Mostly-Sleeping Wireless Sensor Networks: Connectivity, k-Coverage, and α-Lifetime

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Abstract – In this paper, we explore the fundamental limits of a wireless sensor network's lifetime under the guarantee of both connectivity and k-coverage. We consider a wireless sensor network with n sensors deployed independently and uniformly in a square region of unit area. Each sensor is active with probability p, independently from the others, and can sense a disk of radius r_s when active. Two active sensors can communicate with each other if and only if the distance between them is less than communication radius r_c. However, due to the variation of the environment and sensors' characteristics, we model the sensing radius r_s as a random variable with mean r_0 and variance $r_0^2\sigma_s^2$. We first derive the sufficient and necessary condition on the sensing radius in order to maintain the k-coverage with probability one as the number of sensors goes to infinity. Then, we introduce a new definition of network's lifetime, namely α-lifetime, from a probabilistic perspective, which is the expectation of the entire interval during which the probability of guaranteeing connectivity and k-coverage simultaneously is at least α. Finally, we propose a near-optimal scheduling algorithm to maximize the network's α-lifetime, which is verified by simulation results.

I. Introduction

Recently, there has been an increase of interests in large-scale wireless sensor networks [1], [2]. Each sensor in such a network is battery-powered and has a very limited amount of energy. It is, therefore, critical to extend the battery operation time of individual sensors and, consequently, the network's lifetime. Operating each sensor in a low duty-cycle is naturally an attractive idea to achieve this goal. Here, *duty-cycle* is defined as the fraction of time that a sensor device is active. On the other hand, a wireless sensor network typically has two major tasks: *sensing* and *communication*. It is always desirable to have all the active sensors connected and, at the same time, to have the entire sensing region k-covered. The connectivity of all active sensors is necessary if any active sensor wants to deliver its sensing results back to the user. The reason for requiring k-coverage rather than just 1-coverage is to increase the detection probability and accuracy of tracking.

Obviously, the lower the duty-cycles of individual sensors, the longer the wireless sensor network's lifetime, but at the same time, the smaller the number of active sensors at a given time and, hence, more likely either the active sensors are not connected or the k-coverage of the sensing region cannot be guaranteed. So, there are inherent tradeoffs, and the key contribution of this paper is to present an integrated study on connectivity, k-coverage and lifetime of a large-scale wireless sensor network.

Several researchers [3]–[7] have addressed the coverage and connectivity issues in wireless sensor/ad hoc networks. Gupta et al. [3] studied scaling laws for asymptotic connectivity of sensors placed at random over a unit area, and provided bounds on connectivity probability for finite-size networks. In [7], authors studied the relation between k-coverage and k-connectivity when the communication radius is at least twice of the sensing radius, where the sensing radius is deterministic. However, no statistical properties of either k-coverage or k-connectivity were given. In [4] and [5], the asymptotic coverage problem was addressed for mostly-sleeping (unreliable) wireless sensor networks, where 1-coverage was studied in [4] and k-coverage in [5]. In [6], authors presented the sufficient and necessary condition for asymptotic k-coverage. However, none of above work considered the possible heterogeneity of sensing radius due to the variation of the environment and sensors' characteristics.

Recently, research efforts [6], [8] have also been made to analyze the lifetime of wireless sensor/ad hoc networks with coverage requirements. The definitions of network's lifetime

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in these literature are different from ours. In [8], the lifetime was defined as the time it takes for the coverage — defined as the ratio of the area covered by working sensors to the total area — to drop below a pre-defined threshold. In [6], the α-lifetime of a wireless sensor network was defined as the interval during which at least α portion of the sensing region is covered by at least one sensor node. Both [8] and [6] only studied the relation between network's lifetime and the coverage of the sensing region. However, the connectivity is another prerequisite element for the network to function properly. Another issue in optimizing the network's lifetime is how to take connectivity and coverage into consideration. Both of the above definitions of network's lifetime are from the deterministic point of view. Since the deployment and dynamics of wireless sensor networks are random, the coverage of the sensing region and the connectivity of the whole network are also random variables. Therefore, it is more reasonable to study the network's lifetime from a probabilistic perspective.

The focus of this paper is to explore the fundamental limits of a wireless sensor network's lifetime under the guarantee of both connectivity and k-coverage. First, asymptotic results for k-coverage of the sensing region are presented. Under randomized independent sleeping and random sensing radius model, we derive the sufficient and necessary condition on the sensing radius in order to maintain the k-coverage with probability one as the number of sensors goes to infinity. Then, we introduce a new definition of network's lifetime, namely α-lifetime, which is the entire interval during which the probability of guaranteeing connectivity and k-coverage simultaneously is at least α. By solving two convex optimization problems, we obtain a lower bound and an upper bound for the network's maximum α-lifetime. Furthermore, based on the lower bound, we propose a scheduling algorithm as a near-optimal solution to maximize the network's α-lifetime.

The rest of this paper is organized as follows. Section II describes our network model and gives the problem formulation. In Section III, we derive the sufficient and necessary condition for maintaining k-coverage with probability one as the number of sensors goes to infinity. A scheduling algorithm is proposed in Section IV to maximize the α-lifetime for finite-size wireless sensor networks. Section V presents and evaluates the simulation results and, finally, the paper concludes in Section VI.

