Module B3

Three Phase Circuit Analysis

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Prerequisite Competencies: Steady state analysis of circuits using phasors
Module Objectives:
1. Form one line diagrams for three phase circuits
2. Apply per phase analysis in performing power calculations for balanced three phase circuits using actual quantities and per unit quantities.
3. Represent sinusoidally time varying voltages and currents using phasor rotation.
4. Calculate real, reactive and apparent power for single phase and three phase circuits.
5. Identify phase and magnitude relations between all lone and phase voltages (currents for Delta and Wye connected LOADS).

B3.1 Sinusoidal Steady State Analysis

At the nodes of a power system, the voltage waveform can be assumed to be purely sinusoidal and of constant frequency. In this module, we deal with the phasor representation of sinusoidal voltages and currents, and use the boldface quantity V and I to indicate these phasors. $|V|$ and $|I|$ designate the magnitudes of the phasors.

B3.1.1 Phasor Representation

The phasor is a complex number that contains the amplitude and phase angle information of a sinusoidal function. Using Euler's identity, which relates the exponential function to the trigonometric function, the phasor concept can be developed.

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta \qquad \text{(B3.1)}$$

Equation (B3.1) provides us an alternative way of expressing the cosine and sine function. The cosine function can be represented as the real part of the exponential function, and the sine function can be represented as the imaginary part of the exponential function as follows

$$\cos\theta = \mathrm{Re}\{e^{j\theta}\} \qquad \text{(B3.2)}$$

and

$$\sin\theta = \mathrm{Im}\{e^{j\theta}\} \qquad \text{(B3.3)}$$

Where Re represents *the real part of* and Im represents *the imaginary part of.*

A sinusoidal voltage function (we have chosen to use the cosine function in analyzing the sinusoidal steady state), can be written in the form suggested by Eq. (B3.2)

$$\begin{aligned} v &= V_m \cos(\omega t + \phi) \\ &= V_m \mathrm{Re}\{e^{j(\omega t+\phi)}\} \\ &= V_m \mathrm{Re}\{e^{j\omega t} e^{j\phi}\} \end{aligned} \qquad \text{(B3.4)}$$

We can move the coefficient V_m inside the argument, and also reverse the order of the two exponential functions inside the argument without altering the result.

$$v = \mathrm{Re}\{V_m e^{j\phi} e^{j\omega t}\} \qquad \text{(B3.5)}$$

In Eq. (B3.5) the coefficient of the exponential $e^{j\omega t}$ is a complex number that carries the amplitude and phase angle of the given sinusoidal function. This complex number is by definition the *phasor representation* or *phasor transform*, of the given sinusoidal function. Thus

$$\mathbf{V} = V_m e^{j\phi} = \mathrm{P}\{V_m \cos(\omega t + \phi)\} \qquad \text{(B3.6)}$$

Where the notation $\mathrm{P}\{V_m \cos(\omega t + \phi)\}$ depicts "the phasor transform of $V_m \cos(\omega t + \phi)$." Hence, the phasor transform P transforms the sinusoidal function from the time domain to the complex-number domain.

Equation (B3.6) is the polar representation of a phasor, We can also obtain the rectangular representation of the phasor as

$$\mathbf{V} = V_m \cos\phi + jV_m \sin\phi \qquad \text{(B3.7)}$$

B3.1.2 Power Calculation in Single-Phase AC Circuits

Our aim is to determine the average power that is either delivered to or absorbed from a pair of terminals by a sinusoidal voltage and current. Figure B3.1 depicts the problem. Here, *v* and *i* are steady-state sinusoidal signals. With the use of passive sign convention; the power at any instant is

$$p = vi \qquad \text{(B3.8)}$$

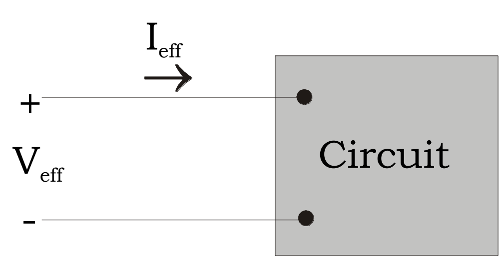


Figure B3.1 The Basic Calculation to Determine Average Power

The power is measured in watts when the voltage is in volts and the current is in amperes. First we write expressions for *v* and *i*

$$v = V_m cos(\omega t + \theta_v) \tag{B3.9}$$

and

$$i = I_m cos(\omega t + \theta_i) \tag{B3.10}$$

Here, θ_v is the voltage phase angle and θ_i is the current phase angle. Using the instant when current is passing through a positive maximum as the reference for zero time, and expressing *v* and *i* with respect to this reference we have

$$\begin{aligned} v &= V_m cos(\omega t + \theta_v - \theta_i) \\ &= V_m cos(\omega t + \theta) \end{aligned} \tag{B3.11}$$

Where $\theta = \theta_v - \theta_i$

$$i = I_m cos\, \omega t \tag{B3.12}$$

The angle θ in these equations is positive for current *lagging* the voltage and negative for *leading* current. A positive value of *p* shows that energy is *absorbed* at the terminals. Substituting Eqs. (B3.11) and (B3.12) into Eq. (B3.8), the instantaneous power is given by

$$p = V_m I_m \cos(\omega t + \theta)\cos(\omega t) \tag{B3.13}$$

The average power associated with sinusoidal signals is given by the average of the instantaneous power over one period

$$p_{avg} = \frac{1}{T}\int_{t_0}^{t_0+T} p dt \tag{B3.14}$$

where T is the period of the sinusoidal function. Substituting Eq. (B3.13) into Eq. (B3.14), the average power can be determined. More information, however can be obtained by expanding Eq. (B3.13) using the trigonometric identity

