Module B3

More on
Three Phase Analysis

B3.2

Balanced Three Phase Circuits

Terminology:

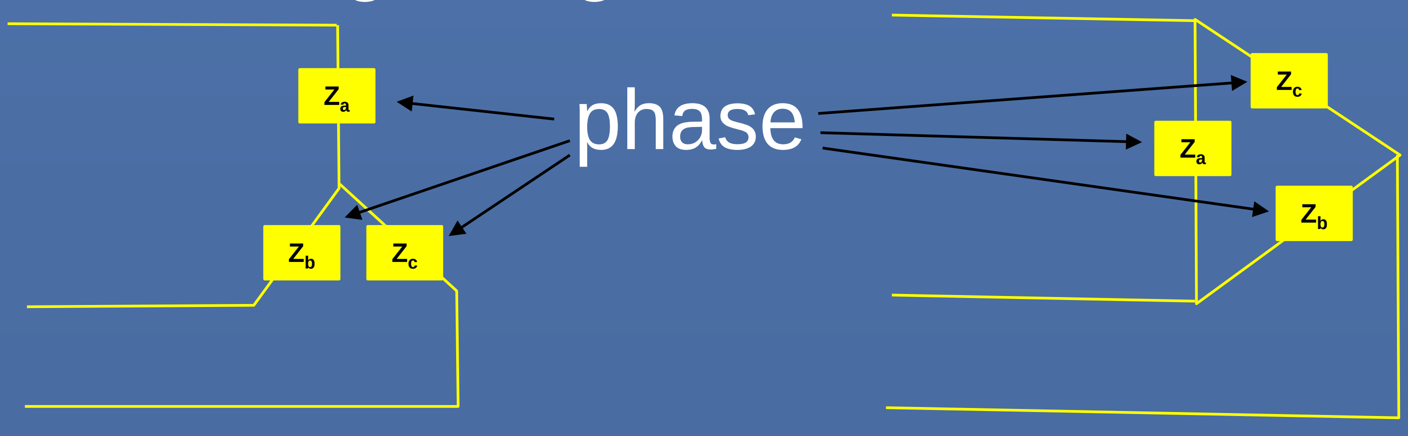
- phase: the impedance or voltage source element in the three-phase connection.
- phase current: current through the phase.
- phase voltage: voltage across the phase.

Notation:

I_ϕ: phase current magnitude;

V_ϕ: phase voltage magnitude;

This notation will be used to express phase current phasor mag.



Wye connected load **Delta connected load**

Terminology:

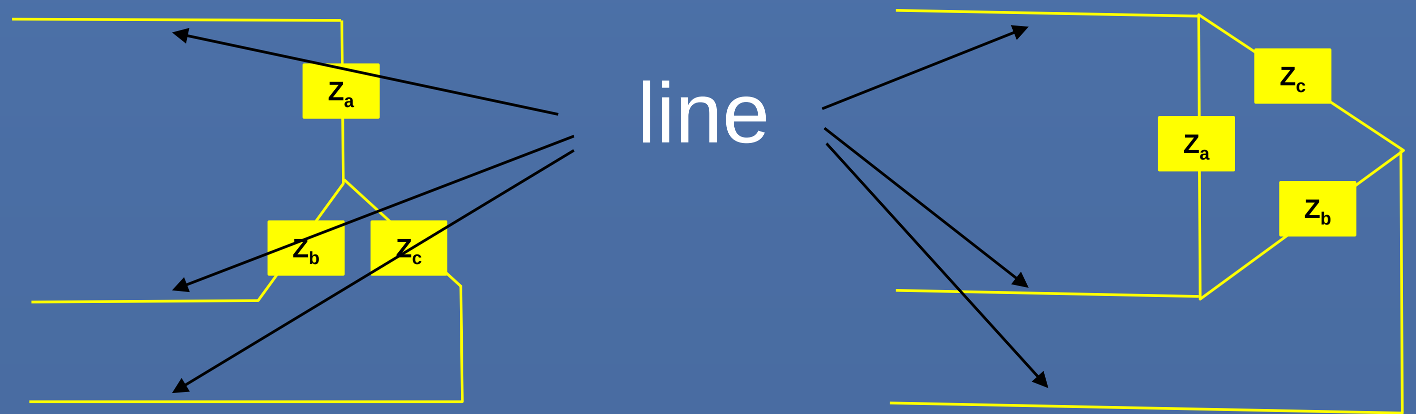
- line: one of the three conductors connecting source to load.
- line current: current through a line.
- line voltage (or line-to-line voltage): voltage between 2 of the 3 lines at a common point.

