$$
$$Q_c = -57\,kVAr = -Q_{net}$$
$$|V_c| = 500V$$
$$\omega = 377\,\frac{rad}{\sec}$$
$$C = -Q_c / \left(\omega |V_c|^2\right) = 605\,\mu F$$

Other compensation configurations and calculations may be required for unique load conditions. Inductors may also be added to compensate for excessive capacitance at the customer site.

Alternatively, automatic compensation may be required if the load characteristics change quickly. A power electronic device called a static VAR or voltage compensator (svc) as mentioned above (D1.1) accomplishes such automatic compensation. The power system specialist should be able to derive all of the equations needed for compensation by shunt inductors, series capacitors, or series inductors.

D1.9 Balancing Power Feeder Demands

Distribution line load demands are "balanced" by dividing the number of customers between the three phases such that the expected demand (kVA) is evenly divided. The amount of demand per customer has to be known or estimated as well as the time schedule for the demand. Then the demand amount and schedule is used to determine to which phase the customer should be connected. If the demand is unknown, then it is typical to use the rated capacity of the pole transformer as the customer demand.

Consider a feeder, which has a loading profile of (A-120, B-60, and C-100), all in kVA at a given point on the line. The next customer closest to the distribution substation to be added is expected to demand 40 kVA. This customer should be placed on the B phase to yield a resulting loading profile of (A-120, B-100, C-100). Thus this process starts at the farthest point of the feeder and completes at the distribution substation. The same process is used if only two of the phases are present. Obviously, if only one phase is present, there is no decision to be made.

An example calculation would be to divide the loads in Table D1.4 among the three phases as shown. Note that the feeder requires only the c phase to extend to the last customer and only the b phase to extend to the second to last customer. Thus, one would only find one phase at the end of the circuit, two phases as the circuit is traced to the source. Note that loading is not equal. Note that the demand is only an estimate and does not contain information as to the time of day when the demand will occur. This data is normally adopted from typical demand patterns for typical homes on typical days. Almost as accurate as a weather forecast!

If the distance between customers is large compared to the cost of line, then the number of phases extended to the last customers can be significantly changed as shown in Table D1.5.

Note that phase A does not extend beyond the fifth customer. Phase B does not extend beyond the ninth customer. Thus, the number of conductors can be reduced to save costs. Additionally, the complexity of the pole top design to carry one conductor is simpler than for three. Also, less expensive. The economics of installation will determine the approach taken.

The student may wish to consider the reliability of the circuit. Would the extension of all three phases to the last customer provide service that is more reliable? Under what conditions would the service be more reliable? Under what conditions would the service be of the same reliability?

Table D1.4 Distribution Customer Data (Plan A)

Customer	Demand (kVA)	Phase A	Phase B	Phase C
1	10			10
2	35	35		
3	40		40	
4	20			20
5	10		10	
6	35	35		
7	15	15		
8	25		25	
9	30			30
10	05	05		
11	25	25		
12	15		15	
13	15		15	
14	30			30
Total		115	105	90

Table D1.5 Distribution Customer Data (Plan B)

Customer	Demand (kVA)	Phase A	Phase B	Phase C
1	10	10		
2	35	35		
3	40	40		
4	20	20		
5	10	10		
6	35		35	
7	15		15	
8	25		25	
9	30		30	
10	05			05
11	25			25
12	15			15
13	15			15
14	30			30
Total		115	105	90

D1.10 Tolerance of Current

Table D1.6 shows the current impact on the human body. Note that the amount of current is measured in milli-amps. Whereas, the current in a distribution system is normally measured in kilo-amps.

Table D1.6 Tolerance of Current - Humans

Current	Effect
1-5 mA	Sensation
10-20 mA	Involuntary muscle contractions
20-100 mA	Pain, breathing difficult
100-300 mA	Ventricular fibrillation, possible death
>300mA	Respiratory paralysis, burns, unconsciousness, permanent loss of short-term memory, compaction of soft tissue (spinal cord, etc.)

