EE523: Random Processes for Communication and Signal Processing

Homework #4

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1. Suppose $X_n \xrightarrow{m.s.} X$ and $E(X_n^2) < L, \forall n$ and $E(X^2) < L$ for some finite L. Show that $E|X_n^2| \to E|X^2|$.

2. Suppose $X_n \xrightarrow{1} X$ i.e. $E(|X_n - X|) \to 0$, as $n \to \infty$. Show that $E(X_n) \to E(X)$. Is the converse true ?

3. Let $E(X_n^2) < \infty, \forall n$. Suppose $X_n \xrightarrow{m.s.} X$. Show that $var(X_n) \to var(X)$. You may find Minkowski's inequality useful. If $p \geq 1$,

$$\{E(|X+Y|^p)\}^{1/p} \leq \{E(|X|^p)\}^{1/p} + \{E(|Y|^p)\}^{1/p}.$$

4. Suppose that $X_n \xrightarrow{p.} X$. Show that $\{X_n\}$ is Cauchy convergent in probability, i.e. for all $\epsilon > 0, P(|X_n - X_m| > \epsilon) \to 0$ as $n, m \to \infty$.

5. Suppose that $X_n \xrightarrow{a.s} X$ and $Y_n \xrightarrow{a.s.} Y$. Show that $X_n + Y_n \xrightarrow{a.s.} X + Y$ and the corresponding result for convergence in probability. Show that the corresponding result for convergence in distribution is not true. *Hint: Think of a simple counter-example.*

6. Let Θ be distributed uniform over$[0, 2\pi]$. (a) Show that $X_n = \cos(n\Theta)$ converges in distribution as $n \to \infty$. (b) Show that $Y_n = |1 - \Theta/\pi|^n$ converges both a.s. and m.s.