Module B4
Per Unit Analysis

B4.1

Per Unit Calculations

WHAT IS A PER UNIT VALUE?

- The numerical per-unit value of any quantity is its ratio to the chosen *base quantity* of the same dimensions.
- A per-unit quantity is a *normalized* quantity with respect to a chosen base value. It has magnitude only.
- Industry uses per-unit representation in all system studies.

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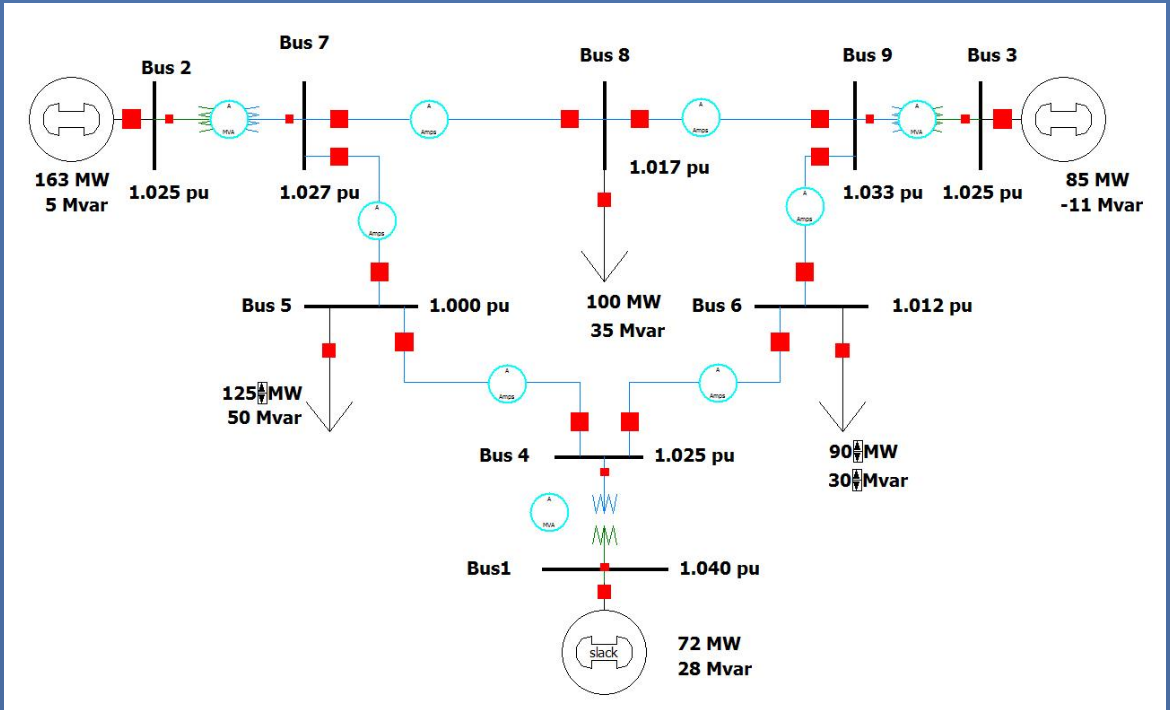
ADVANTAGES OF PU REPRESENTATION:

- device parameters tend to fall in a relatively fixed range, making erroneous values prominent.
- in system analysis, ideal transformers are eliminated as circuit elements, resulting in reduction of the number of components that must be represented in a system study.
- in system analysis, voltage magnitudes throughout a given normally operating power system are close to unity (1.0), providing a way to perform “sanity checks” on calculations.



How to convert to per unit system?

- Choose voltage base and power base.
- Compute current base, impedance base.
- Divide all actual quantities by the respective base to get pu quantity, i.,e.

VOLTAGE $$V_{pu} = \frac{V}{V_{base}},$$

POWER $$S_{pu} = \frac{S}{S_{base}},\ P_{pu} = \frac{P}{S_{base}},\ Q_{pu} = \frac{Q}{S_{base}}$$

CURRENT $$I_{pu} = \frac{I}{I_{base}},$$

IMPEDANCE $$Z_{pu} = \frac{Z}{Z_{base}},\ (\text{or } Y_{pu} = \frac{Y}{Y_{base}} = \frac{1}{Z_{pu}})$$

Always use units of volts, amps, VA, watts, vars, ohms, mhos.

Do not use units of kV, kA, kVA, MVA, kW, MW, etc.

How to choose base voltage and base power?