D1.11 Customer Wiring

The distribution transformer connected between the distribution line and the customers may be configured in a number of ways. The configuration is based on selected practice (preferences) and economic cost. Once in the customer's site, the wiring is the responsibility of the customer. The National Electric code requires all designs to be within 125% of equipment capability. Figure D1.8 shows how a single-phase connection can be transformed into a two-phase connection. Note that the key is to ground the middle of the secondary coil to be the reference for the customer.

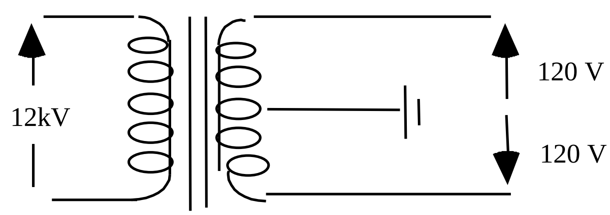


Figure D1.8 Distribution to Customer Wiring Transformation

Figure D1.9 shows a typical distribution box found in most residential and commercial facilities. The incoming phases (2) are connected to a main circuit breaker to isolate the customer from the source if current exceeds a maximum (typically 100 or 200 amps). The main circuit breaker is connected to two (2) bus bars to which individual circuit breakers are attached. The basic circuit breaker will feed one or two rooms and is limited to 15 or 20 amps. Note that the black wire is always assumed hot unless the circuit breaker is open. The white is the normal return (ground). The green wire is the safety ground. The safety ground protects the user by providing a lessor resistance path from the appliance to ground as shown in Figure D1.10.