II. Network model and problem formulation

A. Network Model

We consider a square region $\mathcal{D}$ of unit area where n sensors are deployed independently and uniformly. In order to extend the network's lifetime, an appropriate duty cycle and a sleeping schedule are required. Consider the following sleeping scheme, which we call modified Randomized Independent Sleeping (mRIS): *time is divided into rounds, and at the beginning of a round, each alive sensor decides to be active with probability* p*, independently from the others, or inactive (sleeping) with probability (*$1-p$*).* Here, alive sensors refer to the sensors with enough energy to operate. Note that the active probability (p) vary with the round and may be determined by the specific performance criteria. The mRIS scheme is based on the Random Independent Sleeping (RIS) scheme proposed in [5] and the details of mRIS will be discussed in Section IV.

Two active sensors can communicate directly with each other if and only if the distance between them is no more than r_c. The radius r_c is usually referred to as the *communication radius*. For the purpose of simplicity, we assume that all active sensors have the same and deterministic communication radii. The network is said to be connected if the underlying graph composed of active sensors is connected. A random disc-based sensing model is employed, where each active sensor has a *sensing radius* of r_s, and any object within a disc of radius r_s centered at an active sensor can be reliably detected by the sensor. Due to the variation of the environment and sensors' characteristics, the sensing radius r_s's are assumed to be independently identically distributed (i.i.d) with mean r_0 and variance $r_0^2\sigma_s^2$. The underlying distribution is assumed unknown. A point in the region $\mathcal{D}$ is said to be k-*covered* if it is within the sensing radius of at least k active sensors. The region $\mathcal{D}$ is said to be k-*covered* if every point in $\mathcal{D}$ is k-covered. We assume torus convention (also known as the toroidal model) [9], i.e., each disc (communication or sensing) that protrudes one side of the region $\mathcal{D}$ enters $\mathcal{D}$ again from the opposite side. This eliminates the edge effects and simplifies the problem.

Due to the randomness in sensor deployment and sleeping schedule, it is impossible to guarantee the connectivity and k-coverage with probability one using finite number of sensors, unless the communication disc and sensing disc of each active sensor can cover the entire region. However, the physical limitations prohibit such large communication radius and sensing radius. In other words, there is no deterministic guarantee of connectivity or k-coverage for random networks in practice. Such facts motivate us to define the lifetime of random networks from a probabilistic perspective. We define the *α-lifetime* of a wireless sensor network as the expectation of the entire interval during which the probability of guaranteeing k-coverage of region $\mathcal{D}$ and the connectivity of the network simultaneously is at least α, where $0 < \alpha < 1$. For example, suppose that the mRIS scheduling scheme is employed, the α-lifetime of a random network is said to be $T_\alpha = \mathrm{E}\left[\sum_{i=1}^{M} T_i\right]$, where T_i is the duration of the i-th round, and M is the maximum number of rounds during which the network can function properly. In other words, for any round i ($i \leqslant M$), the probability of guaranteeing both connectivity and k-coverage simultaneously, defined as P_{op}, is at least α, but for round $M+1$, P_{op} is smaller than α.

B. Problem Formulation

The problems we study in this paper are the following:

1) What relation among n, p, r_0 and σ_s^2 would be the sufficient and necessary condition to guarantee that the probability of the entire region $\mathcal{D}$ being k-covered approaches 1 as n goes to infinity?
2) For a finite-size wireless sensor network, how to find the optimal parameters for the mRIS scheme to maximize the α-lifetime of the network?

The first problem above is referred to as critical conditions for asymptotic k-coverage. Although the answer to this problem can not be directly applied to practical wireless sensor networks, such conditions may give us insights on designing large-scale wireless sensor networks. As a comparison, the second problem is a more realistic and the resulting scheduling scheme may serve as a good guideline in deploying finite-size wireless sensor networks.

III. The Sufficient and necessary condition for asymptotic k-coverage

In this section, we investigate the sufficient and necessary condition for asymptotic k-coverage, i.e., the entire sensing region $\mathcal{D}$ is k-covered with probability one as the number of sensors n goes to infinity.

We assume that n sensors are deployed independently and uniformly within the square region $\mathcal{D}$ of unit area centered at the origin of $\mathbb{R}^2$, and each sensor is active with probability p, independently from the others. It is well-known that n nodes, whose locations are distributed independently and uniformly in the region $\mathcal{D}$, form a stationary Poisson point process with density n if n is large. This result is formally stated in Lemma 1 and its proof is given by Hall in [9].

Lemma 1 *Let n points distributed independently and uniformly in a square region $\mathcal{D}$ of unit area, where $\mathcal{D} \subset \mathbb{R}^2$, then these points form a stationary Poisson process with density n for sufficiently large n.*

Let $\mathcal{P} \equiv \{\xi_i, i \geqslant 1\}$ denote the set of active sensors, then it is shown in Lemma 2 that $\mathcal{P}$ is also a stationary Poisson process with density np for sufficiently large n.

