# Kalman Filter Application to Electrical Impedance Tomography (EIT)

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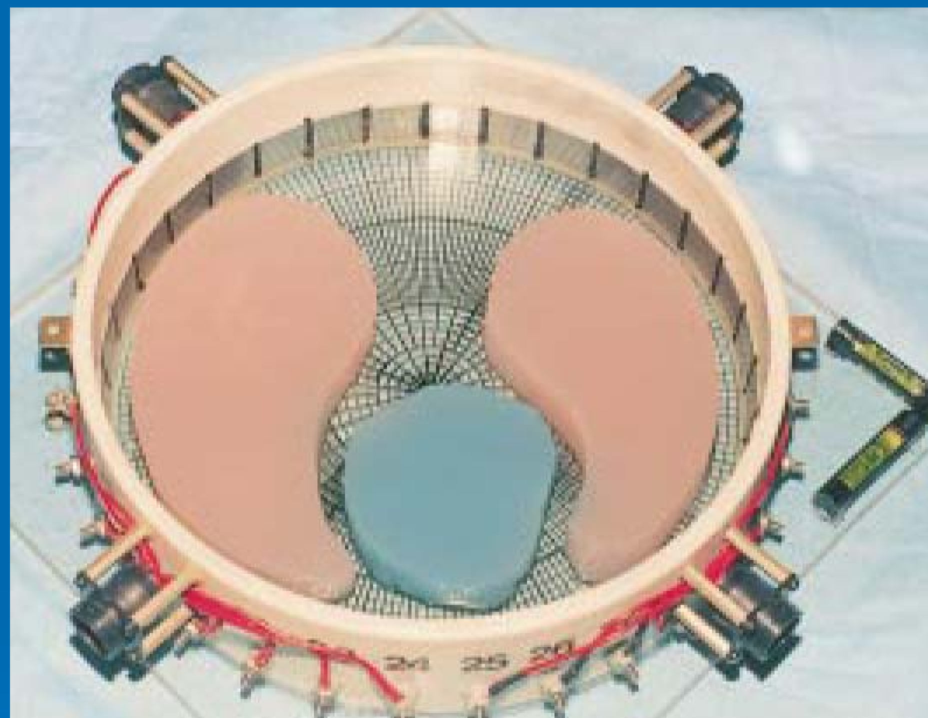
# Electrical Impedance Tomography

- A novel medical imaging technique
- Makes use of large resistivity contrast (up to about 200:1) between a wide range of tissue types in the body
- Basically ‘impedance imaging’ of the interior of the body
- May be used to complement X-ray Tomography (CT), positron emission tomography etc.
- Cheaper, faster and harmless