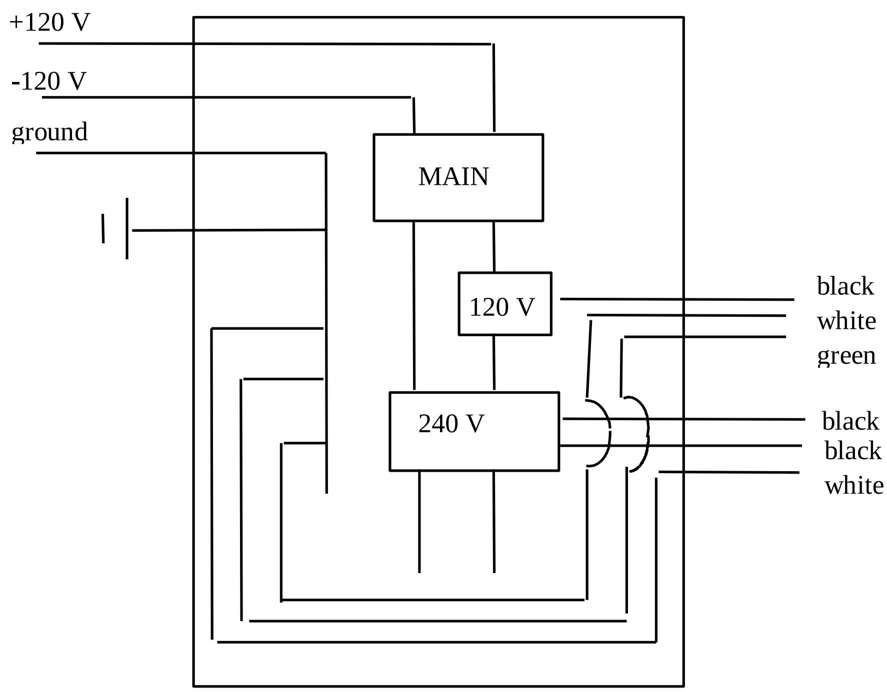


Figure D1.9 Residential Distribution Panel

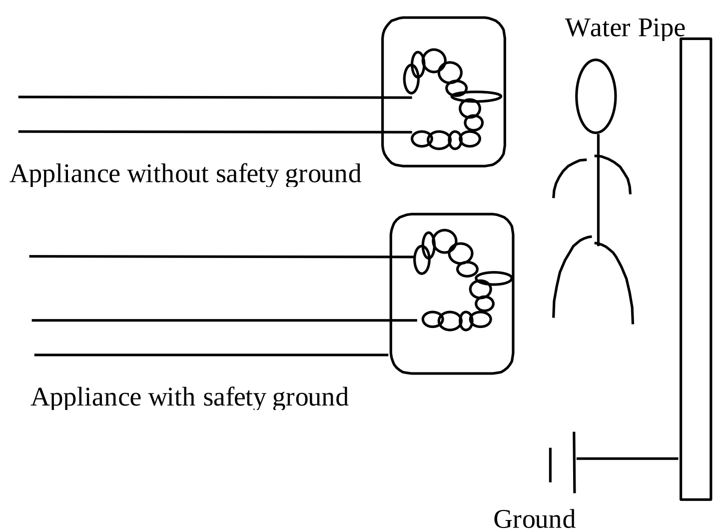


Figure D1.10 Safety Ground Principles

A special circuit breaker, called a Ground Fault Interrupter (GFI), can be used to provide more protection to the user. As shown in Figure D1.11, the GFI senses the amount of current flowing to the appliance(s). If the amount of current flowing on the hot (black) wire is not equal to the current flowing in the white wire, or if there is any current flow through the green wire, then the circuit breaker is opened. Note that many residences have a GFI installed in the wall box instead of in the distribution panel. The circuit breaker in the distribution box is then a normal circuit breaker. The National Electrical Code (NEC) requires GFIs in any room where the user may come in contact with a water pipe or the water tables (kitchen, bath, outdoors, etc.). Consider that almost all water pipes are metal and are connected to ground through a small resistance compared to most building materials. The concern is that the appliance may become defective and the current would return to the ground through the user. Figure D1.11 shows the protection provided by a GFI when a user uses a metal instrument (knife) to remove bread from a toaster. Most users would prefer a GFI to start the day in a better way.

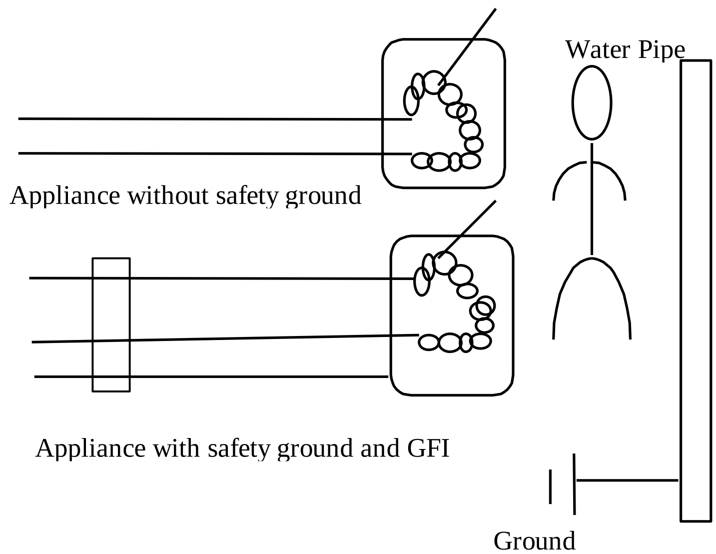


Figure D1.11 Application of Ground Fault Interrupter

The student should consider alternative grounding problems such as the "stray voltage" problem. The stray voltage problem has occurred in many farm facilities. This problem occurs when the ground is of high resistance compared to alternative current paths. An alternative current path may be a building, natural gas pipes, railroad tracks, etc. Metal structures within the building are the primary problem. Consider what would happen if an animal or a human touched a handrail or a metal feeder which is at a higher voltage than the ground underneath the feet. The electrons always find the least resistance path independent of the discomfort to the biological entity. This is often a serious problem, since animals cannot say that they feel the power. The result is often a cow that will not eat or allow anyone to hook up the milking machine. If farm equipment is improperly installed the currents can be sufficiently high to kill.

D1.11 Two Port Circuits

The treatment below is largely adapted from [1]. In performing circuit analysis of single phase distribution systems, it is often convenient to make use of two-port theory. A two-port network is one with two pairs of terminals emerging from it such that, for each pair of terminals, the current entering one terminal is the same as the current leaving the other terminal. Figure D1.12 illustrates a generalized two-port network. We require that there be no independent sources in the network (dependent sources are allowed), and there can be no energy stored in the network.