- Mathematically,
  - one voltage can be chosen to be any number
  - power base can be chosen to be any number

How to choose base voltage and base power?

- Practically,
  - one voltage should be chosen corresponding to nominal voltage of associated part of system.
  - power base should be chosen as a multiple of 10, e.g., 10 kVA, 100 kVA, 1 MVA, 10 MVA, etc., depending on "size" of the system.

We will discuss later why we have emphasized "one voltage"

How to compute base current and impedance? Assuming Y-connection…

$$\text{base current, } I = \frac{\text{base power } S_{l\phi}}{\text{base voltage, } V_{LN}}$$

$$\text{base impedance, } Z = \frac{\text{base voltage, } V_{LN}}{\text{base current, } I} = \frac{(\text{base voltage } V_{LN})^2}{\text{base power } S_{1\phi}}$$

$$\text{base admittance, } Y = \frac{1}{\text{base impedance, } Z}$$

Use the above for 1-phase or 3-phase circuits.

Some comments on per unit in 3 phase circuits:

- Equations on previous page assume Y-connection:
  - voltages (actual & base) must be line to neutral
  - powers (actual & base) must be per phase
  - currents (actual & base) must be line current
    (there is no choice in the Y-connection)

Some comments on per unit in 3 phase circuits:

- We can obtain a line-line base voltage:

$$\text{base voltage}\, V_{LL} = \sqrt{3}\, \text{base voltage}\, V_{LN}$$

- We can obtain a base current for delta:

$$\text{base current } I_{\Delta} = \frac{\text{base current } I}{\sqrt{3}}$$

More comments on per unit in 3 phase circuits:

- We can obtain a base impedance for delta:

$$\text{base impedance } Z_{\Delta} = 3\,\text{base impedance } Z$$

- We can obtain a base power for 3 phase:

$$\text{base power } S_{3\phi} = 3\,\text{base power } S_{1\phi}$$

More comments on per unit in 3 phase circuits:

- Base current can also be computed using line to line voltage:

$$\text{base current } I = \frac{\text{base power } S_{1\phi}}{\text{base voltage}, V_{LN}} = \frac{\text{base power } S_{3\phi}/3}{(\text{base voltage } V_{LL})/\sqrt{3}}$$

$$= \frac{\text{base power } S_{3\phi}}{\sqrt{3}\,\text{base voltage } V_{LL}}$$

More comments on per unit in 3 phase circuits:

- Base impedance can also be computed using line to line voltage. From slide 9:

$$\text{base impedance, } Z = \frac{\text{base voltage, } V_{LN}}{\text{base current, } I} = \frac{(\text{base voltage } V_{LN})^2}{\text{base power } S_{1\phi}}$$

- Then

$$\text{base impedance, } Z = \frac{(\text{base voltage } V_{LL} / \sqrt{3})^2}{\text{base power } S_{1\varphi}}$$

$$= \frac{(\text{base voltage } V_{LL})^2}{3 \text{ base power } S_{1\varphi}} = \frac{(\text{base voltage } V_{LL})^2}{\text{base power } S_{3\varphi}}$$

So base impedance can be computed using $V_{LN}^2/S_{1\phi}$ or using $V_{LL}^2/S_{3\phi}$.

More comments on per unit in 3 phase circuits:

- Base current can also be computed using line to line voltage:

$$\text{base current } I = \frac{\text{base power } S_{1\phi}}{\text{base voltage}, V_{LN}} = \frac{\text{base power } S_{3\phi}/3}{(\text{base voltage } V_{LL})/\sqrt{3}}$$