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

Letting $A = \omega t + \theta$, and $B = \omega t$, we obtain from Eq. (B3.13)

$$p = \frac{V_m I_m}{2} cos\,\theta + \frac{V_m I_m}{2} cos(2\omega t + \theta) \tag{B3.15}$$

Expanding the second term on the right-hand side of Eq. (B3.15) using the trigonometric identity $cos(A+B) = cos\,A\,cos\,B - sin\,A\,sin\,B$, we get

$$p = \frac{V_m I_m}{2} cos\ \theta + \frac{V_m I_m}{2} cos(\theta) cos\ 2\omega t - \frac{V_m I_m}{2} sin\ \theta sin\ 2\omega t \qquad \text{(B3.16)}$$

The average value of *p* is given by the first term on the right hand side of Eq. (B3.16) because the integral of either $cos\,2\,\omega t$ or $sin\,2\omega t$ over one time period is zero. Hence, the average power is

$$P = \frac{V_m I_m}{2} cos\,\theta \qquad \text{(B3.17)}$$

P is also referred to as the *real* or *active* power. The third term on the right hand side of Eq. (B3.16), the term containing $sin\,\theta$, is alternatively positive and negative and has an average value of zero. This component of instantaneous power *p* is called the *instantaneous reactive power*. The maximum value of this pulsating power is designated as *Q*, and is called *reactive power*. Hence,

$$Q = \frac{V_m I_m}{2} sin\theta \qquad \text{(B3.18)}$$

P and *Q* carry the same dimension. However, in order to distinguish between real and reactive power, we use the term *vars* (volt-ampere reactive) for reactive power. Since we have used current as the reference, *Q* is positive for inductors (θ = 90º) and negative for capacitors (θ = -90º). The angle θ is referred to as the *power factor angle*. The cosine of this angle is called the *power factor*. Lagging power factor implies that current lags voltage. Leading power factor implies that the current leads the voltage.

The average power given by Eq. (B3.17) and the reactive power given by Eq. (B3.18) can be written in terms of effective or rms values

$$\begin{aligned} P &= \frac{V_m I_m}{2} \cos\theta \\ &= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\theta \\ &= V_{eff} I_{eff} \cos\theta \end{aligned} \qquad \text{(B3.19)}$$

and, by similar manipulation,

$$Q = V_{eff} I_{eff} sin\ \theta \qquad \text{(B3.20)}$$

Complex power is the complex sum of average real power and reactive power, or

$$S = P + jQ \qquad \text{(B3.21)}$$

Complex power has the same dimensions as real or reactive power. However, in order to distinguish complex power from real and reactive power, we use the term *volt amps*. Thus we use volt amps for complex power, watts for average real power, and vars for reactive power. We can think of *P*, *Q*, and |*S* | as the sides of a right-angled triangle as shown in Figure B3.2.

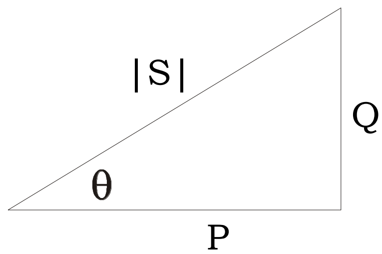


Figure B3.2 The Power Triangle

Combining Eqs. (B3.17), (B3.18), and (B3.21) we get

$$\begin{aligned} S &= \frac{V_m I_m}{2} \cos\theta + j\frac{V_m I_m}{2} \sin\theta \\ &= \frac{V_m I_m}{2} [\cos\theta + j\sin\theta] \\ &= \frac{V_m I_m}{2} e^{j\theta} = \frac{1}{2} V_m I_m \angle\theta \end{aligned} \qquad \text{(B3.22)}$$

Using effective values of the sinusoidal voltage and current, Eq. (B3.22) becomes

$$S = V_{eff} I_{eff} \angle\theta \qquad \text{(B3.23)}$$

Equations (B3.22) and (B3.23) show that if the phasor current and voltage are known at a pair of terminals, the complex power associated with that pair of terminals is either one half the product of the voltage and conjugate of the current or the product of the rms phasor voltage and the conjugate of the rms phasor current.

$$S = \frac{1}{2}\mathbf{VI}^* = P + jQ \qquad \text{(B3.24)}$$

or

$$S = \mathbf{V}_{eff}\mathbf{I}^*_{eff} = P + jQ \qquad \text{(B3.25)}$$

Equations (B3.24) and (B3.25) have several useful variations. In order to demonstrate these variations, we first replace the circuit in the box in Figure B3.1 by equivalent impedance *Z* as shown in Figure B3.3.

