**EE523: Random Processes for Communication and Signal Processing**

Homework #7

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1. Let $X_t = A\cos(2\pi Vt + \Theta)$ where $E(A) = 2, Var(A) = 4$, and $V$ is distributed uniform over $[0, 5]$ and $\Theta$ is uniform over $[0, 2\pi]$ and $A, V$ and $\Theta$ are independent. Find the mean function and the autocorrelation function of $X_t$. Is it wide sense stationary ?

2. Let $\{Z_n\}$ be a sequence of uncorrelated real-valued random variables with zero mean and unit variance and define the moving average,

$$Y_n = \sum_{i=0}^{r} \alpha_i Z_{n-i},$$

for constants $\alpha_0, \alpha_1, \ldots, \alpha_r$. Show that $Y$ is wide sense stationary and find its autocovariance function.

3. *Random telegraph.* Let $\{N(t) : t \geq 0\}$ be a Poisson process of rate $\lambda$, and let $T_0$ be an independent random variable such that $P(T_0 = \pm 1) = \frac{1}{2}$. Define $T(t) = T_0(-1)^{N(t)}$. Find,

   a) $E(T(s))$ and $E(T(s), T(s+\tau))$.
   b) The mean and variance of $X(t) = \int_0^t T(s)ds$.

4. Consider a Poisson process with rate $\lambda > 0$. Recall that $N_t$ represents the number of arrivals in the interval $(0, t]$.

   a) Find the probability of the event $\{N_1 = 1, N_2 - N_1 = 1, N_3 - N_2 = 1\}$.
   b) Find the probability that there are two arrivals in the interval $(0, 2]$ and two arrivals in the interval $(1, 3]$.
   c) Find the probability that there are two arrivals in $(1, 2]$, given that there are two arrivals in $(0, 2]$ and two arrivals in $(1, 3]$.

5. a) Consider a stream of packets arriving at a router according to a Poisson process with rate $\lambda$. Suppose that each packet is independently *Good* with probability 0.1 and *Bad* with probability 0.9. Given that 1000 good packets arrived in the time interval $(0, 1]$ what is the expected number of bad packets that arrived in $(0, 1]$.
   b) Good packets arrive at rate $\lambda_G$ and bad packets arrive at a rate $\lambda_B$ to a router. We are given that 100 packets arrived in the interval $(0, 1]$. Find the probability that exactly 40 of those packets were good.
   c) Suppose that packets arrive at a router with rate $\lambda$. You are given that in the interval $(0, 40]$, 100 packets arrived. What is the probability that 20 of those packets arrived in the interval $(0, 5]$.
   d) Suppose packets arrive at a router with rate $\lambda$. Let $W_n$ represent the time of the arrival of the $n^{th}$ packet. Find the distribution of $W_n$.

6. Let $N_t$ be a Poisson process with rate $\lambda$ and let $X = \{X_t : t \geq 0\}$ be defined by $X_t = N_{t+1} - N_t$. Find the mean function and covariance function for $X$.