Notation:

I_L: line current magnitude;

V_L: line voltage magnitude;

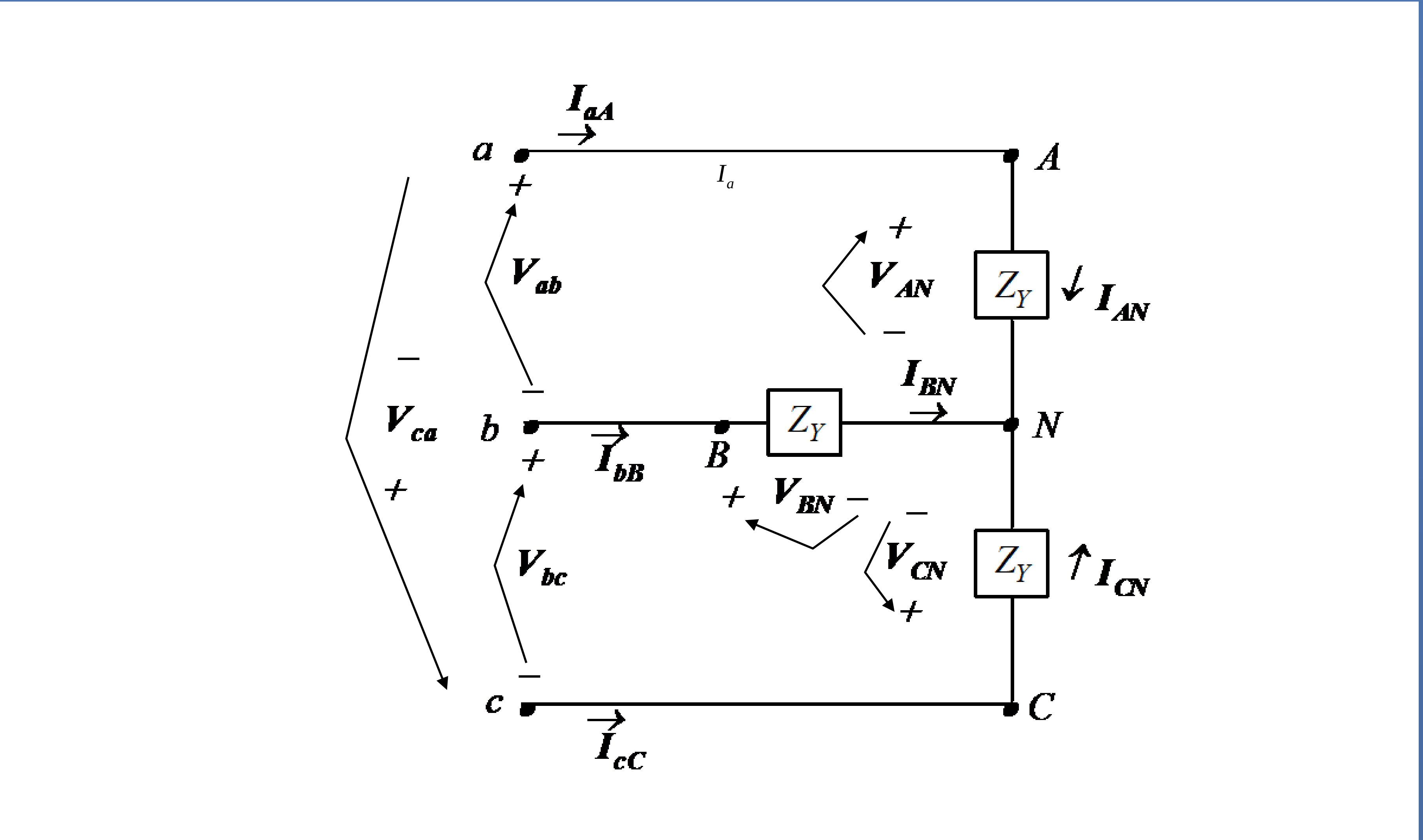
This notation will be used to express line current phasor mag.



