Synchronous Machine Modeling

1.0 Introduction

Our motivation at this point is to put you in a position to understand synchronous machine modeling for power system dynamic analysis.

If you take EE 457, you will be introduced to a conceptual treatment of power system dynamics using what is called the equal area criterion. This treatment is useful for thinking about how power systems respond during the first few seconds following a disturbance, and for understanding the main influencing factors behind power system dynamics. However, this treatment is not very useful for understanding the computer models that are used in time-domain simulation programs such as those offered by Siemens (PSS/E), GE, Powertech, and RTE-France (Eurostag).

Chapter 7 of your text does a reasonable job of introducing you to synchronous machine models. But I hope to improve on this with these notes. Please read both.

One last comment before we proceed. Working hard to understand this material will put you in a very good position to take EE 554 and do well in that course. EE 554 dedicates a full semester to study of synchronous machine models and other computer methods for simulation of power system dynamics. It is a good course, and I strongly recommend it.

What we are after here is a characterization of the machine dynamics. Because a synchronous machine is comprised of both electrical and mechanical dynamics, our interest can be expressed as "electro-mechanical dynamics." We will focus mainly on the electrical dynamics, leaving the mechanical dynamics for another course.

It is of interest that the underlying theory to our work was developed in 1929 by an employee of GE named Robert Park. Below is a snapshot of the first page of the papers that was published in relation to this work.

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Two-Reaction Theory of Synchronous Machines

Generalized Method of Analysis—Part I

BY R. H. PARK*

Associate, A. I. E. E.

Synopsis.—Starting with the basic assumption of no saturation or hysteresis, and with distribution of armature phase m. m. f. effectively sinusoidal as far as regards phenomena dependent upon rotor position, general formulas are developed for current, voltage, power, and torque under steady and transient load conditions. Special detailed formulas are also developed which permit the determination of current and torque on three-phase short circuit, during starting, and when only small deviations from an average operating angle are involved.

In addition, new and more accurate equivalent circuits are developed for synchronous and asynchronous machines operating in parallel, and the domain of validity of such circuits is established.

Throughout, the treatment has been generalized to include salient poles and an arbitrary number of rotor circuits. The analysis is thus adapted to machines equipped with field pole collars, or with amortisseur windings of any arbitrary construction.

It is proposed to continue the analysis in a subsequent paper.

* * * * *

THIS paper presents a generalization and extension of the work of Blondel, Dreyfus, and Doherty and Nickle, and establishes new and general methods of calculating current power and torque in salient† and non-salient pole synchronous machines, under both transient and steady load conditions.

Attention is restricted to symmetrical three-phase† machines with field structure symmetrical about the axes of the field winding and interpolar space, but salient poles and an arbitrary number of rotor‡ circuits is considered.

Idealization is resorted to, to the extent that saturation and hysteresis in every magnetic circuit and eddy

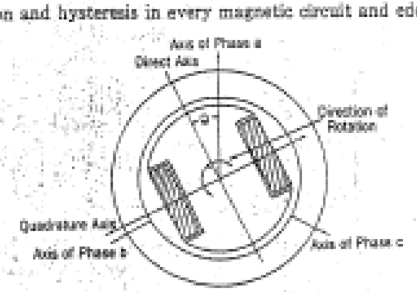

Fig. 1

currents in the armature iron are neglected, and in the assumption that, as far as concerns effects depending on the position of the rotor, each armature winding may be regarded as, in effect, sinusoidally distributed.[1]

A. Fundamental Circuit Equations

Consider the ideal synchronous machine of Fig. 1, and let

i_a, i_b, i_c = per unit instantaneous phase currents

e_a, e_b, e_c = per unit instantaneous phase voltages

ψ_a, ψ_b, ψ_c = per unit instantaneous phase linkages

t = time in electrical radians

$$p = \frac{d}{dt}$$

Then there is

$$\begin{aligned} e_a &= p\,\psi_a - r\,i_a \\ e_b &= p\,\psi_b - r\,i_b \\ e_c &= p\,\psi_c - r\,i_c \end{aligned} \qquad (1)$$