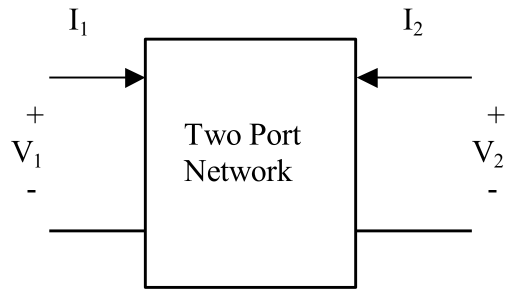


Fig. D1.12: Generalized Two-Port Network

Passive two-port networks are commonly specified in terms of their network parameters, according to the following expressions:

Z (impedance) parameters $$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{(D1.17)}$$

Y (admittance) parameters $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Y \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{(D1.18)}$$

H (hybrid) parameters $$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = H \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \text{(D1.19)}$$

G (inverse hybrid) parameters $$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = G \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \quad \text{(D1.20)}$$

a (transmission) parameters $$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = A \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \text{(D1.21)}$$

b (inverse transmission) parameters $$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = B \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \quad \text{(D1.22)}$$

In using two ports, it is assumed that only voltages and currents at the terminals are of interest, i.e., there is no interest in computing currents and voltages within the circuit. Such emphasis on terminal behavior is common when dealing with operational amplifiers. It is also common when dealing with transformers and transmission lines. In fact, the a-parameters are used in distribution system analysis and are commonly referred to as the ABCD parameters. Typically, the ABCD parameters are defined with the current I_2 of Fig. D1.12 defined in the opposite direction, as shown in Fig. D1.13, where

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = T \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

(D1.23)

The a-transmission parameters may be related to the ABCD-transmission parameters according to $A=a_{11}$, $B=-a_{12}$, $C=a_{21}$, and $D=-a_{22}$.

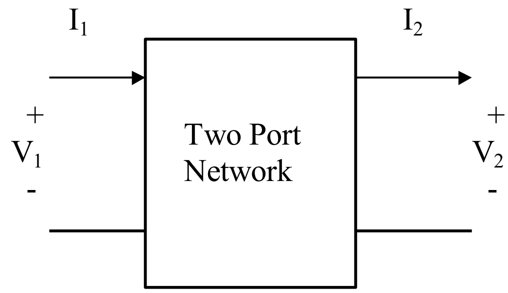


Fig D1.13: Generalized two port network with I_2 direction reversed

Given a two-port network, any particular set of parameters may be computed from measurements if we are clever about the conditions under which we perform the measurements. For example, the z-parameters may be computed from measurements of so-called short-circuit conditions. Specifically,

$$z_{11}=\left.\frac{V_1}{I_1}\right|_{I_2=0}\qquad z_{12}=\left.\frac{V_1}{I_2}\right|_{I_1=0}\qquad z_{21}=\left.\frac{V_2}{I_1}\right|_{I_2=0}\qquad z_{22}=\left.\frac{V_2}{I_2}\right|_{I_1=0}\qquad \text{(D1.24)}$$

Each expression in (D1.24) may be inferred from (D1.17) by solving for the desired parameter and then setting to zero the current or voltage necessary to eliminate the remaining parameter. For example, from (D1.17), we see that:

$$V_1=z_{11}I_1+z_{12}I_2 \Rightarrow z_{11}=\frac{V_1-z_{12}I_2}{I_1}$$

Here, we see if I_2=0, then $z_{11}=\frac{V_1}{I_1}$ which is the first expression of (D1.24). The expressions for the other z-parameters, and indeed for all of the parameters in the other two-port models, may be found in a similar fashion. The expressions for the admittance, hybrid, inverse hybrid, transmission, and inverse transmission are as follows:

$$y_{11}=\left.\frac{I_1}{V_1}\right|_{V_2=0}\qquad y_{12}=\left.\frac{I_1}{V_2}\right|_{V_1=0}\qquad y_{21}=\left.\frac{I_2}{V_1}\right|_{V_2=0}\qquad y_{22}=\left.\frac{I_2}{V_2}\right|_{V_1=0}\qquad \text{(D1.25)}$$