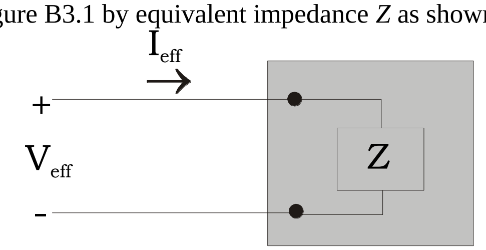


Figure B3.3 General Circuit Replaced with Equivalent Impedance

Expressing the voltage as the product of the current times the impedance, we obtain

$$\mathbf{V}_{\mathbf{e}ff} = Z\mathbf{I}_{\mathbf{e}ff} \tag{B3.26}$$

Substituting Eq. (B3.26) into Eq. (B3.25) yields

$$\begin{aligned} S &= Z\mathbf{I}_{\mathbf{e}ff}\mathbf{I}^{*}_{\mathbf{e}ff} \\ &= \left|\mathbf{I}_{\mathbf{e}ff}\right|^{2} Z \\ &= \left|\mathbf{I}_{\mathbf{e}ff}\right|^{2} (R + jX) \\ &= \left|\mathbf{I}_{eff}\right|^{2} R + j\left|\mathbf{I}_{\mathbf{e}ff}\right|^{2} X = P + jQ \end{aligned} \tag{B3.27}$$

from which,

$$P = \left|\mathbf{I}_{\mathbf{e}ff}\right|^{2} R = \frac{1}{2}\mathbf{I}_{m}^{2} R \tag{B3.28}$$

and

$$Q = \left|\mathbf{I}_{eff}\right|^{2} X = \frac{1}{2}\mathbf{I}_{m}^{2} X \tag{B3.29}$$

In Eq. (B3.29), X is the reactance of either the equivalent inductance or the equivalent capacitance of the circuit; it is positive for inductive circuits and negative for capacitive circuits.

Another useful variation of Eq. (B3.25) is obtained by replacing the current with the voltage divided by the impedance:

$$S = \mathbf{V}_{\mathbf{e}ff}\left(\frac{\mathbf{V}_{\mathbf{e}ff}}{Z}\right)^{*} = \frac{\left|\mathbf{V}_{\mathbf{e}ff}\right|^{2}}{Z^{*}} = P + jQ \tag{B3.30}$$

If the Z is a pure resistance element,

$$P = \frac{\left|\mathbf{V}_{\mathbf{e}ff}\right|^{2}}{R} \tag{B3.31}$$

and if Z is a pure reactive element

$$Q = \frac{\left|\mathbf{V}_{\mathbf{e}ff}\right|^{2}}{X} \tag{B3.32}$$

X is positive for an inductor and negative for a capacitor in Eq. (B3.32).

Example B 3.1

Three loads are connected in parallel across a 660-V (rms) line as shown in Figure B3.4. Load 1 absorbs 18 kW and 10 kVAR. Load 2 absorbs 6 kVA at 0.96-pf lead. Load 3 absorbs 22.4 kW at unity power factor. Find the impedance that is equivalent to the three parallel loads

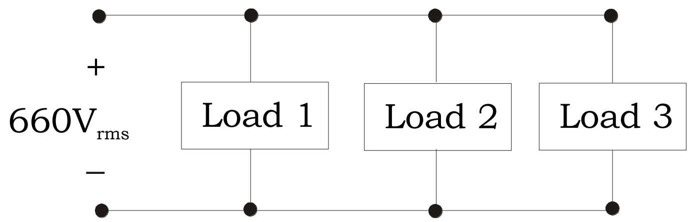


Figure B3.4

Solution

S_1 = 18 + j10 kVA

S_2 = 6(0.96) - j6(sin ($\cos^{-1}$(0.96))) kVA = 5.76 - j1.68 kVA

S_3 = 22.4 + j0 kVA

S_{Total} = 46.16 + j8.32 kVA

Series Combination of R and X in Impedance Z

From Eq. (B3.30)

$$Z^* = \frac{(660)^2 \times 10^{-3}}{46.16 + j8.32} = 9.28\angle -10.217^\circ \Omega$$
$$= 9.1398 - j1.6474\Omega$$

therefore Z = 9.1398 + j1.6474 Ω

Parallel Combination of R and X in Impedance Z

$$R = \frac{(660)^2}{46160} = 9.436\Omega$$
$$X = \frac{(660)^2}{8320} = 52.355\Omega$$
$$Z = 9.43\Omega \;\|\; j52.355\Omega$$

B3.2 Balanced Three-Phase Circuits

Electric power is supplied by three-phase generators. It is then transformed appropriately using transformers and transmitted in the form of three-phase power except at the lowest voltage levels of the distribution system where single phase power is used.

There are two main reasons for using three-phase power. First, the instantaneous power supplied to motors is constant torque and therefore motors run much smoother. Second, three phase power requires less conductor-cost than single-phase power for the same delivered power.

Figure B3.5 represents a three-phase (3ϕ) circuit. The circuit is said to be a balanced 3ϕ circuit if the impedances are equal and the three voltage source phasors are equal in magnitude and are out of phase with each other by exactly 120°. In discussing three-phase circuits, it is standard practice to refer to the three phases as a, b, and c. Furthermore, the a-phase is almost always used as the reference phase. The three voltages that comprise the three-phase set are referred to as the *a-phase voltage*, the *b-phase voltage*, and the *c-phase voltage*.