It has been shown previously[2] that

$$\begin{aligned} \psi_a = {} & I_d \cos\theta - I_q \sin\theta \\ & - \frac{x_0}{3}\,[i_a + i_b + i_c] - \frac{x_d + x_q}{3}\left[i_a - \frac{i_b + i_c}{2}\right] \\ & - \frac{x_d - x_q}{3}\,[i_a \cos 2\theta + i_b \cos(2\theta - 120) \\ & \qquad + i_c \cos(2\theta + 120)] \\ \psi_b = {} & I_d \cos(\theta - 120) - I_q \sin(\theta - 120) \\ & - x_0 \frac{i_a + i_b + i_c}{3} - \frac{x_d + x_q}{3}\left[i_b - \frac{i_a + i_c}{2}\right] \\ & - \frac{x_d - x_q}{3}\,[i_a \cos(2\theta - 120) + i_b \cos(2\theta + 120) \\ & \qquad + i_c \cos 2\theta] \\ \psi_c = {} & I_d \cos(\theta + 120) \\ & - I_q \sin(\theta + 120) - x_0 \frac{i_a + i_b + i_c}{3} \\ & - \frac{x_d + x_q}{3}\left[i_c - \frac{i_a + i_b}{2}\right] \\ & - \frac{x_d - x_q}{3}\,[i_a \cos(2\theta + 120) + i_b \cos 2\theta \\ & \qquad + i_c \cos(2\theta - 120)] \end{aligned} \qquad (2)$$

where,

*General Engg. Dept., General Electric Company, Schenectady, N. Y.

†Single-phase machines may be regarded as three-phase machines with one phase open circuited.

‡Stator for a machine with stationary field structure.

[1]For numbered references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

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2.0 Preliminaries

2.1 Assumed machine construction

We will conduct our modeling exercise for a two-pole salient machine. Results will be generalizable to a smooth rotor machine because such machines can be well approximated using a salient pole model together with proper designation of the machine parameters. Results will be generalizable to machines with p>2 because such machines will have the exact same phenomena, except p/2 times/cycle.

2.2 Defined axes

The magnetic circuit and all rotor winding circuits are symmetrical with respect to the polar and inter-polar (between-poles) axes. This proves convenient, so we give these axes a special name:

- Polar axis: Direct, or D-axis
- Interpolar axis: Quadrature, or Q-axis

These axes are shown in Fig. 1.