Wye connected load **Delta connected load**

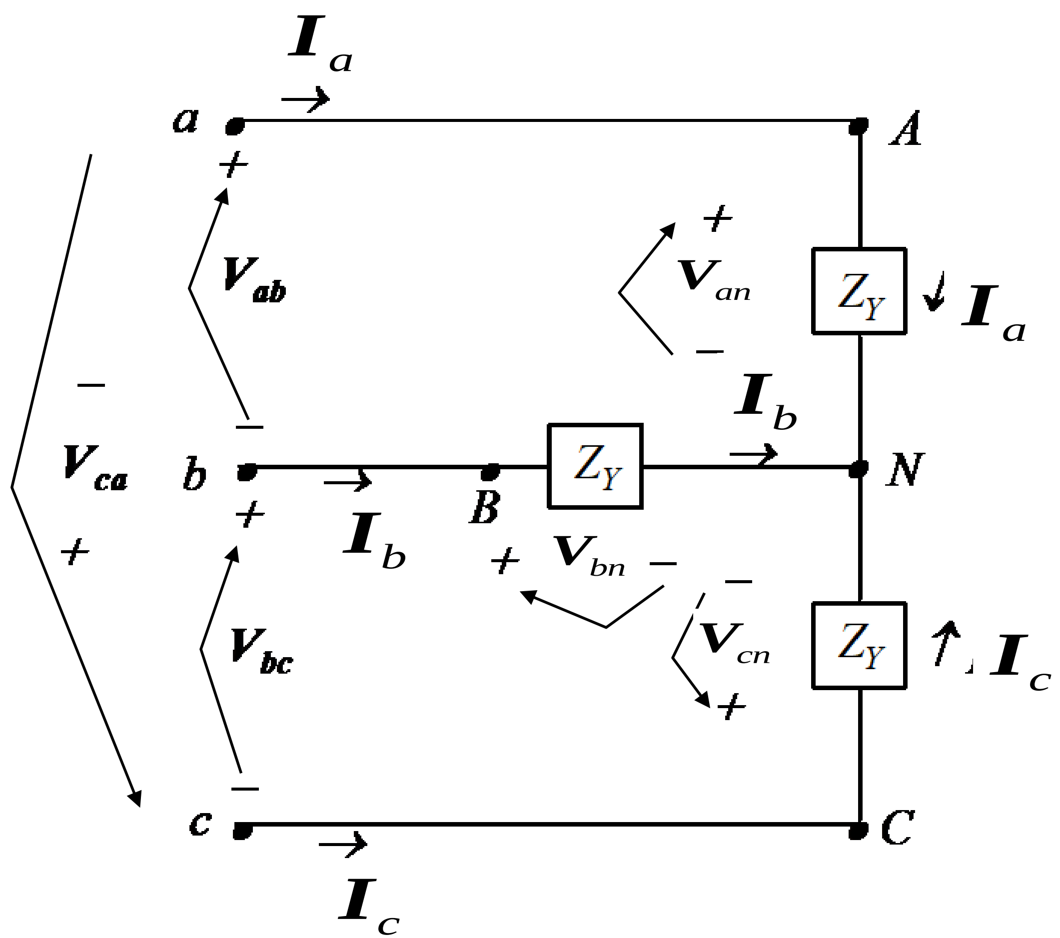
WYE Connection (as shown in B3 notes)

Lower and upper case subscripts correspond to the particular identified nodes. They are used in this figure to be very explicit regarding each quantity. However, it is more common to use only one or the other, as indicated in the next slide.



WYE Connection

Same figure, but with more common nomenclature.



WYE Connection

$$I_{AN} = I_{BN} = I_{CN} = I_{\phi}\,, \qquad V_{AN} = V_{BN} = V_{CN} = V_{\phi}$$

$$\boldsymbol{V_{AN}} = V_{\phi}\angle 0^{\circ}$$

$$\boldsymbol{V_{BN}} = V_{\phi}\angle -120^{\circ}$$

$$\boldsymbol{V_{CN}} = V_{\phi}\angle +120^{\circ}$$

$$\boldsymbol{V_{AB}} = \boldsymbol{V_{AN}} - \boldsymbol{V_{BN}} = V_{\phi} - V_{\phi}\angle -120^{\circ} = \sqrt{3}V_{\phi}\angle 30^{\circ}$$

$$\boldsymbol{V_{BC}} = \boldsymbol{V_{BN}} - \boldsymbol{V_{CN}} = V_{\phi}\angle -120^{\circ} - V_{\phi}\angle 120^{\circ} = \sqrt{3}V_{\phi}\angle -90^{\circ}$$

$$\boldsymbol{V_{CA}} = \boldsymbol{V_{CN}} - \boldsymbol{V_{AN}} = V_{\phi}\angle 120^{\circ} - V_{\phi}\angle 0^{\circ} = \sqrt{3}V_{\phi}\angle 150^{\circ}$$