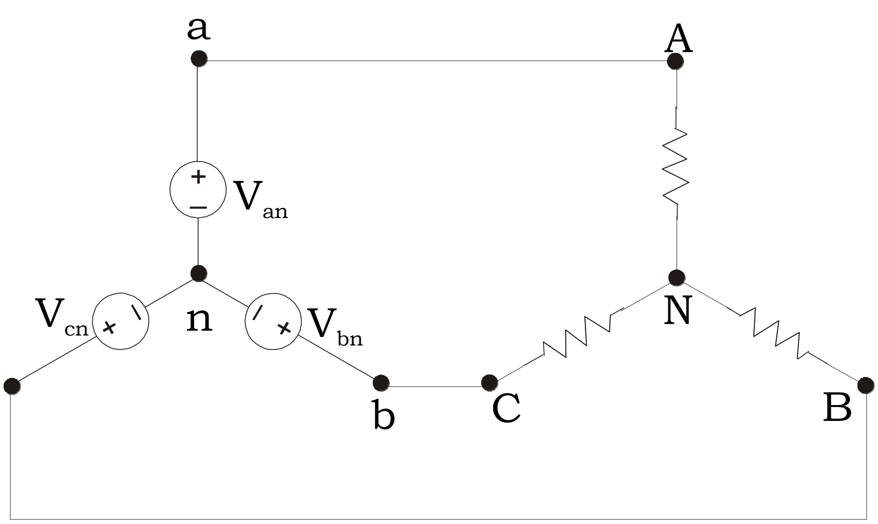


Figure B3.5 Three-Phase Balanced Circuit

Since the phase voltages are out of phase by 120°, two possible phase relationships can exist between the a-phase voltage and the b- and c- phase voltages. One possibility is that the b-phase voltage lags the a-phase voltage by 120°, in which case the c-phase voltage must lead the a-phase voltage by 120°. This phase relationship is known as the *abc*, or *positive phase sequence*. The other possibility is for the b-phase voltage to lead the a-phase voltage by 120°, in which case the c-phase voltage must lag the a-phase voltage by 120°. This phase sequence is known as the *acb*, or *negative phase sequence*. In phasor notation, the two possible sets of balanced phase voltages are

$$\begin{aligned} \mathbf{V_a} &= V_m \angle 0^\circ \\ \mathbf{V_b} &= V_m \angle -120^\circ \\ \mathbf{V_c} &= V_m \angle +120^\circ \end{aligned} \qquad \text{(B3.33)}$$

and

$$\mathbf{V_a} = V_m\angle 0^\circ$$
$$\mathbf{V_b} = V_m\angle +120^\circ \qquad \text{(B3.34)}$$
$$\mathbf{V_c} = V_m\angle -120^\circ$$

The phase sequence of the voltages given by Eq. (B3.33) is the abc, or positive sequence. The phase sequence of the voltages given by Eq. (B3.34) is the acb, or negative sequence. Another important characteristic of a set of balanced three-phase voltages is that the sum of the voltages is zero. Thus, from either Eq. (B3.33) or Eq. (B3.34)

$$\mathbf{V_a} + \mathbf{V_b} + \mathbf{V_c} = 0 \qquad \text{(B3.35)}$$

This relationship holds for any set of balanced three-phase variables. Components of balanced three-phase circuits can be connected either in a Y-connection or a Δ- connection. The phase quantities and the line quantities for these connections are related as follows.

B3.2.1 Delta Connection

Figure B3.6 illustrates a Δ- connected balanced three-phase load. The relationships developed however, can be applied to any component, e.g., generator, transformer, etc.

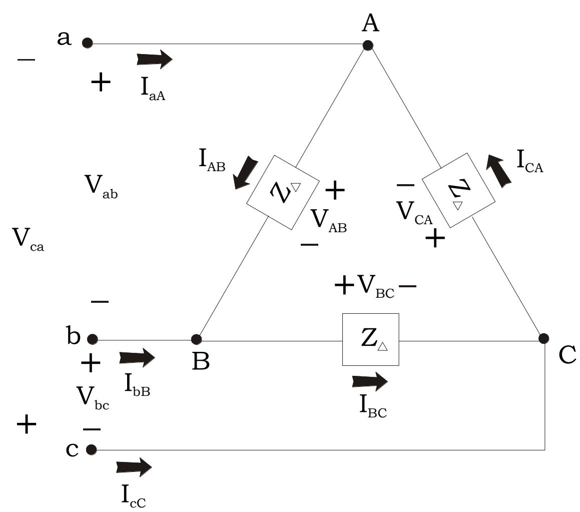


Figure B3.6 Delta Connection

In the Δ- circuit, the line-to-line voltage $\boldsymbol{V_{ab}}$ is equal to the phase voltage $\boldsymbol{V_{AB}} = \boldsymbol{V_\phi}$. To demonstrate the relationship between the phase currents and line currents, we assume a positive phase sequence and let I_ϕ represent the magnitude of the phase current. Then selecting $\boldsymbol{I_{AB}}$ arbitrarily as a reference phasor we have

$$\mathbf{I_{AB}} = I_\phi\angle 0^\circ \qquad \text{(B3.36)}$$
$$\mathbf{I_{BC}} = I_\phi\angle -120^\circ \qquad \text{(B3.37)}$$

and

$$\mathbf{I}_{CA} = I_\phi \angle +120^\circ \tag{B3.38}$$

We can express the line currents in terms of the phase currents by direct application of Kirchoff's current law:

$$\begin{aligned}\mathbf{I}_{aA} &= \mathbf{I}_{AB} - \mathbf{I}_{CA} = I_\phi \angle 0^\circ - I_\phi \angle 120^\circ \\ &= \sqrt{3} I_\phi \angle -30^\circ\end{aligned} \tag{B3.39}$$

$$\begin{aligned}\mathbf{I}_{bB} &= \mathbf{I}_{BC} - \mathbf{I}_{AB} = I_\phi \angle -120^\circ - I_\phi \angle 0^\circ \\ &= \sqrt{3} I_\phi \angle -150^\circ\end{aligned} \tag{B3.40}$$

$$\begin{aligned}\mathbf{I}_{cC} &= \mathbf{I}_{CA} - \mathbf{I}_{BC} = I_\phi \angle 120^\circ - I_\phi \angle -120^\circ \\ &= \sqrt{3} I_\phi \angle 90^\circ\end{aligned} \tag{B3.41}$$