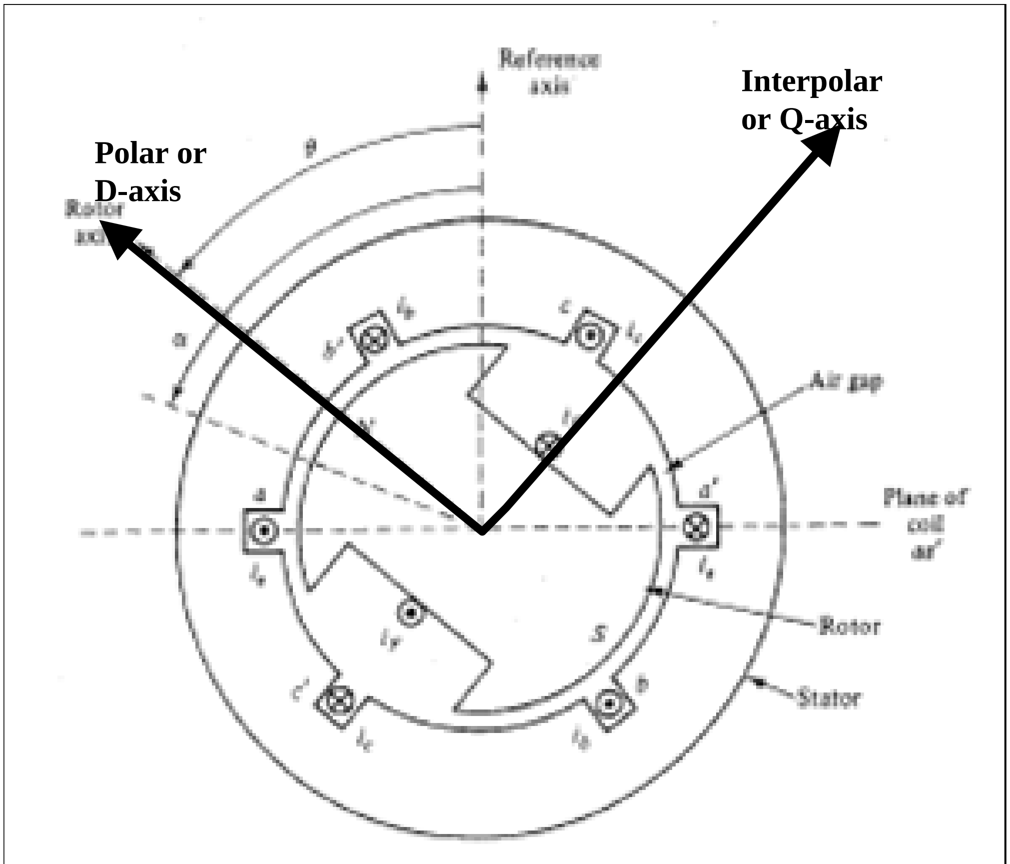

Figure 6.1 Generator cross section.

Fig. 1

The Q-axis is 90° from the D-axis, but which way is the choice of the modeler. But once that choice is made, you must stick with it. We will choose the Q-axis to lag the D-axis.

2.3 Machine windings

There are 5 physical windings on a synchronous generator.

- The 3 stator (phase) windings, denoted a, b, c.
- The main field winding, denoted F.
- So-called "amortessuer" (dead) windings exist on the pole-faces of salient pole machines, which produce damping currents that contribute to the magnetic field. We denote these windings with Q, since they will produce flux along the quadrature axis. Fig. 2 illustrates armortessuer windings.

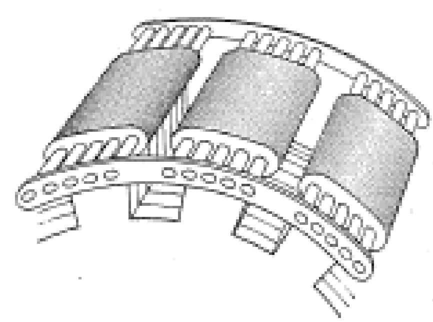

Figure 5.3.13 A sketch of damper bars located on the salient-pole shoes of a synchronous machine.

Fig 2

Although the above covers all of the physical windings, it does provide for modeling a component of flux along the D-axis that is observed. This component of flux is similar to the component of Q-axis flux from the amortessuer windings in that it is observed only after disturbances. In fact, it comes from damping currents in the iron of the rotor.

So we MODEL a fictitious winding placed exactly like the main field winding. We denote this winding with D, since it produces flux along the direct axis.

Fig. 3 illustrates these windings.

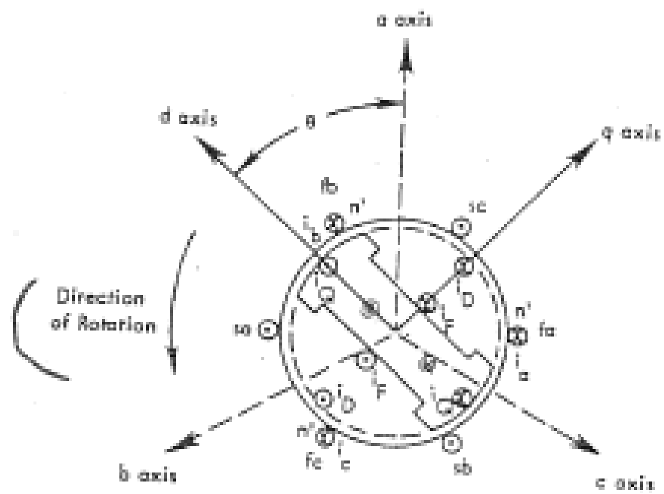

Fig. 3

Each one of these windings will

- have a voltage across their terminals
- carry a current
- have a resistance
- see a flux linkage

Table 1 summarizes the notation to be used.

Table 1

Winding	Voltage	Current	resistance	flux
a	v_a	i_a	r	λ_a
b	v_b	i_b	r	λ_b
c	v_c	i_c	r	λ_c
F	v_F	i_F	r_F	λ_F
D	v_D	i_D	r_D	λ_D
Q	v_Q	i_Q	r_Q	λ_Q

All voltages, currents, and flux linkages in Table 1 are instantaneous time-domain expressions. For example, v_a is really $v_a(t)$.