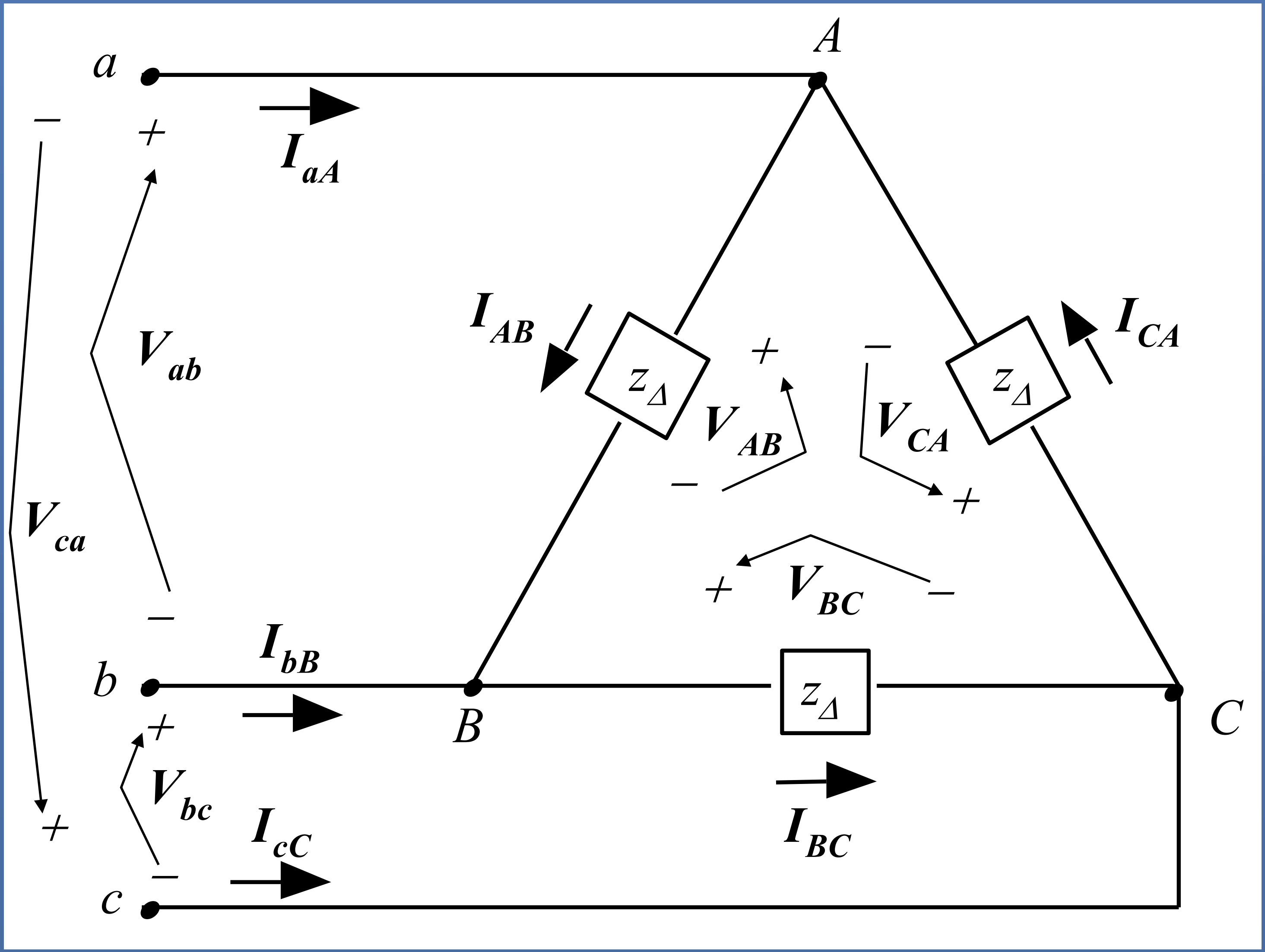
Wye connection:

Line-Line voltages are sqrt(3) times phase voltages in magnitude and lead them by 30 degrees in angle.