Lemma 2 *Let n points distributed independently and uniformly in a square region $\mathcal{D}$ of unit area, where $\mathcal{D} \subset \mathbb{R}^2$. Each point is marked independently as active point with probability p, where $0 < p < 1$. Let $\mathcal{P} \equiv \{\xi_i, i \geqslant 1\}$ denote the set of active points, then $\mathcal{P}$ is a stationary Poisson process with density np for sufficiently large n.*

The proof details of Lemma 2 are omitted due to space limitation. Interested readers can refer to the full version of this paper [10].

Let S_i denote a random disc with radius $r_{s,i}$ centered at the origin of $\mathbb{R}^2$, which is defined as $S_i \equiv \{x \in \mathbb{R}^2 : |x| \leqslant r_{s,i}\}$, where $r_{s,i}$ is the sensing radius of the i-th active sensor ξ_i. Here, we assume all sensing radii are i.i.d random variables with mean r_0 and variance $r_0^2\sigma_s^2$. Then, the sensing disc (abbreviated as disc) centered at active sensor ξ_i can be defined as:

$$D_i \equiv \xi_i + S_i = \{\xi_i + y : y \in S_i\}. \tag{1}$$

Let $I_k(x)$ denote the indicator function of whether a point x is covered by at most $(k-1)$ active sensors, i.e.,

$$I_k(x) = \begin{cases} 1, & \text{if at most } (k-1) \text{ active sensors cover point } x, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Then, the expectation of Bernoulli random variable $I_k(x)$ is

$$\begin{aligned} \mathrm{E}[I_k(x)] = P(x \text{ is at most } (k-1) \text{ covered}) &= \sum_{j=0}^{k-1} P(|\{i : x \in D_i\}| = j) \\ = \sum_{j=0}^{k-1} P(|\{i : \xi_i \in x - S_i\}| = j) &= \sum_{j=0}^{k-1} P(|\{i : \xi_i \in S_i\}| = j), \end{aligned}$$

where $|A|$ denotes the cardinality of set A. The last equality follows from symmetry and homogeneity of the Poisson process and the assumption of toroidal model in Section II.

Suppose that for some $r_1 > 0$, and with probability 1, $|x| \leqslant r_1$ for each $x \in S_i$. Let the region $\mathcal{A}$ be at least as large as the disc of radius r_1 centered at the origin. If a point ξ_i is placed randomly into $\mathcal{A}$, then the probability that the point lies within S_i equals $\mathrm{E}[\|S_i\|/\|\mathcal{A}\|] = a_s/\|\mathcal{A}\|$, where

$$a_s \equiv \mathrm{E}[\|S_i\|] = \pi r_0^2(1+\sigma_s^2). \tag{3}$$

If N points $\xi_i, \ldots, \xi_N$ are placed independently and uniformly into $\mathcal{A}$, then (conditioned on N) the chance that for some j ($j \leqslant N$) indices i, $\xi_i \in S_i$, and for all other indices i, $\xi_i \notin S_i$, equals $\binom{N}{j}(a_s/\|\mathcal{A}\|)^j(1-a_s/\|\mathcal{A}\|)^{N-j}$. If $\xi_1, \ldots, \xi_N$ are from a stationary Poisson process with density np, then

$$P(|\{i : \xi_i \in S_i\}| = j) = \mathrm{E}\left[\binom{N}{j}(a_s/\|\mathcal{A}\|)^j(1-a_s/\|\mathcal{A}\|)^{N-j}\right] = e^{-npa_s}\frac{(npa_s)^j}{j!}, \tag{4}$$

where the expectation is with respect to the Poisson distributed random variable N with mean np. Since the active sensors $\{\xi_i, i \geqslant 1\}$ form a stationary Poisson process with density np, we have

$$\mathrm{E}[I_k(x)] = e^{-npa_s}\sum_{j=0}^{k-1}\frac{(npa_s)^j}{j!}. \tag{5}$$

Let the k-vacancy V_k denote the area that is covered by at most $(k-1)$ active sensors, then the random variable V_k can be expressed as

$$V_k = \int_{\mathcal{D}} I_k(x)\mathrm{d}x. \tag{6}$$

Using Fubini's theorem [11] and exchanging the order of integral and expectation, we obtain the expected value of the k-vacancy:

$$\mathrm{E}[V_k] = \mathrm{E}\left[\int_{\mathcal{D}} I_k(x)\mathrm{d}x\right] = \int_{\mathcal{D}} \mathrm{E}[I_k(x)]\mathrm{d}x = e^{-npa_s}\sum_{j=0}^{k-1}\frac{(npa_s)^j}{j!}. \tag{7}$$

K-coverage of the sensing region $\mathcal{D}$ means that each point in $\mathcal{D}$ should be covered by at least k active sensors, which implies $V_k = 0$. As sensors are deployed independently and uniformly within $\mathcal{D}$, it cannot guarantee $P(V_k = 0) = 1$ with finite n for $a_s < 1$ regardless of the value of n. However, if $np \to \infty$ as $n \to \infty$, it is possible that $P(V_k = 0) \to 1$ as $n \to \infty$. Before studying the asymptotic behavior of $P(V_k = 0)$, we first give an upper bound and a lower bound of $P(V_k = 0)$ for finite n.