# How does EIT work?

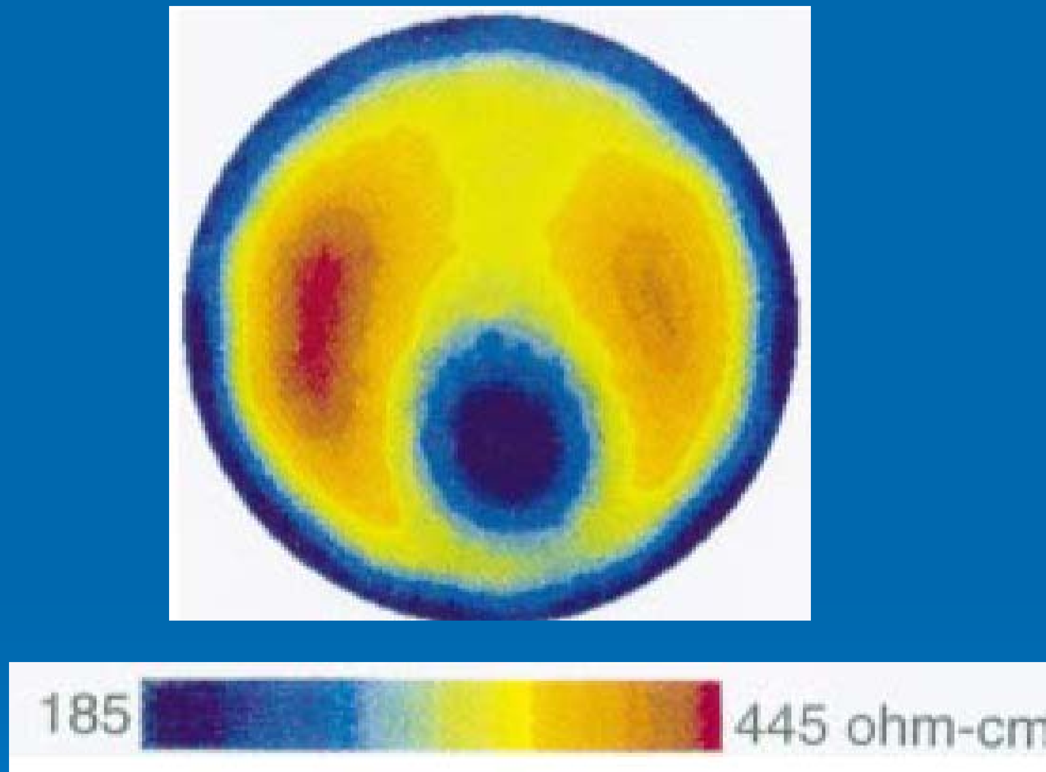
- Electrodes are placed in a transverse plan around the volume of the conductor
- Various Current Patterns are injected through electrodes
- Corresponding voltages between the electrodes are measured
- Construct  impedance map or compute the impedance distribution of the cross-section of the volume using the boundary values ( Voltages) at the surface

# A Diagrammatic View



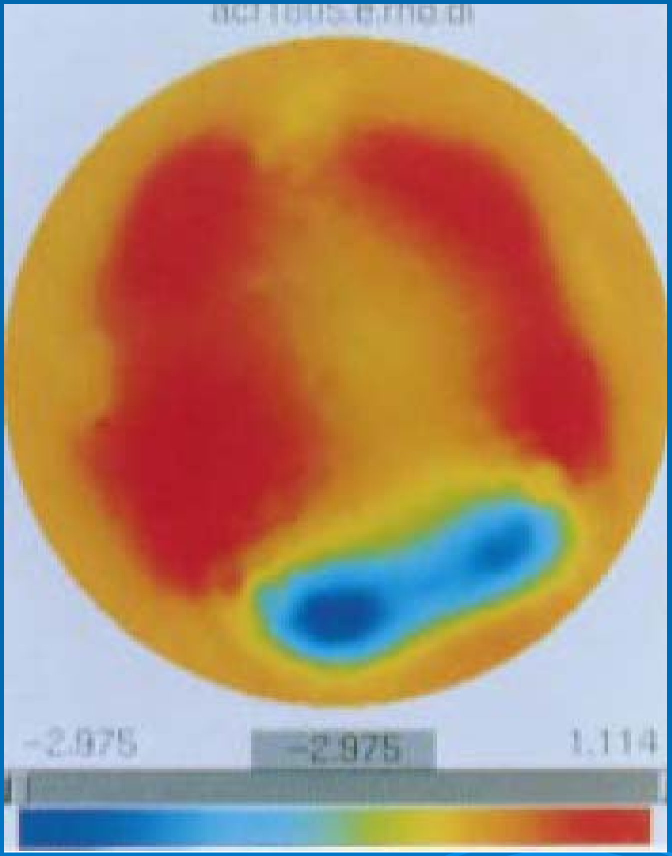
- Electrodes around the circumference of a Cylindrical Volume with artificial Lungs and Heart
- All in the same plane. Impedance map is computed for the corresponding cross-section of the volume

# The Impedance Map




- Impedance Map of the cross-section i.e. the resistivity distribution over the cross-section computed from measurements at the boundary Or the Circumference