In addition, the voltage notation, relative to Fig. 2, are $v_a(t)=v_{aa'}$, $v_b(t)=v_{bb'}$, $v_c(t)=v_{cc'}$, $v_F=v_{FF'}$, $v_D=v_{DD'}$, $v_Q=v_{QQ'}$.

3.0 Voltage (electrical) equation

One important attribute of our work now is that we are not considering an open circuit. As a result, we must use v_a to account for the drop across the resistance due to the current, instead of just the open circuit voltage.

The circuit associated with each circuit, assuming voltages are applied (not generated) and currents flow into the circuit, are illustrated in Fig. 3. We can write a KVL equation for each circuit, resulting in:

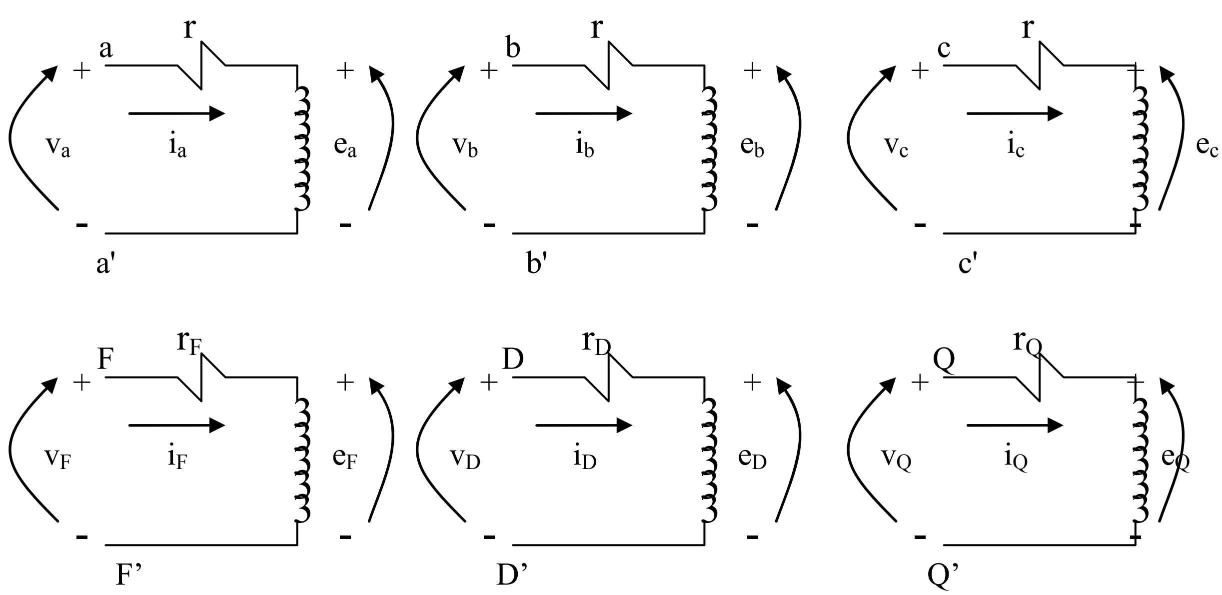

Fig. 3

With an applied voltage, the internal (induced) voltage will oppose it, as shown in each circuit. Using KVL, we may write:

$$\boxed{\begin{aligned} v_a &= i_a r + e_a \\ v_b &= i_b r + e_b \\ v_c &= i_c r + e_c \end{aligned}} \qquad \boxed{\begin{aligned} v_F &= i_F r_F + e_F \\ v_D &= i_D r_D + e_D \\ v_Q &= i_Q r_Q + e_Q \end{aligned}}$$

Each of the induced voltages is a result of the time derivatives of the flux linking the corresponding circuits. Modifying the above equations accordingly, we obtain

$$\boxed{\begin{aligned} v_a &= i_a r + \frac{d\lambda_{aa'}}{dt} \\ v_b &= i_b r + \frac{d\lambda_{bb'}}{dt} \\ v_c &= i_c r + \frac{d\lambda_{cc'}}{dt} \end{aligned}} \qquad \boxed{\begin{aligned} v_F &= i_F r_F + \frac{d\lambda_{FF'}}{dt} \\ v_D &= i_D r_D + \frac{d\lambda_{DD'}}{dt} \\ v_Q &= i_Q r_Q + \frac{d\lambda_{QQ'}}{dt} \end{aligned}}$$

In the above equations, each flux linkage is in a direction consistent with a current in opposite direction to the defined current. For example, the flux linkage $\lambda_{aa'}$ is in a downward direction, consistent with a current in a defined positive direction from a to a', as shown in Fig. 4.