Theorem 1 *For $n > 1$, $0 < p \leqslant 1$, and $a_s < 1$*

$$P_l < P(V_k = 0) < P_u, \tag{8}$$

in which

$$P_u = \frac{4(k+1)!(1+\sigma_s^2)(np)^{-1}(npa_s)^{-k}e^{npa_s}}{1+4(k+1)!(1+\sigma_s^2)(np)^{-1}(npa_s)^{-k}e^{npa_s}}, \tag{9}$$

and

$$P_l = 1 - 2e^{-npa_s}\Big(1 + (n^2p^2a_s' + 2npr)\sum_{i=0}^{k-1}\frac{(npa_s)^i}{i!}\Big), \tag{10}$$

where $a_s' \equiv \pi r_0^2(1+\sigma_s^2/2)$.

The proof of this theorem is similar to the one in [6] and Chapter 3.7 of [9], and the major difference is to take the randomness of r_s into consideration. Again, the proof details are omitted due to space limitation. Interested readers can refer to the full version of this paper [10]. Following the similar procedure in [6], we establish the sufficient and necessary condition on the statistics of the sensing radius for asymptotic k-coverage.

Theorem 2 *Assume $np \to \infty$ as $n \to \infty$ and let*

$$\pi r_0^2(1+\sigma_s^2) = \frac{\log(np) + k\log\log(np) + c_1(np)}{np}, \tag{11}$$

then the entire unit square region $\mathcal{D}$ is k-covered with probability one as $n \to \infty$, if and only if $c_1(np) \to \infty$ as $n \to \infty$.

Proof: As $n \to \infty$, the fact that region $\mathcal{D}$ is k-covered with probability one means $P(V_k = 0) \to 1$. First, we prove if $c_1(np) \to \infty$ as $n \to \infty$, $P(V_k = 0) \to 1$. From Eqs. (8) and (10) in Theorem 1, we have

$$P(V_k = 0) > P_l > 1 - 2e^{-npa_s} - (b_1 + b_2)\cdot(np)(npa_s)^k e^{-npa_s}$$

where $b_1 \equiv 2k\frac{1+\sigma_s^2/2}{1+\sigma^2} > 0$ is independent of n, and $b_2 \equiv \frac{4k}{\pi r(1+\sigma_s^2)np}$. Let $npa_s = \log(np) + k\log\log(np) + c_1(np)$, then $npa_s \to \infty$, $e^{-npa_s} \to 0$, and $b_2 \to 0$, as $n \to \infty$. Since when $c_1(np) \to \infty$,

$$\begin{aligned}\log\Big((b_1+b_2)\cdot(np)(npa_s)^k e^{-npa_s}\Big) =& \log(b_1+b_2) + k\cdot\log\Big(\log(np) + k\log\log(np) + c_1(np)\Big)\\ &+ \log(np) - \log(np) - k\log\log(np) - c_1(np)\\ \to& -\infty,\end{aligned}$$

we have $P(V_k = 0) \to 1$. The first part is proved.

If $c_1(np) \leq C_1$ for some finite C_1 as $n \to \infty$, then for sufficiently large n

$$\begin{aligned}&4(k+1)!(1+\sigma_s^2)(np)^{-1}(npa_s)^{-k}e^{npa_s}\\ =&4(k+1)!(1+\sigma_s^2)e^{npa_s - \log(np) - k\log(npa_s)}\\ =&4(k+1)!(1+\sigma_s^2)e^{c_1(np)} \leqslant 4e^{C_1}(k+1)!(1+\sigma_s^2).\end{aligned}$$

Therefore, by Eqs. (8) and (9) we have

$$P(V_k = 0) < P_u \leqslant \frac{4e^{C_1}(k+1)!(1+\sigma_s^2)}{1+4e^{C_1}(k+1)!(1+\sigma_s^2)} < 1.$$

It means that $P(V_k = 0) \to 1$ only if $c_1(np) \to \infty$ as $n \to \infty$. This completes the proof. □

Remark: Since the bounds obtained in Theorem 1 is valid for finite n, they can be used as performance criteria for deploying finite-size wireless sensor networks.

IV. α-LIFETIME OF FINITE-SIZE WIRELESS SENSOR NETWORKS

The second problem addressed in this paper is how to find optimal parameters for the mRIS scheme to maximize the α-lifetime of a finite-size wireless sensor network. To study the α-lifetime, we first introduce the energy consumption model for each wireless sensor.

We assume that inactive sensors do not consume energy and the communication traffic is evenly distributed across the network. The energy consumption by an active sensor consists of two parts: *communication* and *sensing*. Thus, the power consumption P_0 of each active sensor can be modeled as:

$$P_0 = Q \cdot \frac{1}{r_c} \cdot r_c^{\beta} + \Delta, \tag{12}$$

where

- r_c^{β} is proportional to the energy consumption per bit, and the typical values of β range from 3 to 4 for different propagation models [12];
- $1/r_c$ is proportional to the average traffic rate of active sensors (here, we assume all active sensors have the same traffic rate, i.e., bits per second, following the assumption of evenly distributed traffic.);
- Δ is the power consumption for continuous sensing;
- $Q > 0$ is a constant.