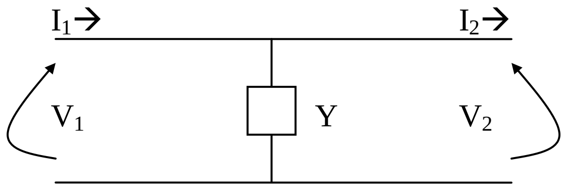
$$h_{11}=\left.\frac{V_1}{I_1}\right|_{V_2=0}\qquad h_{12}=\left.\frac{V_1}{V_2}\right|_{I_1=0}\qquad h_{21}=\left.\frac{I_2}{I_1}\right|_{V_2=0}\qquad h_{22}=\left.\frac{I_2}{V_2}\right|_{I_1=0}\qquad \text{(D1.26)}$$

$$g_{11}=\left.\frac{I_1}{V_1}\right|_{I_2=0}\qquad g_{12}=\left.\frac{I_1}{I_2}\right|_{V_1=0}\qquad g_{21}=\left.\frac{V_2}{V_1}\right|_{I_2=0}\qquad g_{22}=\left.\frac{V_2}{I_2}\right|_{V_1=0}\qquad \text{(D1.27)}$$

$$a_{11}=\left.\frac{V_1}{V_2}\right|_{I_2=0}\qquad a_{12}=\left.\frac{V_1}{I_2}\right|_{V_2=0}\qquad a_{21}=\left.\frac{I_1}{V_2}\right|_{I_2=0}\qquad a_{22}=\left.\frac{I_1}{I_2}\right|_{V_2=0}\qquad \text{(D1.28)}$$

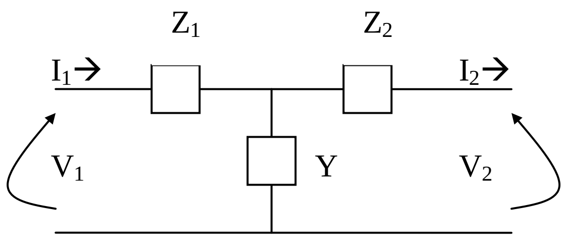
$$b_{11}=\left.\frac{V_2}{V_1}\right|_{I_1=0}\qquad b_{12}=\left.\frac{V_2}{I_1}\right|_{V_1=0}\qquad b_{21}=\left.\frac{I_2}{V_1}\right|_{I_1=0}\qquad b_{22}=\left.\frac{I_2}{I_1}\right|_{V_1=0}\qquad \text{(D1.29)}$$

The ABCD parameters can be derived using eqs. (D1.28) for a given topology. There are three topologies that are frequently encountered in power system models, as shown in Figs. D1.14-D1.16, and it is useful to be able to derive their ABCD parameters. The student should make these derivations for the three topologies shown.



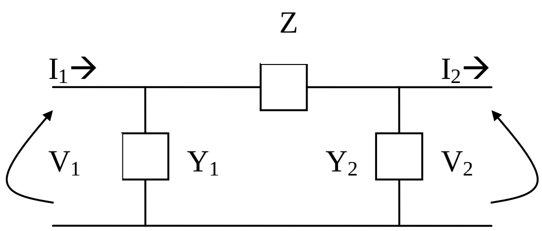
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Fig. D1.14: ABCD parameters for "I" circuit



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1+YZ_1 & Z_1+Z_2+YZ_1Z_2 \\ Y & 1+YZ_2 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Fig. D1.15: ABCD parameters for "T" circuit

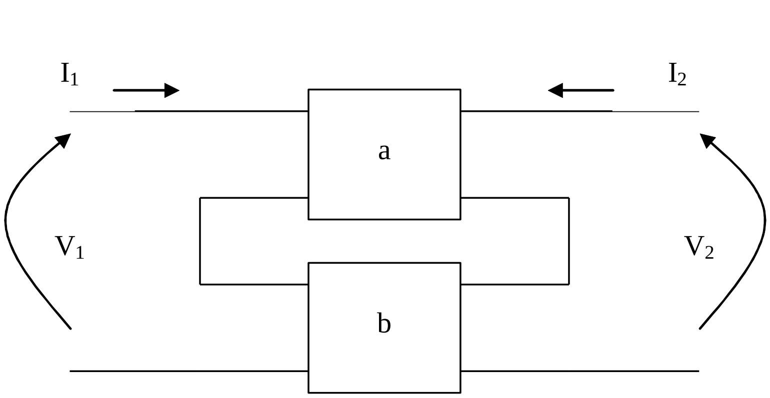


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1+Y_2Z & Z \\ Y_1+Y_2+ZY_1Y_2 & 1+Y_1Z \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Fig. D1.16: ABCD parameters for "π" circuit

