Random Sequential Scheduling for Wireless D2D Communications

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Abstract—This paper proposes a pairwise SIR-based random sequential scheduling algorithm for wireless D2D communications. We derive an upper and a lower bound on the number of scheduled links by identifying the equivalence between the proposed algorithm and the Random Sequential Adsorption (RSA) process in physics. We then study the optimal SIR threshold, which is a key parameter in the proposed algorithm, for achieving the maximum sum rate. We finally extend the algorithm when a minimum SIR is required at each scheduled link. From the simulations, we observe that the proposed algorithm can achieve 24% higher sum rate compared with the aggregate SIR-based scheduling algorithm.

I. INTRODUCTION

In recent years, a number of novel technologies have been proposed for next generation cellular networks, including interference alignment [1], cognitive radio [2]–[4], femtocells [5], and device-to-device (D2D) communications [6]. While all these technologies have the potential to increase the spectrum utilization efficiency by orders of magnitude, D2D communications have become particularly attractive not only because it can dramatically increase the spectral reuse, but also because the physical-layer technology for D2D communications has matured enough for deployment. For example, in 2011, Qualcomm has developed and demonstrated their device-to-device technology, FlashLinq [7], over licensed spectrum .

In current cellular systems, mobiles communicate via base stations. Therefore, even when a sender and its receiver are next to each other, the data have to be sent to a base station and be delivered with two-hop transmissions (uplink and downlink). Using D2D communications, the sender can send the data directly to its receiver when they are close to each other. If the sender and its receiver are at the edge of the cell, the D2D communications reduce two long transmissions to a single short transmission, which reduces the interference to other transmissions and improves the spectral reuse.

Due to the unique and attractive nature of D2D communications for improving the spectral reuse, in the last couple of years, significant progress has been made on system design and resource allocation of D2D communications in cellular networks. However, designing a high performance scheduling

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algorithm for D2D communications remains a challenging problem.

In this paper, we consider the problem of scheduling D2D communications over licensed spectrum with a central scheduler. We focus on single-hop flows, i.e., senders and receivers communicate directly with each other. We study an SIRbased scheduling algorithm that schedules links based on the pairwise interference between links. Our SIR-based scheduling algorithm is inspired by FlashLinq [7], a synchronous scheduling and power control algorithm developed by Qualcomm. The algorithm achieves low-complexity by scheduling the links sequentially and guarantees fairness by randomizing the scheduling sequence. Comparing to 802.11, FlashLing achieves much higher throughput [6]. A key difference between our algorithm and FlashLinq is that our algorithm schedules links based on pairwise SIR instead of aggregate SIR, which turns out to be simpler and more efficient. The main contributions are summarized below:

II. RELATED WORK

Scheduling is one of the fundamental problems in wireless networks. In the ad hoc mode of CSMA/CA with Physical Carrier Sensing or Virtual Carrier Sensing based on RTS/CTS, the energy-based yielding rule is used, which may result in poor spatial reuse since it may be over conservative [8], [9]. To improve the spatial reuse, the SIR-based yielding rule is considered [6], [10]–[13].

In particular, [12] proposes a scheduling algorithm to maximize the number of simultaneously scheduled D2D links with a minimum SIR requirement in a network with an arbitrary topology. The authors developed an algorithm with a constant approximation ratio. In [13], a pairwise-SIR based scheduling algorithm was proposed. In the proposed algorithm, the links are sequentially considered according to randomly generated priorities. Different from the algorithm considered in our work, the algorithm in [13] requires a link to yield to any conflicting links with a higher priority, no matter whether the link has been scheduled or not. This assumption leads to tractable analysis but inefficient spatial reuse.

In both energy-based and SIR-based yielding rules, a key parameter is the energy/SIR threshold. There is a tradeoff in selecting a proper threshold to balance the number of simultaneous transmissions and the average link rate in order to achieve the maximum sum rate. [14], [15] study the problem of choosing optimal physical sensing threshold to maximize the sum rate, and [16] studies the optimal physical sensing threshold to minimize the outage probability.