As the communication radius r_c decreases, the average number of hops required for packets transmitted from one point to another increases proportionally. For this reason, we incorporate the factor of $1/r_c$ into the average traffic rate expression. We further assume that all active sensors choose the same communication radius r_c. Hence, all active sensors have the same individual lifetime:

$$T_0(r_c) = \frac{E_0'}{P_0} = \frac{E_0}{r_c^{\beta-1} + \eta}, \tag{13}$$

where E_0' is the initial energy of each active sensor, $E_0 = \frac{E_0'}{Q}$ and $\eta = \frac{\Delta}{Q}$, respectively. This assumption is typical when analyzing the network's lifetime, e.g., in [13] and [6].

Next, we formally define the mRIS scheme which can extend the α-lifetime of wireless sensor networks. Suppose that time is divided into rounds. At the beginning of round i, there are $n^{(i)}$ alive sensors, and each alive sensor decides independently whether to remain sleeping (with probability $1-p^{(i)}$), or become active (with probability $p^{(i)}$). All active sensors choose the same communication radius of $r_c^{(i)}$. Both $p^{(i)}$ and $r_c^{(i)}$ are chosen such that $P_{\text{op}} \geqslant \alpha$. Next, all active sensors operate continuously until batteries die out. Since we assume all active sensors have the same individual lifetime, they will die out at the same time instant, which is defined as the end of this round. The same procedure is repeated for the next rounds until there are not enough alive sensors to satisfy the "$P_{\text{op}} \geqslant \alpha$" requirement, regardless of the choices of p and r_c.

The major differences between the proposed mRIS scheme and the original RIS scheme in [5] are as follows. In mRIS, p and r_c are determined at the beginning of each round to satisfy both connectivity and k-coverage requirements, and they may vary from round to round. Within each round, all active sensors continuously operate until batteries die out. In contrast, the values of p and r_c in RIS are fixed throughout the rounds, where p is chosen to satisfy only the k-coverage requirement, but there is no optimization on r_c. And in average, no sensor's battery dies out before the end of network's lifetime.

In the rest of this section, we will study the α-lifetime for the mRIS scheme and try to find the optimal parameters to maximize the α-lifetime of the network. Suppose that n sensors are deployed independently and uniformly within a unit-area square region $\mathcal{D}$, and the network can operate M rounds following the mRIS scheduling scheme. Then, the α-lifetime of the wireless sensor network is

$$T_\alpha = \mathrm{E}\left[\sum_{i=1}^{M} T_0(r_c^{(i)})\right] = \mathrm{E}\left[\sum_{i=1}^{M} \left(E_0 \Big/ \left(\left(r_c^{(i)}\right)^{\beta-1} + \eta\right)\right)\right], \tag{14}$$

subject to both connectivity and k-coverage requirements, and the expectation is with respect to M. Define $n_{\text{eff}}^{(i)} = n^{(i)}p^{(i)}$, which is the expected number of active sensors in round i. It is easy to verify that the probability mass function (pmf) of M is

$$P(M = m) = \sum_{\substack{n=n^{(1)} \geqslant n^{(2)} \geqslant \cdots \geqslant n^{(m)} \\ n^{(i)} \geqslant n_{\text{eff}}^{(i)},\ i=1,\ldots,m}} \cdots \sum \sum_{n^{(m+1)}=0}^{n_{\text{eff}}^{(m+1)}-1} \prod_{i=1}^{m} \binom{n^{(i)}}{n^{(i+1)}} \left(1 - p^{(i)}\right)^{n^{(i+1)}} \left(p^{(i)}\right)^{n^{(i)}-n^{(i+1)}}, \quad (15)$$

Let A denote the event that the sensing region $\mathcal{D}$ is k-covered, and let B denote the event that the wireless sensor network is connected. The probability of guaranteeing simultaneously k-coverage of region $\mathcal{D}$ and connectivity of the network is $P_{\text{op}} = P(A \cap B)$. Thus, the problem of maximizing the α-lifetime of the network can be expressed as

$$T_{\alpha}^{\max} = \max_{r_c^{(i)}, n_{\text{eff}}^{(i)}} T_{\alpha} = \max_{r_c^{(i)}, n_{\text{eff}}^{(i)}} \mathrm{E}\left[\sum_{i=1}^{M} \left(E_0 \Big/ \left(\left(r_c^{(i)}\right)^{\beta-1} + \eta\right)\right)\right], \quad (16)$$

$$\text{subject to} \qquad P_{\text{op}} = P(A \cap B) \geqslant \alpha \quad \text{for each round.} \quad (17)$$

Using the union bound, P_{op} can be bounded as

$$\min\{P(A), P(B)\} \geqslant P_{\text{op}} \geqslant P(A) + P(B) - 1. \quad (18)$$

Since it is hard to analyze P_{op} directly, we next focus on finding a lower bound and an upper bound of the optimal α-lifetime, $T_{\alpha}^{\max}$.