One important reason for studying two ports is that a larger system containing multiple two-ports may be efficiently analyzed according to how the various two-ports are interconnected. Connection possibilities include series, parallel, and cascade. A hybrid connection is also possible [1]. Illustration of the series, parallel, and cascade connections are given below together with the relations between the appropriate parameters [1].

So the approach to dealing with interconnected two-ports is:
1. Identify the connection type.
2. Obtain the two-port parameters appropriate to the connection type (Z for series, Y for parallel, and T for cascade)
3. Obtain the composite parameters by performing the operation appropriate to the connection type (addition of Z for series, addition of Y for parallel, and multiplication for cascade).



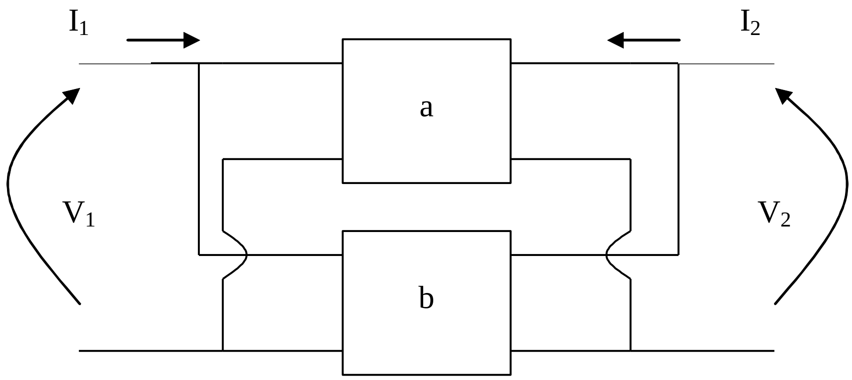
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z = Z_a + Z_b$$

$$Z_a = \begin{bmatrix} z_{a11} & z_{a12} \\ z_{a21} & z_{a22} \end{bmatrix}$$

$$Z_b = \begin{bmatrix} z_{b11} & z_{b12} \\ z_{b21} & z_{b22} \end{bmatrix}$$

Fig. D1.17: Series connection of 2 two-port networks



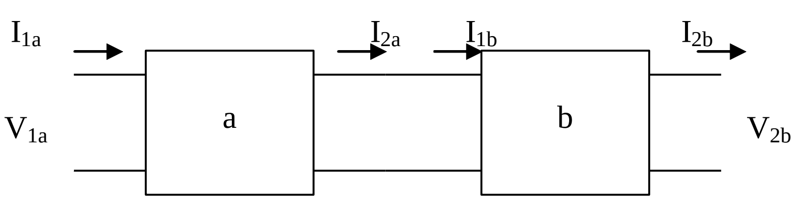
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Y \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y = Y_a + Y_b$$

$$Y_a = \begin{bmatrix} y_{a11} & y_{a12} \\ y_{a21} & y_{a22} \end{bmatrix}$$

$$Y_b = \begin{bmatrix} y_{b11} & y_{b12} \\ y_{b21} & y_{b22} \end{bmatrix}$$

Fig. D1.15: Parallel connection of 2 two-port networks



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$T = T_a T_b$$

$$T_a = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix}$$

$$T_b = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

Fig. D1.16: Cascade connection of 2 two-port networks

In the analysis of operational amplifier circuits, it is frequently of interest to compute input impedance, current gain, and/or voltage gain. Such computations are facilitated by combining the two port-relations with the constraint relations associated with the input and output quantities of the two-port. Such analysis may be found in many circuit analysis texts such as [2]. Power system applications of two ports include calculation of voltage regulation, e.g, example D1.1 may be solved using two-port networks.

