EE523: Random Processes for Communication and Signal Processing

Homework #1

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1. Give an example of a field of sets that is not a σ-field.

2. Let A and B be events with probabilities $P(A) = 3/4$ and $P(B) = 1/3$. Show that $\frac{1}{12} \leq P(A \cap B) \leq \frac{1}{3}$ and show that both upper and lower bounds are possible. Find corresponding bounds for $P(A \cup B)$.

3. Let $A_1, A_2, \ldots, A_n$ be events where $n \geq 2$. Show that

$$P(\bigcup_{i=1}^{n} A_i) = \sum_i P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \ \ldots \ + (-1)^{n+1} P(A_1 \cap A_2 \cap \cdots \cap A_n)$$

4. For events $A_1, A_2, \ldots, A_n$ satisfying $P(\cap_{i=1}^{n} A_i) > 0$, prove that

$$P(\cap_{i=1}^{n} A_i) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \ldots P(A_n|A_1 \cap A_2 \cdots \cap A_{n-1}).$$

5. Show that the conditional independence of A and B given C neither implies, nor is implied by the independence of A and B.

6. We roll a die n times. Let A_{ij} be the event that the i^{th} and the j^{th} rolls produce the same number. Show that the events $\{A_{ij} : 1 \leq i < j \leq n\}$ are pairwise independent but not independent.

7. Show that the probability that *exactly one* of the events A and B occurs is

$$P(A) + P(B) - 2P(A \cap B)$$

8. Show that

$$P(\bigcup_{r=1}^{n} A_r) \geq \sum_{r=1}^{n} P(A_r) - \sum_{r<k} P(A_r \cap A_k)$$

9. There are two roads from city A to city B and two roads from city B to city C. Each of the four roads has probability p of being blocked by snow, independently of the others. What is the probability that there is an open road from A to C.