Comparing Eqs. (B3.39)-(B3.41) with Eqs. (B3.36)-(B3.38) we see that the magnitude of the line currents is $\sqrt{3}$ times the magnitude of the phase currents and that the set of line currents lags (leads) the set of phase currents by 30° for positive (negative) sequences.

B3.2.2 Wye Connection

Figure B3.7 illustrates a Y- connected balanced three-phase load. The relationships developed however, can be applied to any component, e.g., generator, transformer, etc.

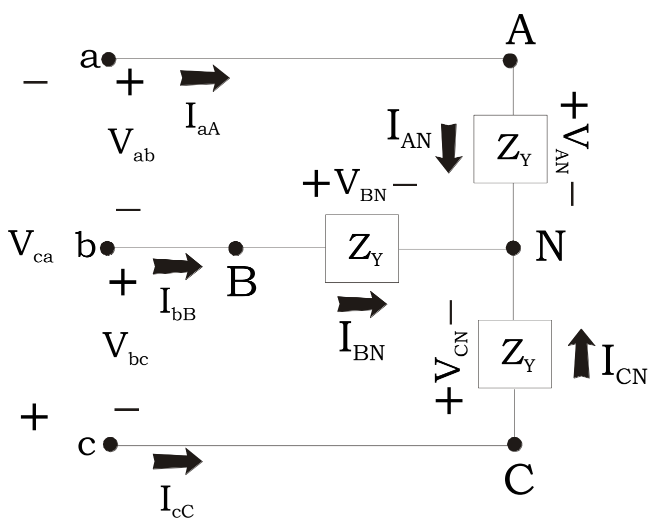


Figure B3.7 Wye Connection

In the Y-circuit, the line current $\boldsymbol{I}_{aA}$ is equal to the phase current $\boldsymbol{I}_{AN} = \boldsymbol{I}_\phi$. To demonstrate the relationship between the line-to-line voltages and the line-to-neutral voltages, we assume a positive, or abc, sequence and let V_ϕ be the magnitude of the line to neutral or phase voltage. We arbitrarily choose the line-to-neutral voltage of a-phase as the reference. We then have

$$\mathbf{V}_{AN} = V_\phi \angle 0^\circ \quad \text{(B3.42)}$$

$$\mathbf{V}_{BN} = V_\phi \angle -120^\circ \quad \text{(B3.43)}$$

$$\mathbf{V}_{CN} = V_\phi \angle +120^\circ \quad \text{(B3.44)}$$

We can express the line-to-line voltages in terms of the line-to-neutral voltages by direct application of Kirchoff's voltage law:

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = V_\phi - V_\phi \angle -120^\circ = \sqrt{3} V_\phi \angle 30^\circ \quad \text{(B3.45)}$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN} = V_\phi \angle -120^\circ - V_\phi \angle 120^\circ = \sqrt{3} V_\phi \angle -90^\circ \quad \text{(B3.46)}$$

$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN} = V_\phi \angle 120^\circ - V_\phi \angle 0^\circ = \sqrt{3} V_\phi \angle 150^\circ \quad \text{(B3.47)}$$

Equations (B3.45) - (B3.47) reveal that the magnitude of the line-to-line voltage is $\sqrt{3}$ times the magnitude of the line-to-neutral or phase voltage, and the set of line-to-line voltages leads (lags) the set of line-to-neutral voltages by 30° for positive (negative) sequences.

B3.2.3 Power Calculations in Balanced Three-Phase Circuits

The total power in a three-phase balanced circuit, i.e., power delivered by a three-phase generator or absorbed by three-phase load is determined by adding the power in each of the three phases. In a balanced circuit this is the same as multiplying the power in any one phase by 3 since the power is the same in all phases.

If the magnitude V_ϕ of the voltages to neutral for a Wye-connected circuit is

$$V_\phi = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}| \quad \text{(B3.48)}$$

and if the magnitude I_ϕ of the phase current for a Wye-connected circuit is

$$I_\phi = |\mathbf{I}_{an}| = |\mathbf{I}_{bn}| = |\mathbf{I}_{cn}| \quad \text{(B3.49)}$$

the total three-phase power is

$$P = 3 V_\phi I_\phi \cos\theta \quad \text{(B3.50)}$$

where θ is the phase angle difference between the phase current $\boldsymbol{I_\phi}$, and the phase voltage $\boldsymbol{V_\phi}$. If V_L and I_L are the magnitudes of line-to line voltage $\boldsymbol{V_L}$ and line current $\boldsymbol{I_L}$, respectively,

$$V_\phi = \frac{V_L}{\sqrt{3}} \quad \textit{and} \quad I_\phi = I_L \quad \text{(B3.51)}$$