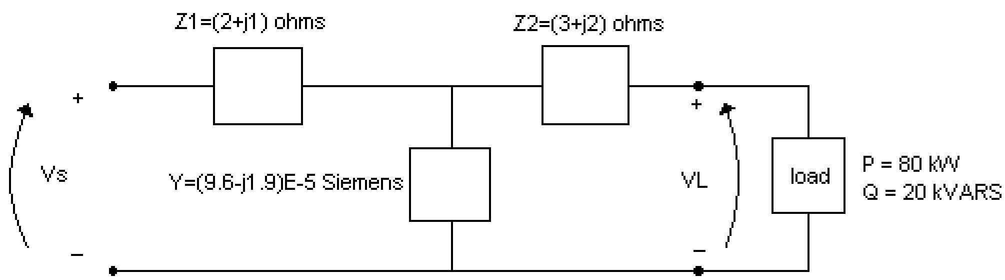
References

[1] P. Anderson, "Analysis of Faulted Power Systems," 1973, Iowa State University Press, Ames, Iowa.
[2] J. Neilsson, "Electric Circuits," second edition, Addison-Wesley Publishing Company, Reading, Mass., 1986.

PROBLEMS

Problem 1

A single-phase distribution system is modeled using the circuit below.



a) Compute the voltage required at the source to hold a voltage of 4kV at the load bus.
b) Determine the additional capacitive reactive power required at the load bus to correct the power factor at the load to 0.98.

Problem 2

A single-phase distribution feeder supplies a load of 80 kW, 20 kVARs. The impedance of the feeder is Z = (5+j2) ohms. It is required to hold a voltage of 12 kV at the load. Compute the minimum sending-end voltage that will satisfy the load-end voltage requirement at this load level. Also compute V_{reg}.

Problem 3

A balanced three-phase industrial load has two components:

1200 kVA at 0.5 power factor lagging
200 kW resistive load

The two loads are supplied by a 13.2 kV feeder.

a) Find the magnitude of the feeder current
b) Determine the capacitive VARs required to correct the power factor of the entire load to 0.80 lagging.

Problem 4

A large industrial customer is connected to the system via a three-phase distribution circuit. The customer consumes 30 MW at 0.95 power factor lagging during peak conditions. The voltage at the customer is 4 kV. In order to obtain a lower electric energy rate form the supplier, the customer must correct the power factor to 0.98 lagging. So the customer has decided to install shunt capacitance. Compute the necessary correction in terms of:

a) 3-phase reactive power
b) susceptance B_c
c) capacitance C

Problem 5

A large industrial facility is consuming 10MW at 0.90 power factor lagging and 5MW at 0.85 power factor lagging. Compute the capacitive VARs necessary to correct the overall plant power factor to 0.95 lagging.

Problem 6

A large industrial load that consumes 8 MW and 6 MVAR must correct its power factor to 0.95. How much additional reactive power is necessary to do this?

Problem 7

A single-phase distribution feeder supplies a load of 1100+j400Ω. The impedance of the feeder is Z = (5+j2) ohms. It is required to hold a voltage of 12 kV at the load. Compute the voltage regulation of this feeder.

Problem 8

Three loads are connected in parallel across a three-phase supply having line-to-line voltage of 12.47 kV. These loads are specified as

Load 1: Inductive Load, 60 kW and 660 kVAR
Load 2: Capacitive Load, 240 kW at 0.8 power factor
Load 3: Resistive Load of 60 kW

(a) Find the total complex power consumed by all three loads, and the current and power factor as seen from the supply. Be sure to indicate whether the power factor is leading or lagging.
(b) A Y-connected capacitor bank is connected in parallel with the loads. Find the total kVAR required from the capacitor to improve the power factor to 0.8 lagging.

Problem 9

A customer at the end of a distribution feeder is experiencing voltage regulation problems. Specifically, under high demand, when the customer's power factor is 0.90 lagging, the voltage magnitude at the customer's meter is too low. The customer comes to you, the engineer, suggesting that it may be feasible to correct this voltage magnitude problem by installing a shunt capacitor at the customer's site. Describe

(a) How a shunt capacitor can help the voltage magnitude problem under the indicated conditions?
(b) What calculations you would make to determine whether this is in fact a feasible solution?

Problem 10

An industrial facility is consuming 100 kW and 48.4 kVAR at a voltage of 480 volts line-to-line.