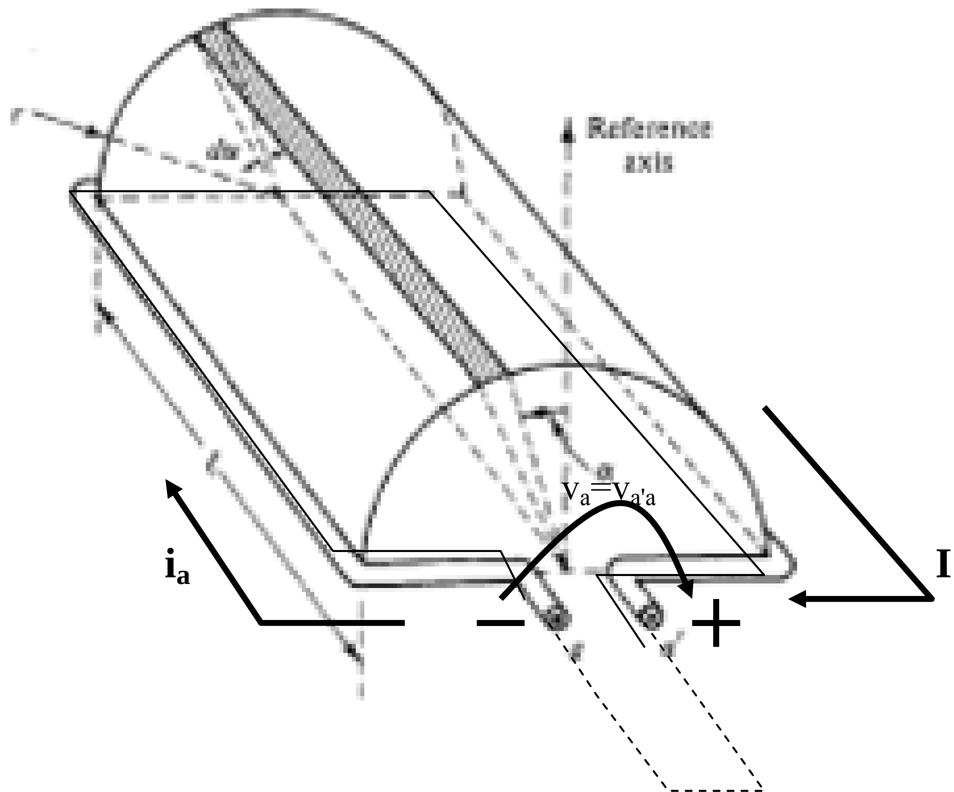

Fig. 4

This convention is fine for the F, D, and Q circuits, because they either have voltage applied as indicated (in the case of F) or they are short circuited and have zero voltage (in the case of D and Q).

This convention is not fine for the phase windings, since the current is actually in the opposite direction, as shown in Fig. 5.