$$= \frac{\text{base power } S_{3\phi}}{\sqrt{3}\,\text{base voltage } V_{LL}}$$

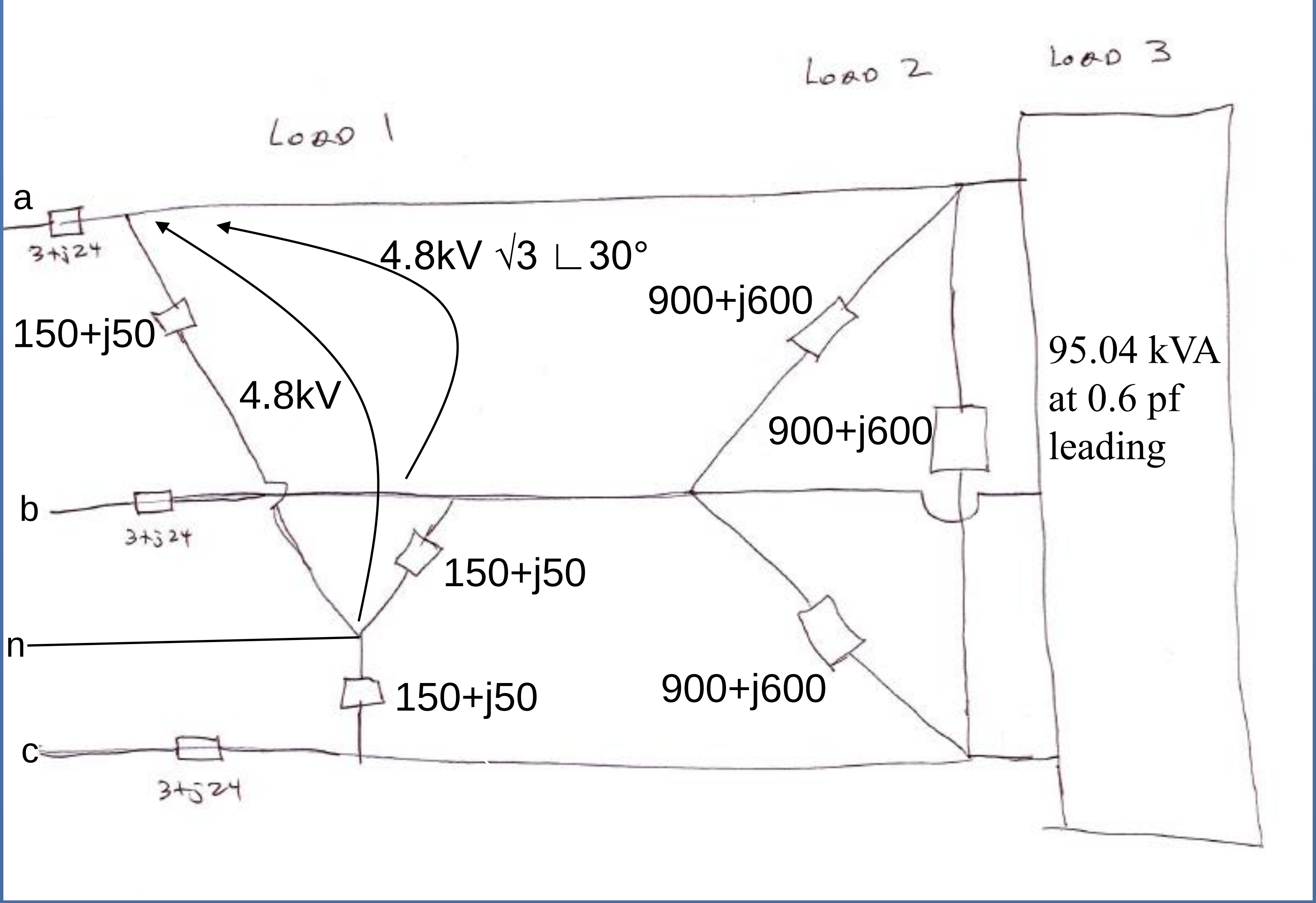
Please study example B4.1, which illustrates per unit analysis for a simple single phase system.

We will look at a different example, for a three phase system, in class. This example is Problem B3.2, Module B3 in your text, where it is worked using per-phase analysis. Here, we will work it in per-unit.

Refer to example B3.2

Three balanced three-phase loads are connected in parallel. Load 1 is Y-connected with an impedance of 150 + j50 ; load 2 is delta-connected with an impedance of 900 + j600 ; and load 3 is 95.04 kVA at 0.6 pf leading. The loads are fed from a distribution line with an impedance of 3 + j24 . The magnitude of the line-to-neutral voltage at the load end of the line is 4.8 kV.

Example B3.2



Load 1
Load 2
Load 3
a
3+j24
4.8kV √3 ∟30°
150+j50
4.8kV
900+j600
900+j600
95.04 kVA
at 0.6 pf
leading
b
3+j24
150+j50
n
150+j50
900+j600
c
3+j24

Let's work this problem in per unit!

