EE523: Random Processes for Communication and Signal Processing

Homework #8

---

1. Let a random process $X(t)$ with mean value 128 and covariance function $C_{XX}(\tau) = 1000\exp(-10|\tau|)$ be filtered by the low pass filter

$$H(\omega) = \frac{1}{1 + j\omega}$$

to produce the output process $Y(t)$.

   a) Find $\mu_Y(t)$.

   b) Find the covariance function $C_{YY}(\tau)$.

2. Consider the system shown in the figure below. Let $X(t)$ and $N(t)$ be WSS and mutually uncorrelated with p.s.d's $S_{XX}(\omega)$ and $S_{NN}(\omega)$ and zero means.

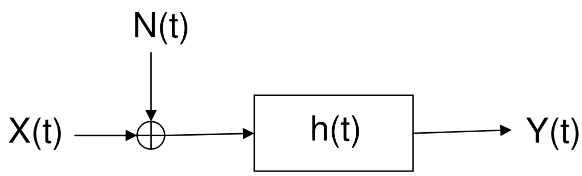

Figure 1:

   a) Find the psd of the output $Y(t)$.

   b) Define the error $\epsilon(t) = Y(t) - X(t)$ and evaluate the psd of $\epsilon(t)$.

   c) Assume that $h(t) = a\delta(t)$ and choose the value of a that minimizes the power of $\epsilon(t)$.

3. Let the random variables, $A_k, B_k, k \geq 0$ be mutually independent with mean zero. Let A_k have variance σ_A^2 and let B_k have variance σ_B^2 for all k. Define a discrete-time random process $\{Y_k\}_{k\geq 0}$ such that $Y_0 = 0$ and $Y_{k+1} = A_k Y_k + B_k$ for $k \geq 0$.

   a) Find a recursive relation for $E(Y_k^2)$.

   b) Does Y have independent increments ?

   c) Find the autocorrelation function of Y (in terms of $E(Y_k^2)$)

4. Consider the random BPSK signal with random initial phase.

$$X(t) = \cos(2\pi f_c t + \Theta(t) + \beta)$$

where β is a random variable distributed uniformly between $[0, 2\pi]$ independent of $\Theta(t)$. The signal $\Theta(t)$ is such that

$$\Theta(t) = B_n \quad \text{if } nT \leq t \leq (n+1)T$$

and B_n is i.i.d. sequence of random variables such that $P(B_n = \pi/2) = P(B_n = -\pi/2) = 0.5$. The time interval T is an integral multiple of $1/f_c$. Show that $X(t)$ is WSS and find its mean and autocorrelation function.

5. Consider a zero mean WSS process $X(t)$ such that $R_{XX}(\tau) = \frac{1}{2\tau_0}\exp(-|\tau|/\tau_0)$. The process $X(t)$ is input to an ideal low-pass filter with frequency response

$$G(\omega) = \begin{cases} 1 & |\omega| \leq \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

to obtain a signal $Y(t)$. Show that when $|\omega_0\tau_0| << 1$ i.e. the product $\omega_0\tau_0$ is much smaller than 1, $S_{YY}(\omega)$ is approximately the psd of white noise band-limited to $[-\omega_0, \omega_0]$.