Substituting Eq. (B3.51) into Eq. (B3.50) yields

$$P = \sqrt{3} V_L I_L \cos\theta \quad \text{(B3.52)}$$

The total vars are

$$Q = 3V_\phi I_\phi \sin\theta \tag{B3.53}$$

$$Q = \sqrt{3}\, V_L I_L \sin\theta \tag{B3.54}$$

and the voltamperes of the load are

$$S = P + jQ$$
$$|S| = \sqrt{P^2 + Q^2} = \sqrt{3} V_L I_L \tag{B3.55}$$

Equations (B3.50), (B3.54), and (B3.55) are used for calculating $P, Q,$ and $|S|$ in balanced three-phase networks since the quantities usually known are line-to-line voltage, line current, and the power factor,

$$pf = \cos\theta$$

When we refer to a three-phase system, balanced conditions are assumed unless otherwise specified, and the terms *voltage*, *current*, and *power*, unless otherwise specified, are understood to mean *line-to-line voltage*, *line current*, and total *three-phase power*, respectively.

If the circuit is Δ-connected, the voltage across each phase is the line-to-line voltage, and the magnitude of the current through each phase is the magnitude of the line current divided by $\sqrt{3}$ (see Section B3.1.1), or

$$V_\phi = V_L \qquad \textit{and} \qquad I_\phi = \frac{I_L}{\sqrt{3}} \tag{B3.56}$$

The total three-phase power is

$$P = 3V_\phi I_\phi \cos\theta \tag{B3.57}$$

Substituting Eq. (B3.56) in Eq. (B3.57) we obtain

$$P = \sqrt{3} V_L I_L \cos\theta \tag{B3.58}$$

which is identical to Eq. (B3.52). It follows that Eqs. (B3.54) and (B3.55) are also valid regardless of whether a particular circuit is connected Δ or Y.

B3.2.4 Per-Phase Analysis

From the analysis in Sections B3.2.1, and B3.2.2, we observe that in balanced three-phase circuits the currents and voltages in each phase are equal in magnitude and displaced from each other by 120°. This characteristic results in a simplified procedure to analyze balanced three-phase circuits. In this procedure it is necessary only to compute results in one phase and subsequently predict results in the other phases by using the relationship that exists among quantities in the other phases. These relationships have been derived in Sections B3.2.1, and B2.2.2. The example below describes the application of the procedure.

Example B 3.2

Three balanced three-phase loads are connected in parallel. Load 1 is Y-connected with an impedance of 150 + j50 Ω/ϕ; load 2 is Δ-connected with an impedance of 900 + j600 Ω/ϕ; and load 3 is 95.04 kVA at 0.6 pf leading. The loads are fed from a distribution line with an impedance of 3 + j24 Ω/ϕ. The magnitude of the line-to-neutral voltage at the load end of the line is 4.8 kV.

a) Calculate the total complex power at the sending end of the line.

b) What percent of the average power at the sending end of the line is delivered to the load?

Solution

The per-phase equivalent circuit is first constructed. For load 1, a Y-connected balanced load, the per-phase impedance is 150 + j50 Ω. For load 2, a Δ-connected balanced load, the per-phase impedance is the equivalent Y-connected load which is $Z_{\Delta}/3$ = 300 + j200 Ω. We will represent load 3 in terms of the complex power it absorbs per-phase. This is given by

$$\begin{aligned} S_{3/\phi} &= \frac{95040}{3}(0.6 - j0.8) \\ &= 19{,}008 - j25{,}344 VA \end{aligned}$$

The voltage across the per-phase equivalents of these loads has been specified as 4800 V which is the line-to-neutral voltage at the load end.

The per-phase equivalent circuit is shown below in Figure 3.8

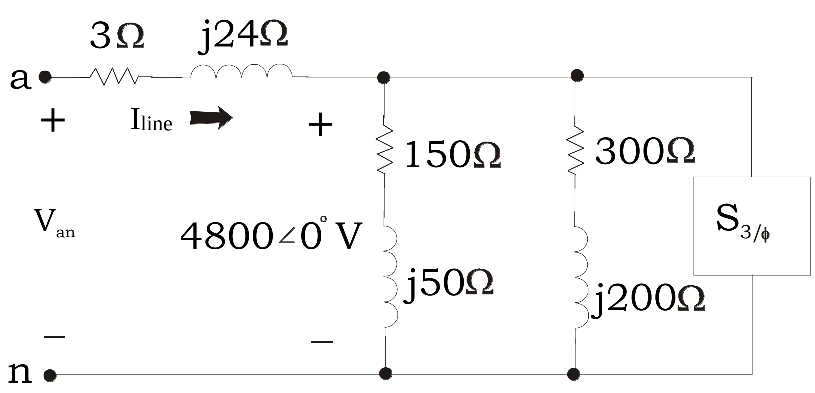


Figure 3.8 Per-Phase Equivalent Circuit

$$
\begin{aligned}
\mathbf{I}_\ell &= \frac{4800}{150+j50}+\frac{4800}{300+j200}+\frac{19{,}008+j25{,}344}{4800} \\
&= 28.8-j9.6+11.0769-j7.3846+3.96+j5.28 \\
&= 43.8369-j11.7046\,A(rms) \\
&= 45.3725\angle -14.949^\circ\,A(rms)
\end{aligned}
$$

In the above step, the total current $\boldsymbol{I}_\ell$ is obtained by summing the individual currents through the three loads. For loads 1&2, we use the expression $\mathbf{I}=\dfrac{\mathbf{V}}{Z}$, and for load 3 the current is determined using $\mathbf{I}=\left(\dfrac{S}{\mathbf{V}}\right)^*$.