III. SYSTEM MODELS AND ASSUMPTIONS

A. Network Model

We consider a wireless network where D2D links are positioned on a two dimensional torus with an area of A. A central scheduler is present in the network. We assume that all links have bidirectional traffic and work in the half-duplex mode. Let s_v and r_v denote the sender and receiver of link l_v , respectively. Let l_{v^-} denote the reverse direction of link l_v . Therefore, we have $s_{v^-} = r_v$, and $r_{v^-} = s_v$. We assume that time is slotted. At each time slot, each link randomly chooses a direction to transmit data. Without loss of generality, we assume that the selected direction for link l_v is l_v . Let $\mathcal{L} = \{l_1, l_2, \ldots\}$ denote the set of links available in the network. Senders of the links (s_v) are located according to a 2-D homogeneous Poisson point process with an intensity of λ . For each s_v , r_v is chosen uniformly at random from the circle centered at s_v with a radius of d_{vv} .

B. Interference Model

In this paper, we assume the physical interference model, and a simple channel model without fading. The channel gain from s_v (i.e., sender of link l_v) to r_w (i.e., receiver of link l_w) is assumed to be

$$g_{vw} = d_{vw}^{-\alpha},\tag{1}$$

where α is the path loss exponent with a typical value between 2 and 4, and d_{vw} is the distance between s_v and r_w . We assume that channel is symmetric, i.e., the channel gain $g_{w^-v^-}$ from $s_{w^-}(r_w)$ to $r_{v^-}(s_v)$ is the same as g_{vw} .

We assume that the transmission rate of a link is a general function of the SINR, i.e.,

$$c_v = C(\gamma_v), \tag{2}$$

where γ_v is the aggregate SINR seen by r_v – the receiver of link l_v . Let $\mathcal{S} \subseteq \mathcal{L}$ denote the set of links that have been scheduled to transmit simultaneously in the network. Therefore, we have:

$$\gamma_v = \frac{P_v/d_{vv}^{\alpha}}{\sum_{l_w \in \mathcal{S} \setminus \{l_v\}} P_w/d_{wv}^{\alpha} + N},$$
(3)

where N is the background noise. We assume that the transmit power is large enough so the thermal noise is negligible comparing to the interference. Hence, we use SIR instead of SINR in the rest of the paper:

$$\gamma_v = \frac{P_v/d_{vv}^{\alpha}}{\sum_{l_w \in S \setminus \{l_v\}} P_w/d_{wv}^{\alpha}}.$$
 (4)

Moreover, we use $\gamma_{v \leftarrow w}$ to denote the *pairwise* SIR seen by r_v due to the interference from s_w :

$$\gamma_{v \leftarrow w} = \frac{P_v / d_{vv}^{\alpha}}{P_w / d_{wv}^{\alpha}}.$$
 (5)

IV. PW: PAIRWISE SIR-BASED SCHEDULING ALGORITHM

A. Problem Statement

We are interested in identifying the set of links (denoted as S) to transmit simultaneously so that the sum rate of the network is maximized:

$$S^* = \arg\max_{S \subseteq \mathcal{L}} C_{\text{sum}}(S), \tag{6}$$

where the sum rate $C_{\text{sum}}(\mathcal{S})$ is defined as the sum of rates of all scheduled links:

$$C_{\text{sum}}(\mathcal{S}) \triangleq \sum_{l_v \in \mathcal{S}} c_v = \sum_{l_v \in \mathcal{S}} C(\gamma_v).$$
 (7)

As this problem has been known as a special case of the MaxWeight problem [17], which is NP-hard, we instead study the following problem by introducing an additional constraint that links are examined sequentially according to a random order. By randomizing the order of links over time, fairness across links can be maintained [6].

B. Algorithm Description

We propose a simple yet efficient scheduling algorithm, referred to as PW, to address the problem defined above. In PW, links are considered sequentially one by one whether it shall be added into the schedule. PW is based on *pairwise* SIR. A link is scheduled if and only if it neither causes nor sees too much pairwise interference to/from the links that have already been scheduled, i.e., the pairwise SIR is higher than a pre-determined SIR threshold, denoted as γ .

Clearly, the choice of γ affects the performance of PW. If γ is too large, scheduled links are far away from each other, which results in a small set of scheduled links and consequently a small sum rate. On the other hand, if γ is too small, even though a large set of links could be scheduled simultaneously, every link would experience severe interference as the network becomes crowded. As a result, the sum rate may also be small. Therefore, there is a tradeoff and it is critical to set a proper SIR threshold in PW.