# EIT with human body




- EIT used to track impedance variation inside Lungs and heart ventricles due to cardiac activity

# EIT comparison with MRI/CT imaging

- For creating an image, the energy signal should proceed linearly through the subject
- MRI/CT satisfies the above condition
- But in EIT current can't be forced to flow linearly. It takes several paths through the volume of interest

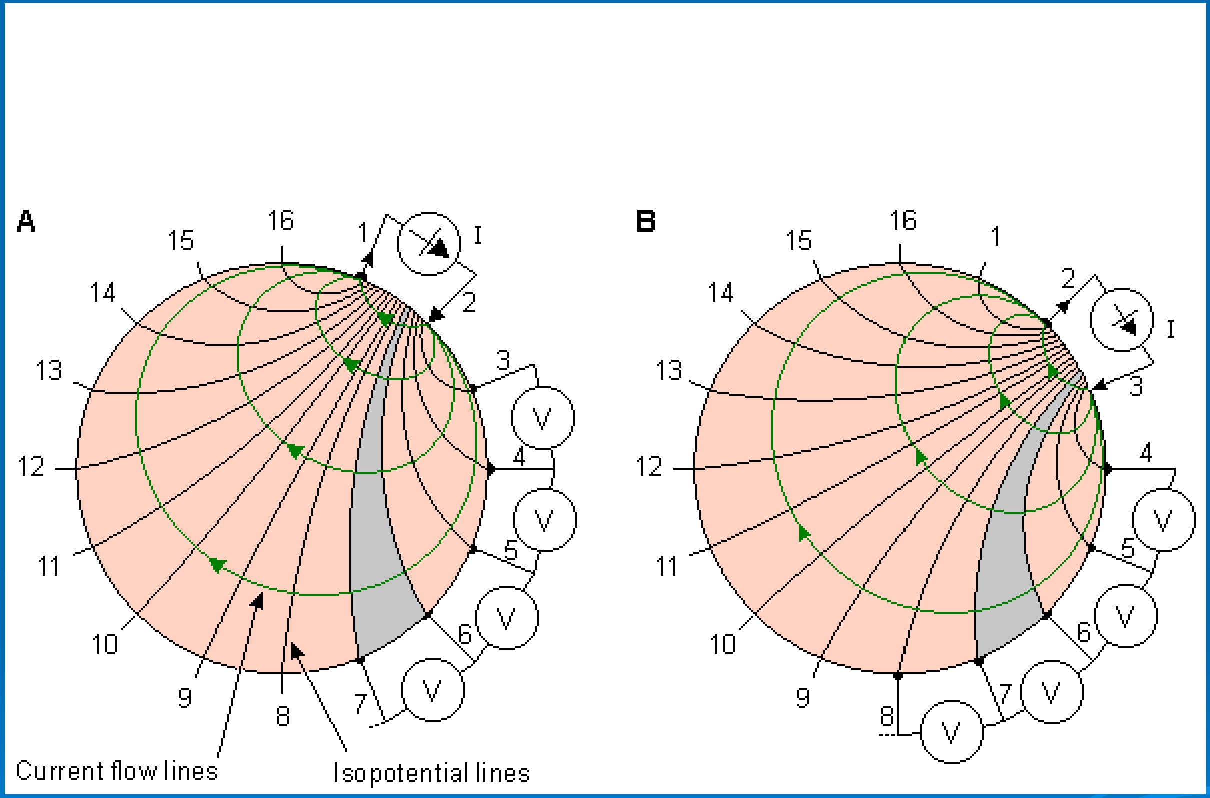
- Spatial resolution of EIT is lesser
- But EIT has good temporal resolution

- EIT can track fast impedance variation inside the body and hence need for a faster algorithm