First, consider the lower bound. Restricting the constraint in Eq. (17) by replacing it with the lower bound in Eq. (18), and assuming all $n_{\text{eff}}^{(i)}$ and $r_c^{(i)}$'s are the same for each round. Then, we obtain a lower bound of $T_{\alpha}^{\max}$ by solving the following optimization problem:

$$\max_{n_{\text{eff}}, r_c, \epsilon} \mathrm{E}[M] \cdot E_0 \big/ \left(r_c^{\beta-1} + \eta\right), \quad (19)$$

$$\text{subject to} \qquad P(A) \geqslant \alpha + \epsilon, \quad P(B) \geqslant 1 - \epsilon, \quad 0 < \epsilon < 1 - \alpha. \quad (20)$$

Using the result $P(A) > P_l$ in Theorem 1, and the following result in [3]:

$$P(B) \approx 1 - P(\text{there exist isolated active sensors}) > 1 - n_{\text{eff}} e^{-n_{\text{eff}} \pi r_c^2}, \quad (21)$$

where the edge effects are avoided by the toroidal model assumption, we can rewrite the constraints in Eq. (20) as:

$$P_l \geqslant \alpha + \epsilon, \quad r_c \geqslant \sqrt{[\log(n_{\text{eff}}/\epsilon)]/(\pi n_{\text{eff}})}, \quad 0 < \epsilon < 1 - \alpha. \quad (22)$$

Notice that the value of α is usually larger than 90% in practice, then the P_l defined in Eq. (10) can be approximated as

$$P_l \approx 1 - g(n_{\text{eff}}) \equiv 1 - 2n_{\text{eff}}^2 a_s' e^{-a_s n_{\text{eff}}} \sum_{i=0}^{k-1} (a_s n_{\text{eff}})^i / i!. \quad (23)$$

Due to the complicated expression of $\mathrm{E}[M]$, it is extremely difficult to optimize Eqs. (19) and (20) directly. Monte Carlo simulation results showed that the pmf of M are mostly concentrated at 3 points: $\left\lfloor \frac{n}{n_{\text{eff}}} \right\rfloor - 1$, $\left\lfloor \frac{n}{n_{\text{eff}}} \right\rfloor$, and $\left\lfloor \frac{n}{n_{\text{eff}}} \right\rfloor + 1$, for n and n_{eff} in the range of our interests, where the floor function $\lfloor x \rfloor$ denotes the largest integer that is equal to or smaller than x. Therefore, we have the lower bound of $\mathrm{E}[M]$ as

$$\mathrm{E}[M] \geqslant \left\lfloor \frac{n - n_{\text{eff}}}{n_{\text{eff}}} \right\rfloor. \quad (24)$$

The rigorous proof of this result is omitted due to space limitation. Interested readers can refer to the full version of this paper [10]. Since $E_0/(r_c^{\beta-1} + \eta)$ is a decreasing function in

r_c, using Eqs. (22), (23) and (24), we obtain a new lower bound of $T_\alpha^{\max}$ as

$$T_\alpha^L = \max_{n_{\text{eff}},\epsilon} T_1(n_{\text{eff}},\epsilon) \equiv \max_{n_{\text{eff}},\epsilon} \left\lfloor \frac{n-n_{\text{eff}}}{n_{\text{eff}}} \right\rfloor \cdot E_0 \Big/ \Big((\frac{1}{\pi n_{\text{eff}}} \log \frac{n_{\text{eff}}}{\epsilon})^{(\beta-1)/2} + \eta \Big), \tag{25}$$

$$\text{subject to} \qquad 0 < \epsilon \leqslant 1 - \alpha - g(n_{\text{eff}}). \tag{26}$$

Then, we temporarily remove the floor function $\lfloor \cdot \rfloor$, and have the following convex optimization problem (given $\beta > 3$)

$$\max_{n_{\text{eff}}} E_0 \cdot (n - n_{\text{eff}}) \Big/ \Big(n_{\text{eff}} \big(\frac{1}{\pi n_{\text{eff}}} \log \frac{n_{\text{eff}}}{1-\alpha-g(n_{\text{eff}})} \big)^{(\beta-1)/2} + \eta \cdot n_{\text{eff}} \Big), \tag{27}$$

$$\text{subject to} \qquad n_{\text{eff}} > g^{-1}(1-\alpha), \tag{28}$$

where $g^{-1}(\cdot)$ is the inverse function of $g(n_{\text{eff}})$. The verification of the concavity of the objective function is omitted here due to space limitation.