Next, we describe how PW works in detail. The pseudocode of PW is given in Alg. 1, and the structure of a time slot is shown in Fig. 1.

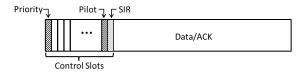


Fig. 1: Structure of a time slot.

Lines 1-2: The algorithm starts with the central scheduler arranging all the available links in a random order. The order is then broadcast to all devices in control slot 0 of the time slot, as shown in Fig. 1.

Line 8: Tx-Yielding Test: This is an important step in the algorithm, which tests whether link l_v or l_{v-} causes too much

Algorithm 1 PW

Input: \mathcal{L} – the set of available links; γ – the SIR threshold **Output:** \mathcal{S} – the set of scheduled links

The central scheduler selects a permutation (denoted as P) uniformly at random from all possible permutations of the available links in L;

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2: The central scheduler broadcasts \mathcal{P} to all devices in the network;
 3: S = \emptyset;
 4: i = 0;
 5: for the first pair of links \{l_v, l_{v^-}\} in {\mathcal P} do
           \mathcal{P} = \mathcal{P} \setminus \{l_v, l_{v^-}\};
            i = i + 1;
 7:
            if \operatorname{Tx-Yielding}(l_v, \mathcal{S}, \gamma) or \operatorname{Tx-Yielding}(l_{v^-}, \mathcal{S}, \gamma) then
 8:
                 continue
 9:
10:
                 s_v broadcasts beacon with power P_v in control slot 4i-3;
11:
                  s_{v^{-}} broadcasts with P_{v^{-}} in control slot 4i-2;
12:
                 if Rx-Yielding(l_v, \mathcal{S}, \gamma) or Rx-Yielding(l_{v^-}, \mathcal{S}, \gamma) then
13:
14:
15:
                 else
                      \begin{split} \mathcal{S} &= \mathcal{S} \cup \{l_v, l_{v^-}\}; \\ r_v \text{ broadcasts with } Kd_{vv}^\alpha/P_v \text{ in control slot } 4i-1; \end{split}
16:
17:
                       r_{v^-} broadcasts with K d_{vv}^{\dot{\alpha}}/P_{v^-} in control slot 4i;
18:
19:
20:
            end if
21:
     end for
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- Senders of all scheduled links broadcast a pilot signal simultaneously in the same control slot;
- 23: Each receiver measures the received power of the pilot signal, estimates the actual SIR, and reports it to the sender;
- 24: Each sender chooses a proper modulation to transmit data, based on the SIR reported by the receiver;

pairwise interference to the links that have already scheduled. It works as follows. As we will explain later in Lines 17-18, for each link $l_w \in \mathcal{S}$, its receiver r_w broadcasts a beacon with power Kd_{ww}^α/P_w , where K is a constant known to all devices in the network. Therefore, s_v receives this beacon at power $P_{\text{measured}} = Kd_{ww}^\alpha/(P_wd_{vw}^\alpha)$. As we assume that channel is symmetric, by manipulating the measured power, s_v can obtain the pairwise SIR that it would cause to r_w as follows:

$$\gamma_{w \leftarrow v} = \frac{P_w/d_{ww}^{\alpha}}{P_v/d_{vw}^{\alpha}} = \frac{(P_{\text{measured}}/K)^{-1}}{P_v}.$$
 (8)

 s_v calculates the pairwise SIR for all the links that have already been scheduled. If any of them is below γ , Tx-Yielding occurs and link l_v will not be scheduled. Similarly, link l_{v-} is also tested for Tx-Yielding.

Lines 11-12: If both l_v and l_{v^-} have passed the Tx-Yielding test, s_v and s_{v^-} broadcast beacons, which will be used in the following Rx-Yielding test.