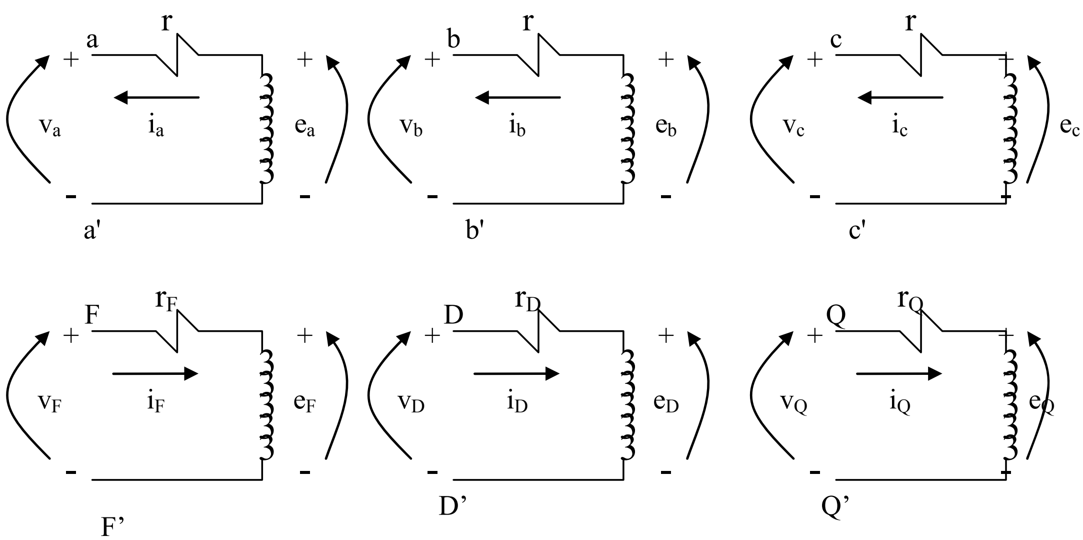

Fig. 5

With the current directions reversed for the phase windings, we need to change the signs of terms that depend on these currents. At the same time, we will multiply the F, D, and Q equations through by -1 to make all right-hand-sides look the same.

$$v_a = -i_a r - \frac{d\lambda_{aa'}}{dt}$$
$$v_b = -i_b r - \frac{d\lambda_{bb'}}{dt}$$
$$v_c = -i_c r - \frac{d\lambda_{cc'}}{dt}$$

$$-v_F = -i_F r_F - \frac{d\lambda_{FF'}}{dt}$$
$$-v_D = -i_D r_D - \frac{d\lambda_{DD'}}{dt}$$
$$-v_Q = -i_Q r_Q - \frac{d\lambda_{QQ'}}{dt}$$

The D and Q windings have no voltage source but carry current ONLY when there is flux linkage variation. So their left-hand-sides must be zero. This results in:

$$v_a = -i_a r - \frac{d\lambda_{aa'}}{dt}$$
$$v_b = -i_b r - \frac{d\lambda_{bb'}}{dt}$$
$$v_c = -i_c r - \frac{d\lambda_{cc'}}{dt}$$

$$-v_F = -i_F r_F - \frac{d\lambda_{FF'}}{dt}$$
$$0 = -i_D r_D - \frac{d\lambda_{DD'}}{dt}$$
$$0 = -i_Q r_Q - \frac{d\lambda_{QQ'}}{dt}$$

We will change the flux-linkage notation to use only a single subscript:

$$v_a = -i_a r - \frac{d\lambda_a}{dt}$$
$$v_b = -i_b r - \frac{d\lambda_b}{dt}$$
$$v_c = -i_c r - \frac{d\lambda_c}{dt}$$

$$-v_F = -i_F r_F - \frac{d\lambda_F}{dt}$$
$$0 = -i_D r_D - \frac{d\lambda_D}{dt}$$
$$0 = -i_Q r_Q - \frac{d\lambda_Q}{dt}$$

Finally, we replace the differentiation notation from fractional form to dot-form:

$$v_a = -i_a r - \dot{\lambda}_a$$
$$v_b = -i_b r - \dot{\lambda}_b$$
$$v_c = -i_c r - \dot{\lambda}_c$$

$$-v_F = -i_F r_F - \dot{\lambda}_F$$
$$0 = -i_D r_D - \dot{\lambda}_D$$
$$0 = -i_Q r_Q - \dot{\lambda}_Q$$

Putting all these equations in a single vector relation, we have:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ -v_F \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & r & 0 & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} - \begin{bmatrix} \dot{\lambda}_a \\ \dot{\lambda}_b \\ \dot{\lambda}_c \\ \dot{\lambda}_F \\ \dot{\lambda}_D \\ \dot{\lambda}_Q \end{bmatrix} \qquad (1)$$

In compact form, (1) becomes:

$$\underline{v} = -\underline{R}\underline{i} - \dot{\underline{\lambda}} \qquad (2)$$

where

$$\underline{v} = \begin{bmatrix} v_a \\ v_b \\ v_c \\ -v_F \\ 0 \\ 0 \end{bmatrix} \qquad \underline{R} = \begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & r & 0 & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix}$$

$$\underline{i} = \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} \qquad \underline{\lambda} = \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix}$$

4.0 Flux-linkages

A beginning point for understanding how to express flux linkages will observe 3 facts:

Fact 1: $\lambda = Li$

Fact 2: Each circuit will see a flux contribution from every current. Therefore, the flux linking a circuit will need to be computed from a summation of applications of Fact 1.

