EE523: Random Processes for Communication and Signal Processing

Homework #9

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1. Suppose X is a zero mean random process. For each choice of autocorrelation below, indicate which of the following properties X has: m.s. continuity, m.s. differentiability, m.s. integrability over finite intervals and mean-ergodicity in the m.s. sense.

   a) X is WSS with $R_{XX}(\tau) = \max((1 - |\tau|), 0)$.

   b) X is WSS with $R_{XX}(\tau) = 1 + \max((1 - |\tau|), 0)$.

   c) X is WSS with $R_{XX}(\tau) = \cos(20\pi\tau)\exp(-10|\tau|)$.

   d) X is not WSS and has autocorrelation given by

   $$R_{XX}(s,t) = \begin{cases} 1 & \text{if} \lfloor s \rfloor = \lfloor t \rfloor \\ 0 & \text{otherwise} \end{cases}$$

   You don't need to check for mean-ergodicity in this case.

   e) X is not WSS with $R_{XX}(s,t) = \sqrt{\min(s,t)}$. You don't need to check for mean-ergodicity in this case.

2. Suppose X is a zero-mean WSS random process such that R_{XX} is twice differentiable. Guided by Taylor series approximations for deterministic functions we may propose to estimate $X(t)$ by $\hat{X}(t) = X(0) + tX^{'}(0)$ where $X^{'}(0)$ denotes the m.s. derivative of $X(t)$ evaluated at $t = 0$.

   - Express the covariance matrix of $(X(0), X^{'}(0), X(t))^T$ in terms of R_{XX} and its derivatives.
   - Express the mean square error $E[(X(t) - \hat{X}(t))^2]$ in terms of R_{XX} and its derivatives.

3. A WSS process X is called correlation ergodic (in the m.s. sense) if for all h

   $$\lim_{t\to\infty} m.s \frac{1}{t}\int_0^t X(s+h)X(s)ds = E(X(s+h)X(s))$$

   Suppose X is zero mean, real-valued Gaussian process such that $R_{XX}(\tau) \to 0$ as $|\tau| \to \infty$. Show that X is correlation-ergodic. You may want to use the following property of four jointly-Gaussian random variables. If $Y_i, i = 1, \ldots, 4$ are jointly Gaussian, then

   $$E(Y_1Y_2Y_3Y_4) = E(Y_1Y_2)E(Y_3Y_4) + E(Y_1Y_3)E(Y_2Y_4) + E(Y_1Y_4)E(Y_2Y_3)$$

   Bonus: Try to prove the hint by using the characteristic function of a Gaussian random vector and by noting that the expectation of the product $Y_1Y_2Y_3Y_4)$ *can be written in terms of derivatives of the characteristic function evaluated at certain points.*

4. Suppose X is a m.s. differentiable random process and $f(t)$ is a differentiable function. Show that the product $X(t)f(t)$ is m.s. differentiable and that

   $$\frac{d}{dt}(X(t)f(t)) = X^{'}(t)f(t) + X(t)f^{'}(t)$$