The above convex optimization problem can be solved easily by numerical methods. Suppose the solution is $\bar{n}_{\text{eff}}$, then

$$\begin{aligned} T_\alpha^L &= \max\{T_1(n_{\text{eff}}^1, 1-\alpha-g(n_{\text{eff}}^1)),\ T_1(n_{\text{eff}}^2, 1-\alpha-g(n_{\text{eff}}^2))\}, \\ n_{\text{eff}}^L &= \arg\max_{n_{\text{eff}}} T_1(n_{\text{eff}}, 1-\alpha-g(n_{\text{eff}})), \end{aligned} \tag{29}$$

where $n_{\text{eff}}^1 = n / \left\lfloor \frac{n}{\bar{n}_{\text{eff}}} \right\rfloor$, $n_{\text{eff}}^2 = n / \left\lceil \frac{n}{\bar{n}_{\text{eff}}} \right\rceil$, and $\lceil x \rceil$ denotes the smallest integer that is equal to or larger than x. We can also obtain the corresponding n_{eff}^L and

$$r_c^L = \sqrt{[\log(n_{\text{eff}}^L/(1-\alpha-g(n_{\text{eff}}^L)))]/(\pi n_{\text{eff}}^L)}. \tag{30}$$

Next, we present an approximated upper bound of $T_\alpha^{\max}$. Relaxing the constraint in Eq. (17) with the upper bound in Eq. (18), we obtain the relaxed constraints as

$$P(A) \geqslant \alpha, \quad P(B) \geqslant \alpha. \tag{31}$$

Then, we use the lower bounds to approximate $P(A)$ and $P(B)$ as

$$P(A) \approx P_l \approx 1 - g(n_{\text{eff}}^{(i)}), \quad P(B) \approx 1 - n_{\text{eff}}^{(i)} e^{-n_{\text{eff}}^{(i)} \pi \left(r_c^{(i)}\right)^2}, \tag{32}$$

Next, we assume that the number of active sensors in round i is approximately equal to $n_{\text{eff}}^{(i)}$. Then the maximum number of rounds, M, is a deterministic quantity, and satisfy the constraint $\sum_{i=1}^{M} n_{\text{eff}}^{(i)} \leq n$. Using Eqs. (31) and (32), we obtain an approximated upper bound of $T_\alpha^{\max}$ by solving the following optimization problem

$$\max_{n_{\text{eff}}^{(i)}} \sum_{i=1}^{M} \left(E_0 \Big/ \Big((\frac{1}{\pi n_{\text{eff}}^{(i)}} \log \frac{n_{\text{eff}}^{(i)}}{\alpha})^{(\beta-1)/2} + \eta \Big) \right), \tag{33}$$

$$\text{subject to} \qquad n_{\text{eff}}^{(i)} \geqslant g^{-1}(1-\alpha), \quad \sum_{i=1}^{M} n_{\text{eff}}^{(i)} \leqslant n. \tag{34}$$

It is easy to verify that this is a convex optimization problem. By Lagrange multiplier, we obtain a new upper bound of $T_\alpha^{\max}$ as

$$T_\alpha^U = \max_{n_{\text{eff}}} T_2(n_{\text{eff}}) \equiv \max_{n_{\text{eff}}} \left\lfloor \frac{n}{n_{\text{eff}}} \right\rfloor \cdot E_0 \Big/ \Big((\frac{1}{\pi n_{\text{eff}}} \log \frac{n_{\text{eff}}}{1-\alpha})^{(\beta-1)/2} + \eta \Big), \tag{35}$$

$$\text{subject to} \qquad n_{\text{eff}}^{(i)} \geqslant g^{-1}(1-\alpha). \tag{36}$$

Similarly, we temporarily remove the floor function $\lfloor\cdot\rfloor$. It is easy to verify that the resulting optimization problem is a convex problem, and suppose the solution of such problem is $\tilde{n}_{\text{eff}}$, then

$$T_\alpha^U = \max\{T_2(n_{\text{eff}}^1),\ T_2(n_{\text{eff}}^2)\}, \qquad n_{\text{eff}}^U = \arg\max_{n_{\text{eff}}} T_2(n_{\text{eff}}), \tag{37}$$

where $n_{\text{eff}}^1 = n/\left\lfloor \frac{n}{\tilde{n}_{\text{eff}}} \right\rfloor$ and $n_{\text{eff}}^2 = n/\left\lceil \frac{n}{\tilde{n}_{\text{eff}}} \right\rceil$.

With $E_0 = 1$, $\beta = 3.5$, $\eta = 0.001$, $\alpha = 0.92$, and $k = 1$, the numerical results show that the relative difference between the lower and upper bounds is at the level of 10% for n is from 10000 to 40000, which suggests that the derived lower bound is a good approximation of the optimal α-lifetime.

Finally, we propose to choose the operational parameters for mRIS scheme according to the lower bound of the optimal α-lifetime, i.e., choosing $p^{(i)}$ and $r_c^{(i)}$ for round i as

$$p^{(i)} = n_{\text{eff}}^L / n^{(i)}, \quad r_c^{(i)} = r_c^L, \tag{38}$$

where $n^{(i)}$ is the number of alive sensors at the beginning of round i.

V. Simulation results

In this section, we use simulation results to demonstrate the performance of the proposed mRIS scheduling scheme. The performance criterion is the α-lifetime of the network. As a comparison, we also include the result of another mRIS-like scheme, namely scheme A. The difference from the proposed mRIS scheme is that scheme A chooses communication radius $r_c = 2r_0$, twice of the mean sensing radius, and chooses n_{eff} according to Eq. (32) such that $P(A) \geqslant \alpha$. This scheme is based on the strategy in [6] that ignores the connectivity requirement by choosing sufficiently large r_c, and chooses the smallest $n_{\text{eff}}^{(i)}$ to satisfy the coverage requirement.