Choose:

$$\text{base voltage}\, V_{LN} = 4.8\,\text{kV} = 4800\,\text{volts},$$
$$\text{base power}\, S_{1\phi} = 10\,\text{kVA} = 10{,}000\,\text{VA}$$

1. Compute remaining base quantities:

$$\text{base voltage}\, V_{LL} = \sqrt{3}(\text{base}\, V_{LN}) = \sqrt{3}(4800) = 8313.8\,\text{volts}$$
$$\text{base power}\, S_{3\phi} = 3(\text{base power}\, S_{1\phi}) = 3(10{,}000) = 30{,}000\,\text{VA}$$
$$\text{base current}\, I = \frac{\text{base power}\, S_{3\phi}}{\sqrt{3}\,\text{base voltage}\, V_{LL}} = \frac{30{,}000\,\text{VA}}{\sqrt{3}(8313.8)} = 2.0833\,\text{A}$$
$$\text{base impedance}\, Z = \frac{(\text{base voltage}\, V_{LL})^2}{\text{base power}\, S_{3\phi}} = \frac{(8313.8)^2}{30{,}000} = 2304\,\Omega$$

2. Convert from SI units into per unit:

$$Z_{Lpu} = \frac{Z_L}{\text{base impedance Z}} = \frac{3 + j24}{2304} = 0.0013 + j0.0104$$

$$Z_{1pu} = \frac{Z_1}{\text{base impedance Z}} = \frac{150 + j50}{2304} = 0.0651 + j0.0217$$

$$Z_{2pu} = \frac{Z_2}{\text{base impedance Z}} = \frac{300 + j200}{2304} = 0.1302 + j0.0868$$

Note we converted the delta Z into a Wye Z.

2. Convert from SI units into per unit (cont):

$$S_{3pu} = \frac{S_{3,3\phi}}{\text{power base } S_{3\phi}} = \frac{95040(0.6 - j0.8)}{30{,}000} = 3.168(0.6 - j0.8)$$

$$= \frac{S_{3,1\phi}}{\text{power base } S_{1\phi}} = \frac{31680(0.6 - j0.8)}{10{,}000} = 3.168(0.6 - j0.8)$$

$$V_{Load,pu} = \frac{V_{Load,LN}}{\text{voltage base } V_{LN}} = \frac{4800}{4800} = 1.0$$

$$= \frac{V_{Load,LL}}{\text{voltage base } V_{LL}} = \frac{\sqrt{3}(4800)}{8314} = 1.0$$

You can compute pu powers using 3 phase or per phase quantities.

You can compute pu voltages using LN or LL quantities.

But be ***consistent*** that numerator is same type as denominator!

For example, if the numerator is a 1-ϕ power, the denominator needs to be the 1-ϕ power base.

Or if the numerator is a LL voltage, the denominator needs to be the LL voltage base.

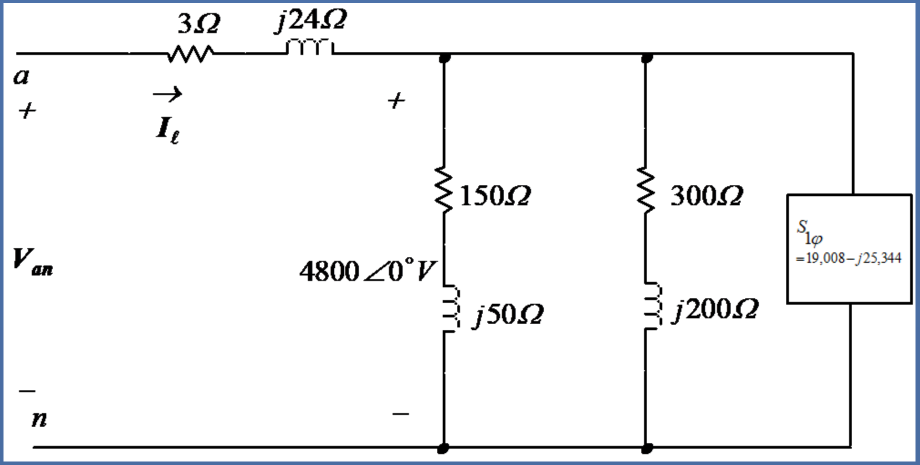
3. Compute the current into the load in pu.