# EIT system design

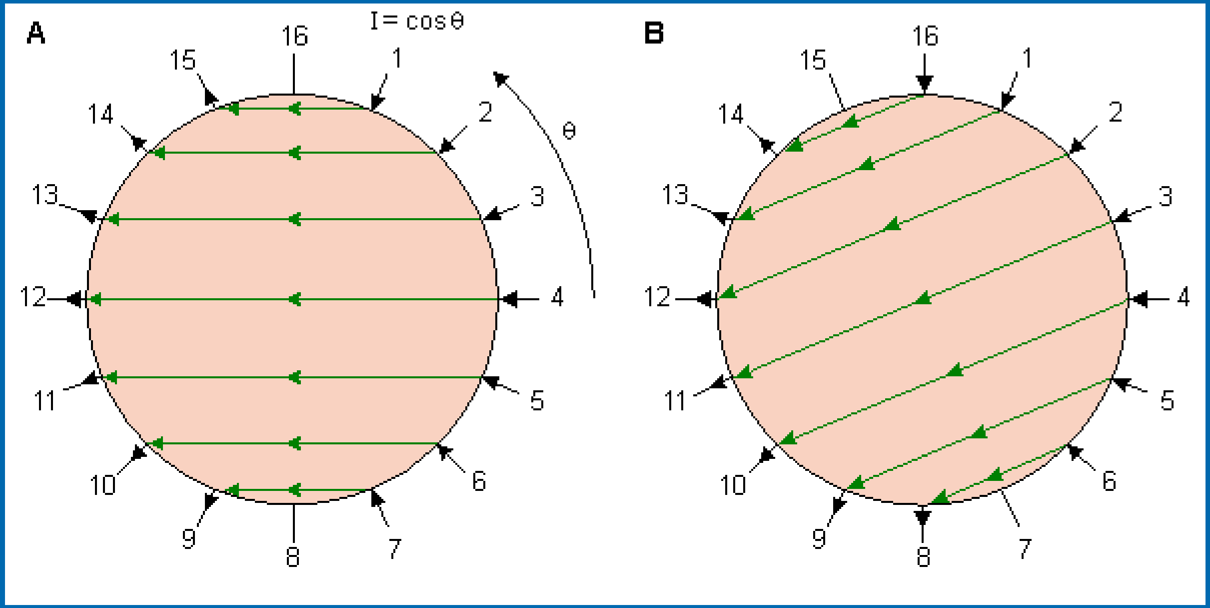
- Basic question : How to measure the impedance? i.e. the 'Reconstruction Problem'
- How to fed the current patterns and how to measure the voltages?
- How to choose the current patterns ?
- Is it possible to create a homogeneous current distribution?
- How to mathematically relate the measured boundary values( Voltages) with the cross-sectional impedance distribution?

# The current patterns and Voltage measurements




The Neighboring method

# The current patterns and Voltage measurements (Adaptive method)




- Current injected through all 16 electrodes simultaneously.
- Voltages are measured w.r.t a common grounded electrode.
- New current pattern generated by rotating the distribution with one Electrode increment.
- Total 8*15=120 voltage measurements

# Reconstruction: A mathematical Perspective

- Determination of impedance distribution from voltage values measured at the boundary
- Condition: There's NO source inside the volume
- Consider the volume space with cross-section as $\Omega$
- Basic equation: $$\nabla \cdot (\sigma \nabla u) = 0 \quad \text{in } \Omega$$
- u=u(x), x=(x1,x2) in the cross-section and $\sigma = \sigma(x)$
- Solution for conductivity will give us the impedance distribution