We simulate a square sensing region $\mathcal{D}$ of unit area in which n sensors are independently and uniformly deployed. The sensing radius r_s is assumed to be a uniformly distributed random variable on $[0.0384, 0.1216]$, which corresponds to that $r = 0.08$ and $\sigma_s = 0.3$. Let $E_0 = 1$, $\beta = 3.5$, $\eta = 0.001$, $\alpha = 0.92$, and $k = 1$, i.e, we considerer 1-coverage as an example. With this network setup, the proposed mRIS scheme selects r_c and n_{eff} according to Eqs. (29) and (30), and scheme A selects the smallest n_{eff} according to $P(A) \approx P_l = \alpha$. The corresponding $p^{(i)}$'s are chosen according to Eq. (38).

First, we simulate the operation of a network with $n = 10000$ using different scheduling schemes. We divide the region $\mathcal{D}$ into a grid of size 62×62, and approximate that the region $\mathcal{D}$ is k-covered if all grid points are k-covered. For the connectivity, we approximate that the network is connected if there is no isolated active sensor. The torus convention is also employed for simulations to avoid edge effects. Then, the P_{op} at each round of the network operation is estimated as follows: given a deployment, the network is operated according to the particular scheduling scheme ($p^{(i)}$ and r_c) for M rounds until the batteries of all sensors die out. Repeat this experiment 2500 times with the same deployment. For round i of experiment j, define $\delta_j^i = 1$ if the region $\mathcal{D}$ is k-covered and active sensors are connected, 0 otherwise. Then, P_{op} of round i can be approximated as $P_{\text{op}}^i = \frac{1}{2500}\sum_{j=1}^{2500} \delta_j^i$.

Fig. 1 shows two snapshots of the network operation with $n = 10000$ using the mRIS scheduling scheme and scheme A, respectively. It is seen that both scheduling schemes can guarantee that the network satisfies the connectivity and k-coverage requirements as long as the number of alive sensors is greater than n_{eff}. Therefore, in the simulation of the network's α-lifetime, we only need to simulate how many rounds a network can operate properly following a particular scheduling scheme. Note that the lifetimes of each round are different for these two scheduling schemes.

Fig. 2 shows the α-lifetime of a network using different scheduling schemes versus the total number of deployed sensors. In addition to the α-lifetime of different scheduling schemes, the derived lower bound and upper bound for the mRIS scheme are also shown in Fig. 2. We observe that for mRIS scheme, the simulation result is very close to the theoretical result,

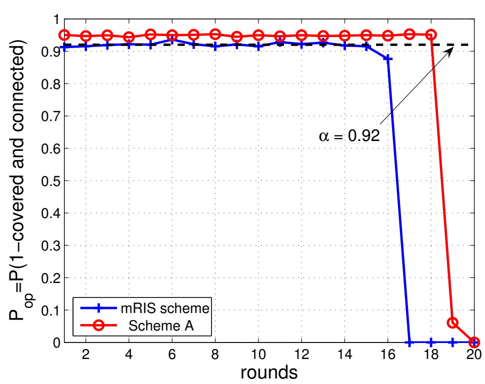

Fig. 1. Two snapshots of the network operation.

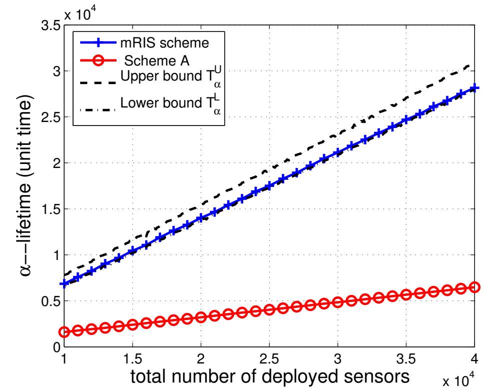

Fig. 2. α-lifetime of different scheduling schemes.

T_α^L, which was derived in Section IV. By comparing mRIS scheme and scheme A, we clearly see that mRIS scheme's α-lifetime is much longer than that of scheme A, and the difference becomes larger with more deployed sensors. Such fact demonstrates the importance of joint optimization of lifetime, connectivity, and coverage.

VI. Conclusions

In this paper, we have investigated the fundamental problems of connectivity, coverage and lifetime in large-scale wireless sensor networks. We first derive the sufficient and necessary condition on the sensing radius for asymptotic k-coverage of the sensing region. Then, we introduce the concept of α-lifetime of wireless sensor networks from probabilistic point of view, which is defined as the entire interval during which the probability of guaranteeing connectivity and k-coverage simultaneously is at least α. Next, we propose a scheduling scheme to maximize the α-lifetime of finite-size wireless sensor networks. Finally, we demonstrate the validity of theoretic results by simulations.

One promising direction of future research is to extend the results to more generic scenarios when only a portion of the sensing region needs to be k-covered. Another possible direction is to design practical distributed networking protocols to maintain the desired connectivity and sensing coverage based on the observations in this paper.

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