Line 13: Rx-Yielding Test: This is another important step in the algorithm, which tests whether link l_v or l_{v^-} sees too much interference from the links that have already been scheduled. It works as follows. For each link $l_w \in \mathcal{S}$, its sender s_w broadcasts a beacon with power P_w . Therefore, r_v receives this beacon at power $P_{\text{measured}_1} = P_w/d_{wv}^{\alpha}$, which is exactly the interference level r_v that would be seen from s_w . On the other hand, as s_v has just sent a beacon with power P_v , r_v can measure the received power of this beacon

as $P_{\text{measured}_2} = P_v/d_{vv}^{\alpha}$. Then, r_v can obtain the pairwise SIR it would see due to the pairwise interference from s_w as follows:

$$\gamma_{v \leftarrow w} = \frac{P_v / d_{vv}^{\alpha}}{P_w / d_{vv}^{\alpha}} = \frac{P_{\text{measured } 2}}{P_{\text{measured } 1}}.$$
 (9)

 r_v calculates the pairwise SIR for all the links that have already been scheduled. If any of them is below γ , Rx-Yielding occurs and link l_v will not be scheduled. Link l_{v^-} is also tested for Rx-Yielding by following the same procedure.

Similar Tx-Yielding and Rx-Yielding tests have been used in FlashLinq [6]. However, there is a key difference between PW and FlashLinq: while PW is based on pairwise SIR, FlashLinq relies on aggregate SIR.

Line 16: If both l_v and l_{v^-} have passed the Rx-Yielding test, both links will be added into the schedule.

Lines 17-18: r_v broadcasts a beacon with power Kd_{vv}^{α}/P_v . This is possible as r_v can get P_v/d_{vv}^{α} by measuring the power of the received beacon from s_v , while K is a constant known to all devices. Similarly, r_{v^-} also broadcasts a beacon in the next control slot with power Kd_{vv}^{α}/P_{v^-} . These beacons will be used by the subsequent links in \mathcal{P} for Tx-Yielding tests.

Lines 22-24: All the scheduled links send a pilot signal simultaneously, and measure the actual aggregate SIR, based on which to decide the proper modulation scheme to transmit data, as shown in Fig. 1.

We define the number of links scheduled by PW as f(n), i.e., $f(n) = \|\mathcal{S}\|$, where $n = \|\mathcal{L}\|$ is the total number of links available in the network, which is assumed to be a Poisson random variable with mean λ . We next model PW as a Random Sequential Adsorption (RSA) process [18], and leverage the prior results on RSA to obtain the average number of scheduled links under PW.

Random Sequential Adsorption [18] is an idealized model for monolayer particle adsorption on a surface. In this process, each particle with a random orientation is sequentially and randomly placed on the surface and gets adsorbed if it does not overlap with the particles already adsorbed. This process has been investigated in depth in chemical physics. Specifically, researchers have studied the number of adsorbed particles, and the fraction of the area covered by the adsorbed particles, given the total number of arrivals of particles. The dynamics of the number of adsorbed particles in RSA have been extensively studied in [19], [20]. Yet, no closed-form expression was derived even for the simplest model, in which the spatial distribution of particles is uniform, the distribution of the particle's orientation is uniform, and the particles are disk-shaped with the same size. In [19], the accurate fitting functions of the number of adsorbed particles have been obtained using diagrammatic algebra and Monte Carlo simulations.

We define the union of two disk areas centered at s_{v} and r_{v} and with a radius of

$$R_{\rm IR} \triangleq \frac{1}{2} d\gamma^{1/\alpha} \tag{10}$$

as the interference region (IR) of the bidirectional link l_v , where γ is the interference threshold and d is the distance

of the link. Notice this interference region of a link is nonconvex. And it is extremely hard to characterize the dynamics of RSA of particles with such non-convex shapes.

Fortunately, the non-convex interference region can be approximated by its inscribed spherocylinder and circumscribed spherocylinder, as shown in Fig. 2. Each scheduled link occupies a interference region which an inscribed spherocylinder can fit in, without overlapping with other interference regions or their corresponding inscribed spherocylinders. Hence, the number of scheduled links is upper bounded by the number of adsorbed inscribed spherocylinders. Similarly, the number of scheduled links is lower bounded by the number of adsorbed circumscribed spherocylinders.

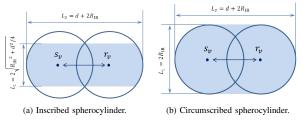


Fig. 2: Spherocylinder approximation of a interference region.