Y: Line-Line Voltages Lead: Y-LV-Lead

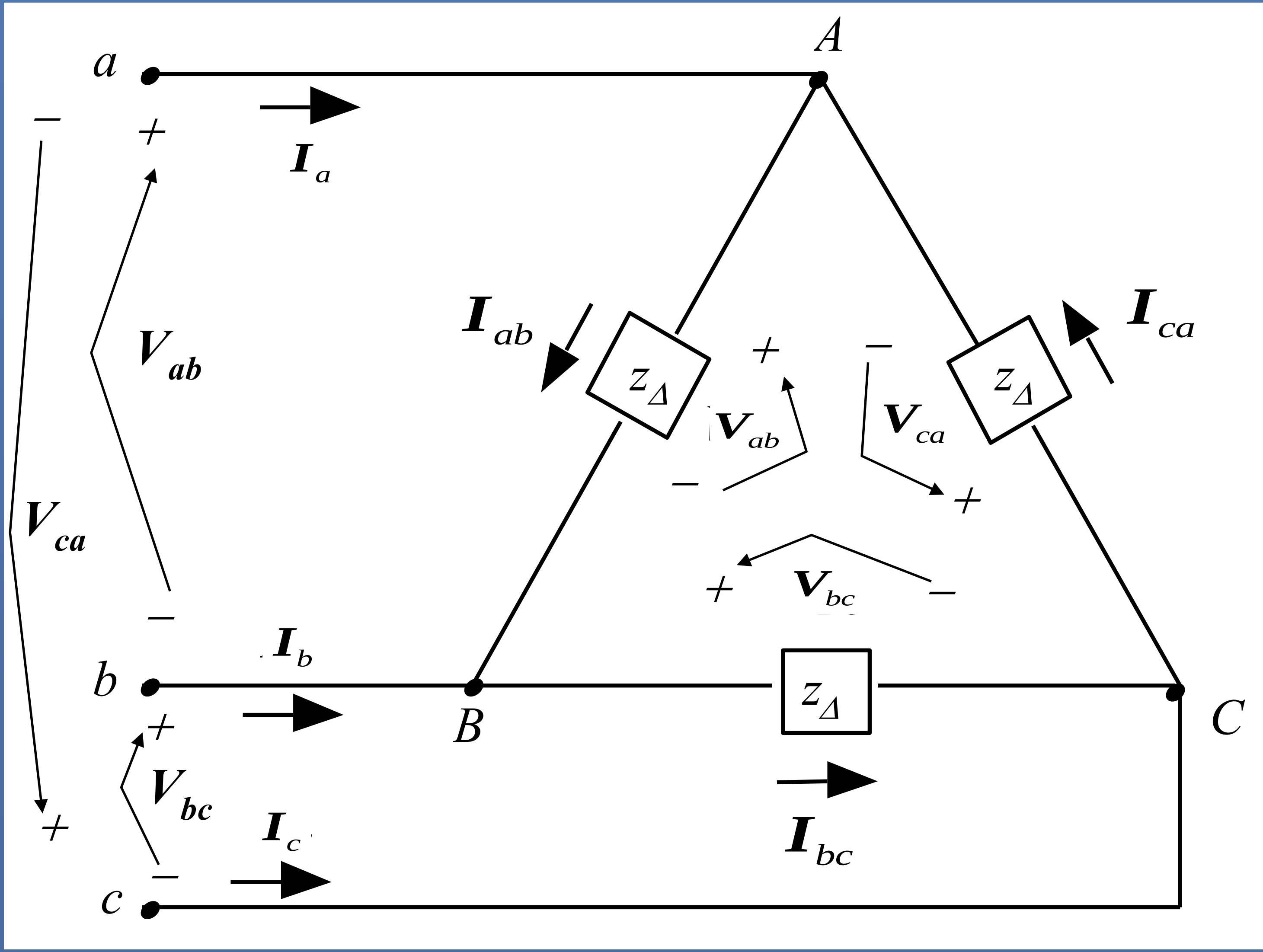
Line currents equal phase currents

Delta Connection (as shown in B3 notes)



Delta Connection

Same figure, but with more common nomenclature.



Delta Connection

$$I_{AB} = I_{BC} = I_{CA} = I_\phi, \qquad V_{AB} = V_{BC} = V_{CA} = V_\phi$$

$$\boldsymbol{I_{AB}} = I_\phi \angle 0^\circ$$

$$\boldsymbol{I_{BC}} = I_\phi \angle -120^\circ$$

$$\boldsymbol{I_{CA}} = I_\phi \angle +120^\circ$$

$$\mathbf{I_{aA}} = \mathbf{I_{AB}} - \mathbf{I_{CA}} = I_\varphi \angle 0^\circ - I_\varphi \angle 120^\circ = \sqrt{3} I_\varphi \angle -30^\circ$$

$$\mathbf{I_{bB}} = \mathbf{I_{BC}} - \mathbf{I_{AB}} = I_\varphi \angle -120^\circ - I_\varphi \angle 0^\circ = \sqrt{3} I_\varphi \angle -150^\circ$$

$$\mathbf{I_{cC}} = \mathbf{I_{CA}} - \mathbf{I_{BC}} = I_\varphi \angle 120^\circ - I_\varphi \angle -120^\circ = \sqrt{3} I_\varphi \angle 90^\circ$$

Delta connection:

Line-line voltages equal phase voltages

Line currents are sqrt(3) times
phase currents in magnitude
and lag them by 30 degrees in angle.

