EE523: Random Processes for Communication and Signal Processing

Homework #4

---

1. Suppose $X_n \xrightarrow{m.s.} X$ and $E(X_n^2) < L, \forall n$ and $E(X^2) < L$ for some finite L. Show that $E|X_n^2| \to E|X^2|$.

2. Suppose $X_n \xrightarrow{1} X$ i.e. $E(|X_n - X|) \to 0$, as $n \to \infty$. Show that $E(X_n) \to E(X)$. Is the converse true ?

3. Let $E(X_n^2) < \infty, \forall n$. Suppose $X_n \xrightarrow{m.s.} X$. Show that $var(X_n) \to var(X)$. You may find Minkowski's inequality useful. If $p \geq 1$,
$$\{E(|X + Y|^p)\}^{1/p} \leq \{E(|X|^p)\}^{1/p} + \{E(|Y|^p)\}^{1/p}.$$

4. Show that convergence in distribution to a constant implies convergence in probability i.e. $X_n \xrightarrow{d.} c$ where c is a constant implies $X_n \xrightarrow{p.} c$.

5. Let $X_1, X_2 \ldots$ be i.i.d. random variables with common mean μ and finite variance. Show that
$$\binom{n}{2}^{-1} \sum_{1 \leq i < j \leq n} X_i X_j \xrightarrow{p.} \mu^2 \quad \text{as } n \to \infty$$

6. Let Θ be distributed uniform over$[0, 2\pi]$. (a) Show that $X_n = \cos(n\Theta)$ converges in distribution as $n \to \infty$. (b) Show that $Y_n = |1 - \Theta/\pi|^n$ converges both a.s. and m.s.