In the distribution line,

$$
\begin{aligned}
P_{loss} &= 3\left|I_{eff}\right|^2 R = 3(45.3725)^2\,(3) = 18{,}528.04\,W \\
Q_{loss} &= 3\left|I_{eff}\right|^2 X = 3(45.3725)^2\,(24) = 148{,}224.34\,VAR
\end{aligned}
$$

In each load,

$$
\begin{aligned}
P_1 &= 3\left|28.8-j9.6\right|^2(150) = 414{,}720\,W \\
Q_1 &= 3\left|28.8-j9.6\right|^2(50) = 138{,}240\,VAR\,(abs) \\
P_2 &= 3\left|11.0769-j7.3846\right|^2(300) = 159{,}507.02\,W \\
Q_2 &= 3\left|11.0769-j7.3846\right|^2(200) = 106{,}338.02\,VAR\,(abs)
\end{aligned}
$$

$$
\begin{aligned}
P_3 &= 95{,}040\,(0.6) = 57{,}024W \\
Q_3 &= -95{,}040(0.8) = -76{,}032VAR \\
S_{total} &= 636{,}251+j168{,}546\,VA\;(load\ end)
\end{aligned}
$$

Check :

$$
\begin{aligned}
S_{total} &= 3(4800)(43.8369+j11.74046) = 631{,}251+j168{,}546\,VA \\
S_{sending} &= 631{,}251+j168{,}546+18{,}528.04+j148{,}224.34VA \\
&= 649{,}779.04+j316{,}770.34VA
\end{aligned}
$$

$$
\%P = \frac{631{,}251}{649{,}779.04}\times 100 = 97.148
$$

Example B 3.3

A balanced 230 volt (rms) three phase source is furnishing 6 kVA at 0.83 pf lagging to two Δ - connected parallel loads. One load is a purely resistive load drawing 2 kW. Determine the phase impedance of the second load.

Solution

The total complex power absorbed by the load is given by

$$S = 6\times10^{3}\left(0.83 + j0.5577\right)VA$$

Note it is specified in the problem that

$$\cos\theta = 0.83$$
$$\sin\theta = \sin\left(\cos^{-1}\left(0.83\right)\right) = 0.5577$$
$$S = 4980 + j3346.58VA$$

Load 1 absorbs $S_1 = 2000 + j0VA$

As a result, Load 2 must absorb $S - S_1 = 2980 + j3346.58VA$

The power absorbed by Load 2 per phase is $S_{2/\phi} = \frac{1}{3}\left(2980 + j3346.38\right) = 993.33 + j1115.5266VA$

$$S_{2/\phi} = \frac{V_{LL}^2}{Z_\phi^*}$$

Hence, $Z_\phi^* = \dfrac{\left(230\right)^2}{993.33 + j1115.5266} = 23.552 - j26.449\Omega$

$$\therefore Z_\phi = 23.552 + j26.449\Omega$$

PROBLEMS

Problem 1

The 3-phase loads are connected in parallel. One is a purely resistive load connected in wye. It consumes 300kW. The second is a purely inductive 300kVAR load connected in wye. The third is a purely capacitive 300kVAR load connected in wye. The line-to-line voltage at the load is 5kV. A 3-phase distribution line supplying this load has an impedance of 10+j5 ohms per phase.

(a) Calculate the currents drawn by each load (magnitude and phase).
(b) Indicate the power factor of each load. Remember that non-unity power factors must also include whether they are lagging or leading.
(c) What is the power factor of the entire load? That is, what is the power factor seen by the transmission line at the load end?
(d) Calculate the real and reactive power supplied at the sending end of the distribution line.

Problem 2

A three phase load has a per phase impedance, connected in Y, of $100 + j30\Omega$. The line-to-line voltage magnitude at the load is 1500V. The three-phase distribution line supplying this load has an impedance of $10 + j5\Omega/\phi$

(a) Calculate the line-to-line voltage magnitude at the sending end of the distribution line.
(b) Calculate the real and reactive power supplied at the sending end of the distribution line.

Problem 3

A three-phase load consumes 100kVA at 0.7 pf lagging. The line-to-line voltage magnitude at the load is 1500V. The three-phase distribution line supplying this load has an impedance of $10 + j5\Omega/\phi$

(a) Calculate the line-to-line voltage magnitude at the sending end of the distribution line.
(b) Calculate the real and reactive power supplied at the sending end of the distribution line.

Problem 4

The complex power absorbed by a three-phase load is 1500kVA at 0.8 pf lag

$P_{/\phi} =$ ______________ $Q_{/\phi} =$ ______________

If the Line voltage at the load in problem 1 is 8660.2540 V, what is the voltage magnitude across each phase of the load, if the load is connected in delta or if the load is connected in wye?

Delta: $|V_d| =$ ______________ Wye: $|V_y| =$ ______________

What is the magnitude of line current drawn by this load?