Delta: Line Currents Lag: Delta-LC-LAG

Power relations for three phase circuits

$$P = 3V_\phi I_\phi \cos\theta$$

$$P = \sqrt{3} V_L I_L \cos\theta$$

$$Q = 3V_\phi I_\phi \sin\theta$$

$$Q = \sqrt{3} V_L I_L \sin\theta$$

$$S = P + jQ$$

$$|S| = \sqrt{P^2 + Q^2} = \sqrt{3} V_L I_L$$

- I_ϕ: phase current magnitude;
- V_ϕ: phase voltage magnitude;
- V_L: line-to-line voltage magnitude
- I_L: line current magnitude

- These formulae can be used for Wye or Delta
- All quantities are magnitudes
- θ is the angle for which phase voltage leads phase current
- Positive Q is for power flowing into L load
- Negative Q is for power flowing “into” C load

Per-phase analysis of 3 phase circuits

- Convert all delta connections to Y connections using

$$Z_Y = \frac{Z_\Delta}{3}$$

(We can prove this via application of a Y-Δ transformation)

- "Lift out" the a-phase to neutral circuit
- Perform single phase analysis using phase quantities and per phase powers
- Multiply all powers by 3 to get solution in terms of three phase powers.

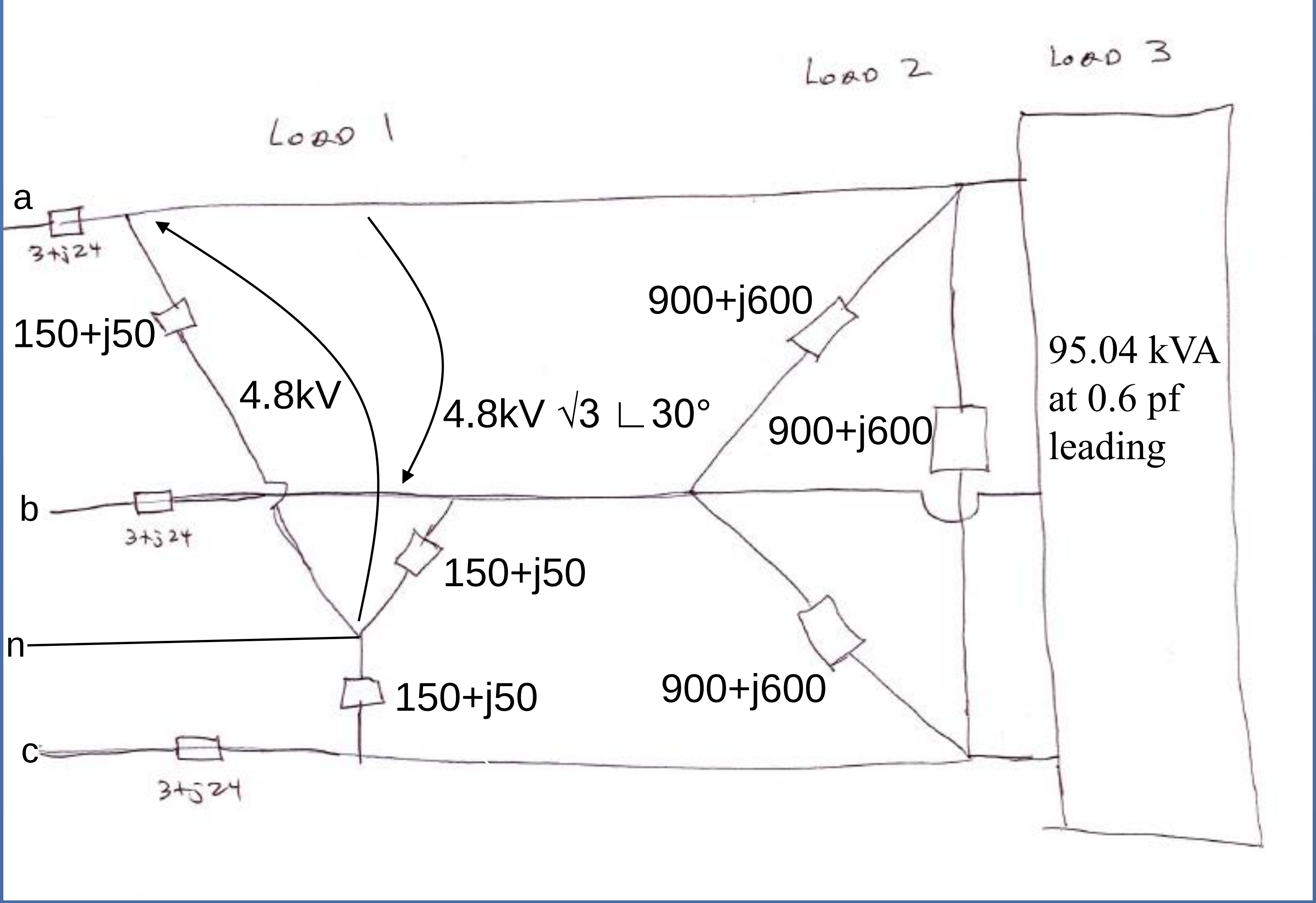
Example B3.2

Three balanced three-phase loads are connected in parallel. Load 1 is Y-connected with an impedance of 150 + j50 ; load 2 is delta-connected with an impedance of 900 + j600 ; and load 3 is 95.04 kVA at 0.6 pf leading. The loads are fed from a distribution line with an impedance of 3 + j24 . The magnitude of the line-to-neutral voltage at the load end of the line is 4.8 kV.

a) Calculate the total complex power at the sending end of the line.

b) What percent of the average power at the sending end of the line is delivered to the load?

Example B3.2


Load 1
Load 2
Load 3
a
b
n
c
3+j24
3+j24
3+j24
150+j50
150+j50
150+j50
4.8kV
4.8kV √3 ∟30°
900+j600
900+j600
900+j600
95.04 kVA
at 0.6 pf
leading

Solution:
First thing to do is to obtain a per-phase circuit.

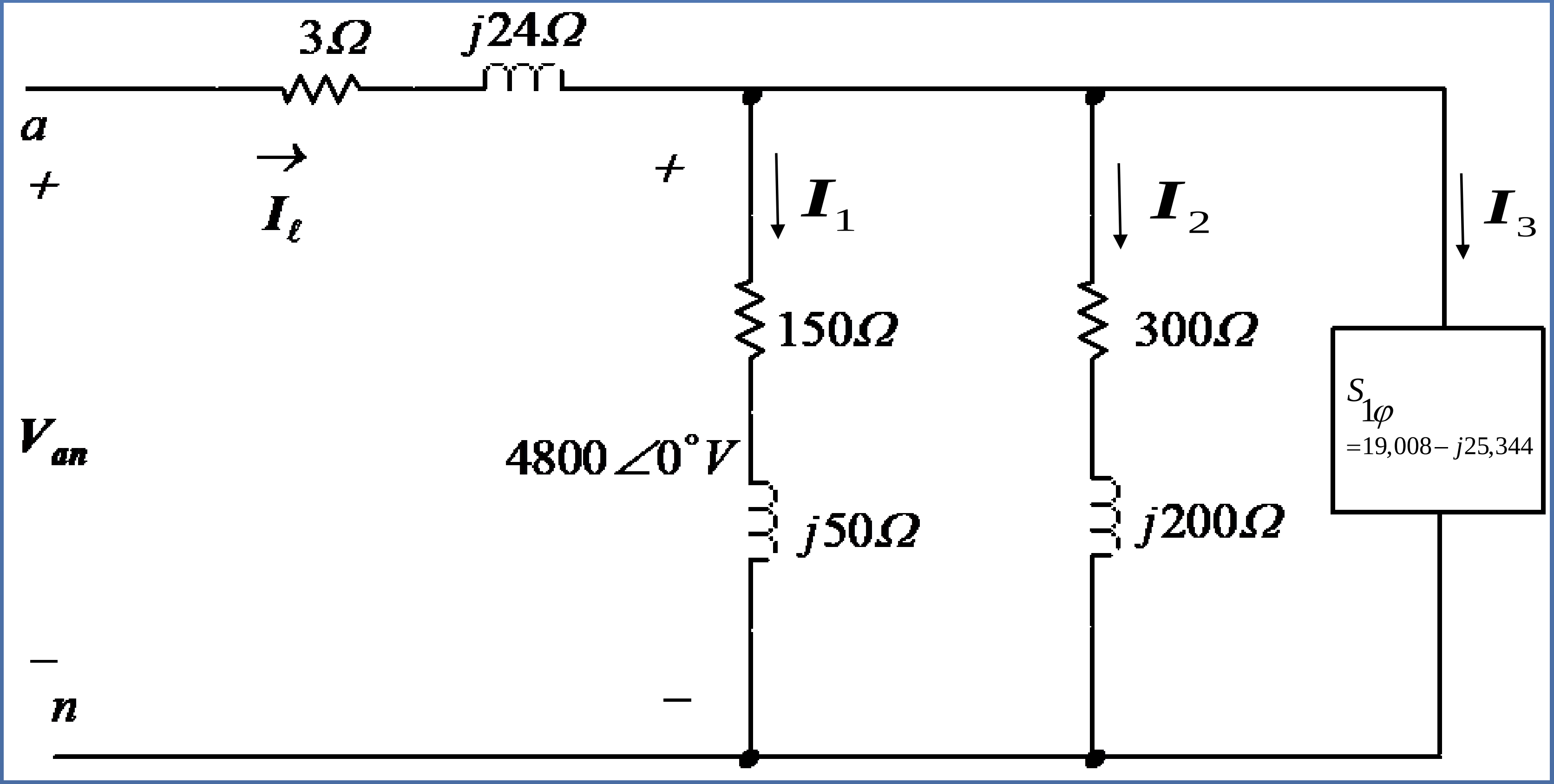
Load 1: Z1=150+j50
Load 2: Z2=(900+j600)/3=300+j200
Load 3:

$$S_{1\varphi} = \frac{95040}{3}(0.6 - j0.8)$$
$$= 19{,}008 - j25{,}344$$

And we need a voltage.
What voltage do we take from the three-phase circuit to use in the per-phase circuit?

$$\mathbf{V}_{an,load} = 4800 \angle 0°$$

Per phase equivalent circuit



How do you compute the four currents?

Compute current from source

$$\mathbf{I}_\ell = \frac{4800}{150 + j50} + \frac{4800}{300 + j200} + \frac{19{,}008 + j25{,}344}{4800}$$

$$= 28.8 - j9.6 + 11.0769 - j7.3846 + 3.96 + j5.28$$

$$= \quad I_1 \quad + \quad \mathrm{I}_2 \quad + \quad \mathrm{I}_3$$

$$= 43.8369 - j11.7046\,A(rms) = 45.3725\angle -14.949^\circ\,A(rms)$$

Compute losses in the transmission line

$$P_{loss} = 3\left|I_{eff}\right|^2 R = 3(45.3725)^2(3) = 18{,}528.04 \text{ W}$$

$$Q_{loss} = 3\left|I_{eff}\right|^2 X = 3(45.3725)^2(24) = 148{,}224.34 \text{ VAR}$$

Compute power consumed by load 1.

$$P_1 = 3\left|28.8 - j9.6\right|^2(150) = 414{,}720 \text{ W}$$

$$Q_1 = 3\left|28.8 - j9.6\right|^2(50) = 138{,}240 \text{ VAR}$$

Compute power consumed by load 2:

$$P_2 = 3\left|11.0769 - j7.3846\right|^2 (300) = 159{,}507.02 \text{ W}$$

$$Q_2 = 3\left|11.0769 - j7.3846\right|^2 (200) = 106{,}338.02 \text{ VAR}$$

Compute power consumed by load 3:

$$P_3 = 95{,}040(0.6) = 57{,}024 \text{ W}$$

$$Q_3 = -95{,}040(0.8) = -76{,}032 \text{ VAR}$$

Add the powers to the three loads

$$S_{totalload,3\phi} = 631{,}251 + j168{,}546 \ \text{ VA (load end)}$$

We could have also obtained this from 3VI* (see the “check” in the text)

But which “V” to use?

Add in the losses to get sending end power:

$$S_{sending,3\phi} = 631{,}251 + j168{,}546 + 18{,}528.04 + j148{,}224.34 \ \text{ VA}$$

$$= 649779.04 + j316{,}770.34 \ \text{ VA}$$

Part b of the problem wants the percent of the power from the sending end that is actually delivered at the load.

$$\%P \text{ delivered} = \frac{631{,}251}{649{,}779.04} \times 100 = 97.148$$

This is a measure of efficiency.
Why is 100% of the power not delivered?

Wye-connections

- Provides for two different voltages; loads may use the line-to-neutral or line-to-line (e.g., 277/480 for industrial loads);
- Requires less number of turns in a transformer or generator for the same line-to-line voltage.

Delta-connections

- Transformers may still operate if one phase is lost (called an “open delta” connection)
- Lower currents in windings and so winding ampacity may be lower.
- Third harmonics caused by nonlinear magnetizing current in transformer circulates among windings but is not passed through.