Fact 3: The flux contribution seen by a circuit from its own current is related to that current through the mutual inductance.

Therefore, if we have 6 circuits and number them 1-6, the flux linkage seen by any given circuit *i* will be

$$\lambda_i = \sum_{j=1}^{6} L_{ij} i_j \tag{3}$$

The notation of (3) has

- L_{ii} is the self-inductance for circuit i.

- L_{ij} is the mutual inductance between circuit i and j.

However, we are using letters to denote each circuit, i.e., a, b, c, F, D, and Q.
Therefore, for example, the flux linking the field winding would be

$$\lambda_F = L_{af} i_a + L_{bF} i_b + L_{cF} i_c + L_{FF} i_F + L_{FD} i_D + L_{FQ} i_Q \qquad (4)$$

The self inductance L_{FF} may be written as just L_F, resulting in

$$\lambda_F = L_{af} i_a + L_{bF} i_b + L_{cF} i_c + L_F i_F + L_{FD} i_D + L_{FQ} i_Q \qquad (5)$$

We can express the flux linkages for the other windings in a similar way. Gathering these six equations into a single matrix relation results in

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \hline \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \left[\begin{array}{ccc:ccc} L_a & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} \\ L_{ab} & L_b & L_{bc} & L_{bF} & L_{bD} & L_{bQ} \\ L_{ac} & L_{bc} & L_c & L_{cF} & L_{cD} & L_{cQ} \\ \hdashline L_{aF} & L_{bF} & L_{cF} & L_F & L_{FD} & L_{FQ} \\ L_{aD} & L_{bD} & L_{cD} & L_{FD} & L_D & L_{DQ} \\ L_{aQ} & L_{bQ} & L_{cQ} & L_{FQ} & L_{DQ} & L_Q \end{array}\right] \begin{bmatrix} i_a \\ i_b \\ i_c \\ \hline i_F \\ i_D \\ i_Q \end{bmatrix} \qquad (6)$$

Or

$$\begin{bmatrix} \underline{\lambda}_{abc} \\ \underline{\lambda}_{FDQ} \end{bmatrix} = \begin{bmatrix} \underline{L}_{11} & \underline{L}_{12} \\ \underline{L}_{21} & \underline{L}_{22} \end{bmatrix} \begin{bmatrix} \underline{i}_{abc} \\ \underline{i}_{FDQ} \end{bmatrix} \tag{7}$$

where

$$\begin{bmatrix} \underline{\lambda}_{abc} \\ \underline{\lambda}_{FDQ} \end{bmatrix} = \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} \qquad \begin{bmatrix} \underline{L}_{11} & \underline{L}_{12} \\ \underline{L}_{21} & \underline{L}_{22} \end{bmatrix} = \begin{bmatrix} L_a & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} \\ L_{ab} & L_b & L_{bc} & L_{bF} & L_{bD} & L_{bQ} \\ L_{ac} & L_{bc} & L_c & L_{cF} & L_{cD} & L_{cQ} \\ L_{aF} & L_{bF} & L_{cF} & L_F & L_{FD} & L_{FQ} \\ L_{aD} & L_{bD} & L_{cD} & L_{FD} & L_D & L_{DQ} \\ L_{aQ} & L_{bQ} & L_{cQ} & L_{FQ} & L_{DQ} & L_Q \end{bmatrix}$$

$$\begin{bmatrix} \underline{i}_{abc} \\ \underline{i}_{FDQ} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

One notices the partitioning of the matrix in (6) into 4 different blocks. We identify these blocks by nomenclature and by whether they capture inductances between windings on the stator or on the rotor or between them:

- $\underline{L}_{11}$ (stator-stator inductances): These give the self inductances of each phase winding and the mutual inductances between each pair of phase windings.

- $\underline{L}_{12}$ (stator-rotor inductances): These give the mutual inductances between each stator winding and each winding on the rotor.
- $\underline{L}_{21}$ (rotor-stator inductances): These give the mutual inductances between each winding on the rotor and each stator winding. Note that $\underline{L}_{12}=[\underline{L}_{21}]^T$.
- $\underline{L}_{22}$ (rotor-rotor inductances): These give the self inductances of each rotor winding and the mutual inductances between each pair of rotor windings.

5.0 Inductances

The different inductances of (6)

6.0 lj