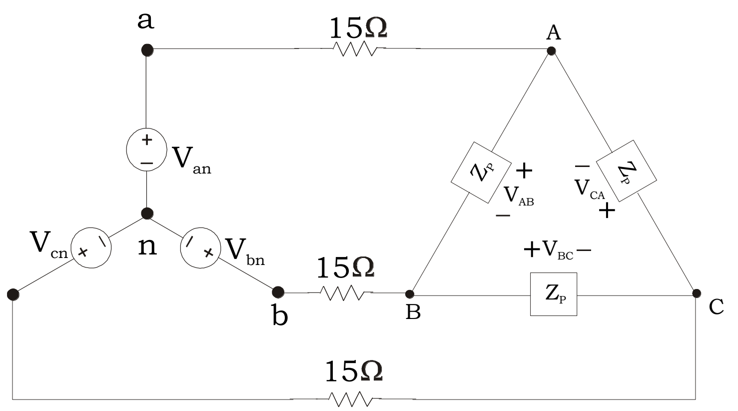
$|I_L| =$ ______________

Problem 5

In the circuit shown below, V_{an} = 12,000 + j 0 V (rms). Assume positive phase sequence. The balanced source supplies 1.5 MW and 0.3 MVAR from its terminals to the three phase balanced line and load. Find:

a) The rms line current.

b) Z_p



Problem 6

A three-phase source is supplying a balanced three phase load over a transmission line having impedance of Z_L=2+j20 ohms per phase. The voltage at the source end of the transmission line is 2887 ∠0 volts line to neutral. The current supplied through the transmission line is I_L=100 ∠-30° amperes.

1. Determine the power factor seen by the source, and specify whether it is leading or lagging.
2. Determine the voltage (line to neutral) at the load.
3. Determine the power factor of the load, and specify whether the load is
   a. leading or lagging
   b. inductive or capacitive
4. Determine the real and reactive power consumed by the load.

Problem 7

A balanced, three-phase load having a power factor of 0.8 lagging is supplied by a transmission line carrying 300 amps at 115 kV line-to-line. Compute the three-phase real and reactive power *delivered* to the load.

Problem 8

A balanced, three-phase, delta-connected load consumes 50-j20 kVA at a line-to-line voltage of 13.8 kV. Compute the per-phase impedance of this load assuming a series connection between R and X.

Problem 9

A three-phase wye-connected load having impedance of Z_1=200+j50 ohms per phase is connected in parallel with a three phase delta-connected load having impedance of Z_2=600+j300 ohms per phase. The load is supplied by a three-phase wye-connected generator that is directly interconnected with the loads (i.e., there is no transmission line between the generator and the loads). The voltage magnitude of the generator is 13.8 kV line-to-line. Assume that the phase to neutral voltage at the generator is the angle reference.

1. Draw the three-phase circuit. Clearly identify the numerical values of one line to neutral source voltage phasor and one-phase impedance for each of loads 1 and 2.
2. Draw the per-phase circuit. Clearly identify the numerical values of the source voltage phasor and the per-phase impedances of loads 1 and 2.
3. Compute the three-phase complex power consumed by each load and the total, complex three-phase power consumed by the two loads.
4. Show that the total, complex three-phase power consumed by the two loads can be computed using the line current and the line-to-line value of the source voltage.

Problem 10

Consider a balanced three-phase source supplying a balanced Y- or Δ- connected load with the following instantaneous voltages and currents.

$$v_{an} = \sqrt{2}\left|V_p\right|\cos(\omega t + \theta_v) \qquad i_a = \sqrt{2}\left|I_p\right|\cos(\omega t + \theta_i)$$

$$v_{bn} = \sqrt{2}\left|V_p\right|\cos(\omega t + \theta_v - 120°) \qquad i_b = \sqrt{2}\left|I_p\right|\cos(\omega t + \theta_i - 120°)$$

$$v_{cn} = \sqrt{2}\left|V_p\right|\cos(\omega t + \theta_v - 240°) \qquad i_c = \sqrt{2}\left|I_p\right|\cos(\omega t + \theta_i - 240°)$$

where $|V_p|$ and $|I_p|$ are the magnitudes of the rms phase voltage and current, respectively. Show that the total instantaneous power provided to the load, as the sum of the instantaneous powers of each phase, is a constant.

Problem 11

A three-phase line has an impedance of 2+j4 ohms/phase, and the line feeds two balanced three-phase loads that are connected in parallel. The first load is Y-connected and has an impedance of 30+j40 ohms/phase. The second load is delta-connected and has an impedance of 60-j45 ohms/phase. The line is energized at the sending end from a three-phase balanced supply of line voltage 207.85 volts. Taking the phase voltage V_{an} at the supply as reference, determine:

a. The current, real power, and reactive power drawn from the supply.
b. The line voltage at the combined loads.
c. The current per phase in each load.
d. The total real and reactive powers in each load and the line.

Problem 12

A three-phase line has an impedance of 0.4+j2.7 ohms per phase. The line feeds two balanced three-phase loads that are connected in parallel. The first load is absorbing 560.1kVA at 0.707 power factor lagging. The second load absorbs 132 kW at unity power factor. The line-to-line voltage at the load end of the line is 3810.5 volts. Determine:

a. The magnitude of the line voltage at the source end of the line.
b. Total real and reactive power loss in the line.
c. Real power and reactive power supplied at the sending end of the line.