$$I_{L,pu} = I_{1pu} + I_{2pu} + I_{3pu}$$

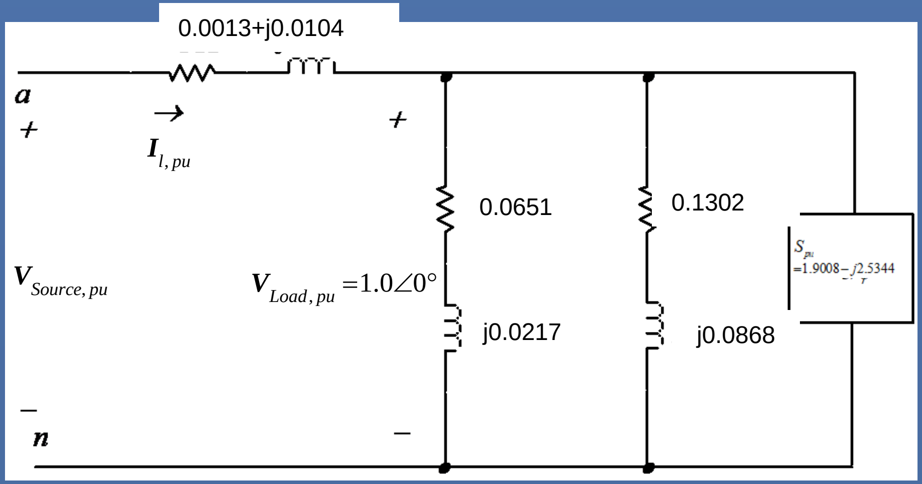
$$= \frac{1.0}{0.0651 + j0.0217} + \frac{1.0}{0.1302 + j0.0868} + \frac{3.168(0.6 + j0.8)}{1.0}$$

$$= 13.825 - j4.608 + 5.317 - j3.545 + 1.901 + j2.534$$

$$= 21.043 - j5.619 \text{ pu} = 21.780\angle{-14.951} \text{ pu}$$



In SI units; a per-phase circuit



In per-unit; a pu circuit

In both cases, the circuit is single-phase.

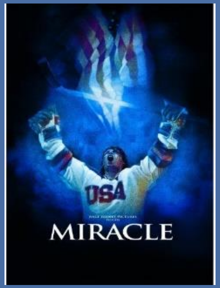
3. Compute the current into the load in pu.

Compare this answer, 21.780 |_ -14.951 amperes, with the one we obtained in module B3, example B3.2, which was 45.3725 |_ -14.949 amperes. You see that we obtain different magnitudes, same angle.

Let’s convert the pu magnitude to amps.

$$I_L = I_{Lpu} \times \text{base current } I = (21.780\angle -14.951)2.0833 = 45.375\angle -14.951$$

So it is the same!



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This only works if bases are chosen ***consistently*** (slide 22) and per-unitization is done ***correctly***:

Correctly? ➔Computing bases must use same relations used for computing magnitudes of electrical quantities (e.g., Ohm’s law, power relations, impedance relations).

4. Compute losses in distribution line.

$$P_{loss,pu} = \left|I_{Lpu}\right|^2 R_{Lpu} = \left|21.78\right|^2 \times 0.0013 = 0.6167 \text{ pu}$$

$$Q_{loss,pu} = \left|I_{Lpu}\right|^2 X_{Lpu} = \left|21.78\right|^2 \times 0.0104 = 4.9334 \text{pu}$$

Let's check to see if these losses are the same as what we computed in Module B3.

Note carefully that multiplication by per phase power base gives us per phase power, and multiplication by 3 phase power base gives us 3 phase power, which is what we want.

Previously, we obtained 18,528 watts and 148,224 vars.

$$P_{loss} = \text{power base } S_{3\phi} \times P_{loss,pu} = (30{,}000)0.6167 = 18{,}501 \text{ watts}$$

$$Q_{loss} = \text{power base } S_{3\phi} \times Q_{loss,pu} = (30{,}000)4.9334 = 148{,}002 \text{ vars}$$

There is small difference due to round-off error.

5. Let's compute source voltage.

$$V_{an,pu} = V_{load,pu} + I_{Lpu}(Z_{Lpu})$$
$$= 1.0\angle 0 + 21.78\angle -14.951(0.0013 + j0.0104)$$
$$= 1.0858 + j0.2115 = 1.1062\angle 11.0245$$

This approach differs from the solution approach we took in Ex. B3.2 (there we computed powers in loads, added to losses, and then computed source voltage from V=S/I*), but the approach here is an equally valid way to obtain the source voltage.