(a) Compute the power factor of the load. Indicate whether it is leading or lagging.
(b) Compute the additional reactive power necessary from a capacitor bank to correct the power factor to 0.95 lagging.
(c) Compute the per-phase reactance of the capacitors needed to perform this correction. Assume the capacitors will be Y-connected at 480 volts.

Problem 11

A balanced three phase industrial facility consists of two parallel loads, as follows:

- 1200 kVA at 0.5 power factor lagging
- 200 kW (entirely resistive load)

The two loads are supplied by a three phase, distribution feeder circuit having impedance of 4+j2 ohms per phase. The load voltage is 13.2 kV line-to-line.

a. Find the magnitude and angle of the feeder current
b. Find the magnitude of the line-to-line voltage at the sending end of the distribution feeder circuit.
c. Determine the capacitive VARS required to correct the power factor of the entire load to 0.80 lagging.
d. Determine the susceptance of the capacitor necessary to supply these vars at the stated load voltage.

Problem 12

Show that ABCD parameters for the following circuits are given as indicated.

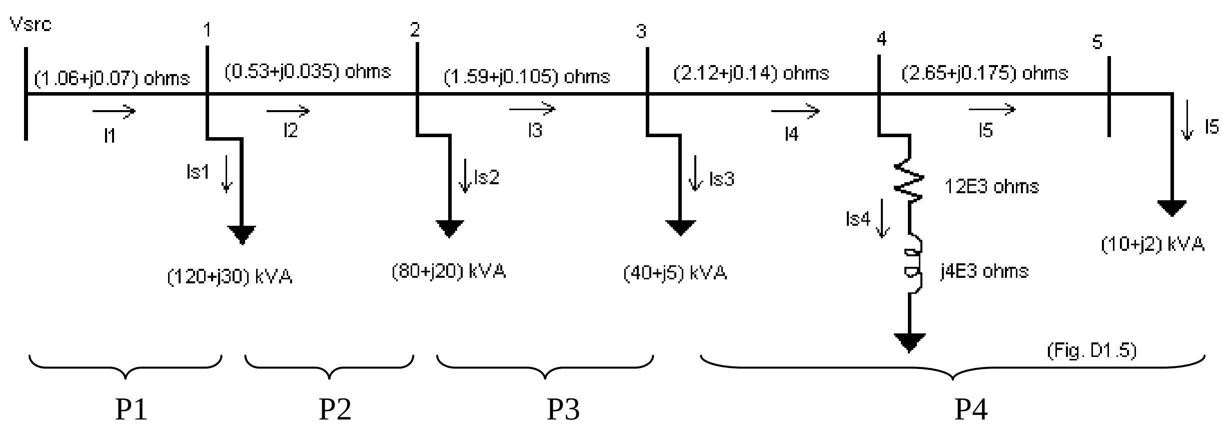
a. Shunt or "I" circuit as given in Figure D1.14.

b. T-circuit as given in Figure D1.15.

c. Pi-circuit as given in Figure D1.16. Note the circuit configuration and matrix when $Y_1=Y_2=0$.

Problem 13

Consider the following circuit partitioned according to P1,…,P4, where a 3-phase radial feeder is supplying 5 customer loads. The line-to-line voltage of the last bus (#5) is 11kV. Line impedances and load values are given in the figure. This problem is solved in Example D1.1 using KCL and KVL. In the exercise below, the goal is to use ABCD parameters to solve it, i.e., obtain V_{src} given V_5=11,000/sqrt(3).



a. P1: Compute I_5. Get ABCD parameters. Compute $[V_3 \quad I_4]^T$.
b. P2: Compute I_{s3} and then I_3. Get ABCD parameters. Compute $[V_2 \quad I_3]^T$. (Note that you will already have I_3 so really this calculation just gives you V_2).
c. P3: Compute I_{s2} and then I_2. Get ABCD parameters. Compute $[V_1 \quad I_2]^T$. (Note that you will already have I_2 so really this calculation just gives you V_1).
d. P4: Compute I_{s1} and then I_1. Get ABCD parameters. Compute $[V_{src} \quad I_1]^T$. (Note that you will already have I_1 so really this calculation just gives you V_{src}).