The result is summarized the following theorem and detailed analysis can be found in [21].

Theorem 1: The average number of scheduled links satisfies

$$\frac{\theta(n, \eta_{\text{circum}})A}{A_{\text{circum}}} \le f(n) \le \frac{\theta(n, \eta_{\text{in}})A}{A_{\text{in}}},$$
 (11)

where $A_{\rm in}$ and $A_{\rm circum}$ are the area of the inscribed and circumscribed spheorcylinder of the interference region, respectively, η is the aspect ratio of a spherocylinder (the ratio between its width and height), and $\theta(t,\eta)$ is determined by the following differential equation

$$\frac{d\theta(t,\eta)}{dt} = k_{\eta} \frac{1 - \left(\theta(t,\eta)/\theta(\infty,\eta)\right)^4}{1 + c_1\theta(t,\eta)/\theta(\infty,\eta) + c_2\left(\theta(t,\eta)/\theta(\infty,\eta)\right)^2}.$$

V. PWQoS: PW WITH MINIMUM SIR REQUIREMENT

In PW, a link can be scheduled if and only if the pairwise SIR is above the SIR threshold γ . However, when all the scheduled links transmit simultaneously, the actual interference experienced by a link is the aggregate interference from all other links, which is larger than the pairwise interference from the nearest neighbor. Therefore, the actual SIR may be smaller than the SIR threshold. In this section, we extend our study on the max sum rate problem by introducing an additional QoS (Quality of Service) requirement that all scheduled links shall meet a minimum SIR requirement (denoted as $\gamma_{\rm req}$). In other words, the goal becomes to maximize the sum rate of the scheduled links whose actual SIR meets the SIR requirement. We propose a modified PW algorithm in Alg. 2.

The algorithm uses PW as a subroutine, and adds an SIR offset (denoted as $\gamma_{\rm offset}$) to $\gamma_{\rm req}$ as the SIR threshold for PW. As long as $\gamma_{\rm offset}$ is larger than the upper bound of the gap between the actual SIR and the SIR threshold, we can

Algorithm 2 PWQoS

Input: \mathcal{L} – the set of available links; $\gamma_{\rm req}$ – min SIR requirement **Output:** \mathcal{S} – the set of scheduled links

- 1: $\gamma = \gamma_{\text{req}} + \gamma_{\text{offset}}(\gamma_{\text{req}})$ (dB);
- 2: $PW(\mathcal{L}, \gamma)$;

guarantee that all scheduled links meet the SIR requirement. The detailed design of $\gamma_{\rm offset}$ can be found in [21].

VI. PERFORMANCE COMPARISON

Next, we compare the performance of PWOoS with an algorithm that is based on aggregate SIR, referred to as AGG. Similar to PWQoS, AGG arranges the available links according to a random order and examines them sequentially one by one. However, different from PWOoS, a link is scheduled in AGG if and only if it neither causes nor sees too much aggregate interference to/from all links that have already been scheduled. By using $\gamma_{\rm req}$ as the SIR threshold, AGG can guarantee that all scheduled links satisfy the minimum SIR requirement. Fig. 3 compares the sum rate obtained by PWQoS with $\gamma_{\rm optOffset}$ and the sum rate obtained by AGG, under different SIR requirements. As shown in the figure, PWQoS outperforms AGG significantly in all scenarios. For example, when $\gamma_{\rm req}=0{
m dB},$ the sum rate yielded by PWQoS is 24%higher than that by AGG. More simulations results can be found in [21].

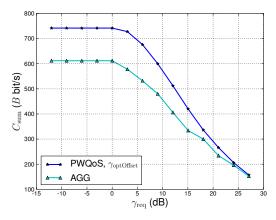


Fig. 3: Sum rate under different SIR requirements.

VII. CONCLUSION

In this paper, we proposed PW - a pairwise SIR-based random sequential scheduling algorithm for wireless D2D communications. We obtained an upper bound and a lower bound on the number of scheduled links based on the existing results on RSA. We further extended the proposed algorithm when a minimum SIR is required, and showed that the pairwise SIR-based algorithm improves the sum rate by 24% comparing to the aggregate SIR-based algorithm due to a better spatial reuse.

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