Now we can obtain total source power and total load power.

$$S_{sending,pu} = V_{an,pu}(I_{Lpu})^* = 1.1062\angle 11.0245(21.78\angle +14.951)$$

$$= 21.659 + j10.553 \text{ pu}$$

$$S_{totalload,pu} = V_{Lpu}(I_{Lpu})^* = 1.0\angle 0(21.78\angle +14.951)$$

$$= 21.043 + 5.619 \text{ pu}$$

6. Convert powers into watts & vars.

$$S_{sending,3\phi} = S_{sending,pu}(\text{power base } S_{3\phi}) = (21.659 + j10.553)30{,}000$$
$$= 649{,}770 + \text{j}316{,}590$$
$$S_{totalload,3\phi} = S_{totalload,pu}(\text{power base } S_{3\phi}) = (21.043 + 5.619)30{,}000$$
$$= 631{,}290 + \text{j}168{,}570$$

Previously, we obtained

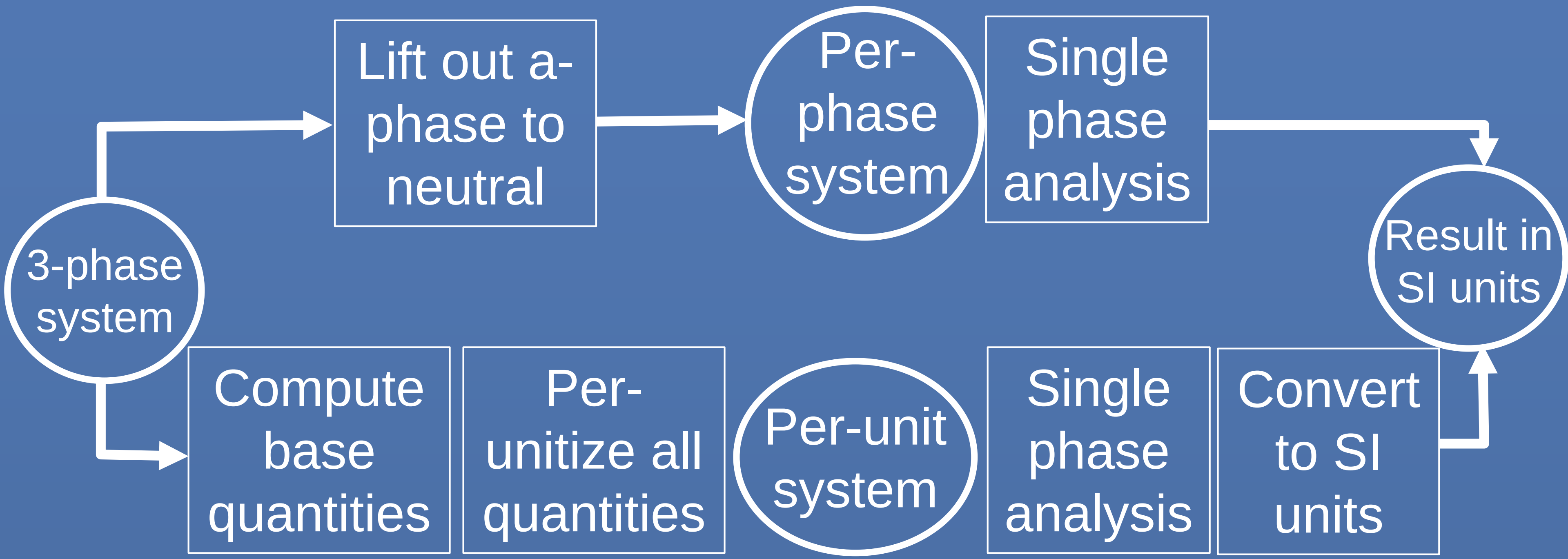
sending end power = 649,779+j316,770

receiving end power = 631,251+168,546

So again, difference is very small due to round-off error.

Re-emphasize: answers from pu will agree with answers from per phase if bases are *consistent* and quantities have been per-unitized *correctly*.

Big Picture: per-phase analysis vs. per-unit analysis



Per-unit looks like more work, and it is for problems where you do everything manually. For large system analysis, we must have computers to do the work, and in this case, there is no disadvantage to “more work” in this sense. In addition, because the datasets are all already in pu, the first steps (compute base quantities & per-unitize) have already been done. Therefore, we get the advantages of the pu system with no additional “cost” in terms of effort (see slide 4: data error detection; no ideal xfmrs; voltages~1.0).

Homework problem: (will be due with next assignment).

Choose your voltage base and power base as

$$\text{base voltage } V_{LN} = 4.8\,\text{kV} = 5000\,\text{volts},$$
$$\text{base power } S_{1\phi} = 10\,\text{kVA} = 100{,}000\,\text{VA}$$

Rework parts 1-6 of the previous problem. You will know you have it correct if the answers in SI units are the same.

Question: Will the per unit values of impedances, voltages, currents, and powers be the same?