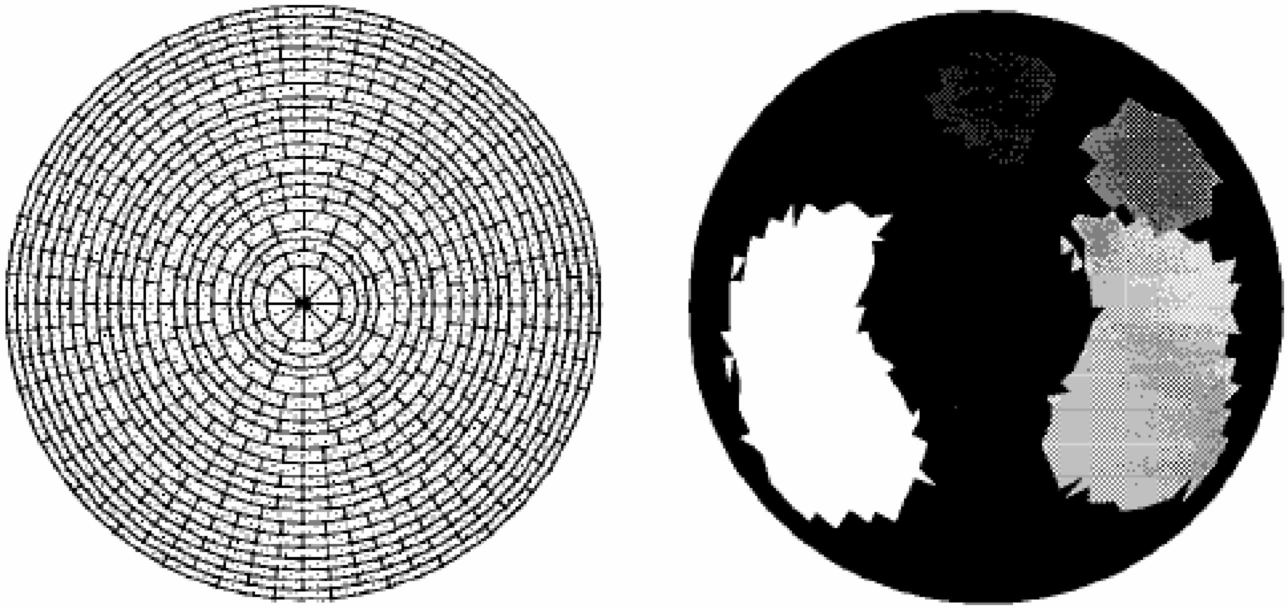
# Computation of Impedance Distribution

- Computation is done by relating measured set of voltages with the impedance distribution in the cross-section
- A linearized problem formulation
- Let $\rho$ be the impedance distribution
- Since we have only a finite number of measurements so we'll be able to recover only limited number of degrees of freedom of $\rho$
- Introduction of Finite Element Method (FEM) discretization of N elements and Consider $\rho \in R^N$

# FEM discretization and ROI

FEM : nodes and grid elements

ROI : Region Of Interest



(a) (b)

# Formulation of State Space model

- Let U be the vector containing the voltage measurements corresponding to all current patterns, where U=U($\rho$ )
- Let 'Uo' be voltage measurements corresponding to a distribution $\rho$ o
- Linearization of mapping U at $\rho$ o is

$$U(\rho) = U_0 + J(\rho_0)(\rho - \rho_0)$$

- J =J($\rho$o) is computed from FEM discretization of associated PDEs (Beyond our scope)

# State-space model (Contd..)

- L voltage measurements corresponding to each current pattern
- $I_k$, K-th current pattern: $I_k$ is L dimensional
- For total K current patterns, U is KL dimensional
- We can rewrite the mapping U as,

$$U = \begin{pmatrix} U_1 \\ \vdots \\ U_K \end{pmatrix} = \begin{pmatrix} U_{0,1} \\ \vdots \\ U_{0,K} \end{pmatrix} + \begin{pmatrix} J_1 \\ \vdots \\ J_K \end{pmatrix} (\rho - \rho_0)$$

$U_{o,k}$ is L-D, $J_k$ is (LxN)-D, K-th block corresponds to current pattern $I_k$ where N is the number of FEM elements

# The time varying model

- Consider at time t current pattern is $I_{k(t)}$ and corresponding voltage measurement U(t)
- $\rho = \rho(t)$ is considered as the state that evolves with time ( State Equation Formulation)

- Observation equation:

$$U(t) = U_{0,k(t)} + J_{k(t)}(\rho(t) - \rho_0) + w(t)$$

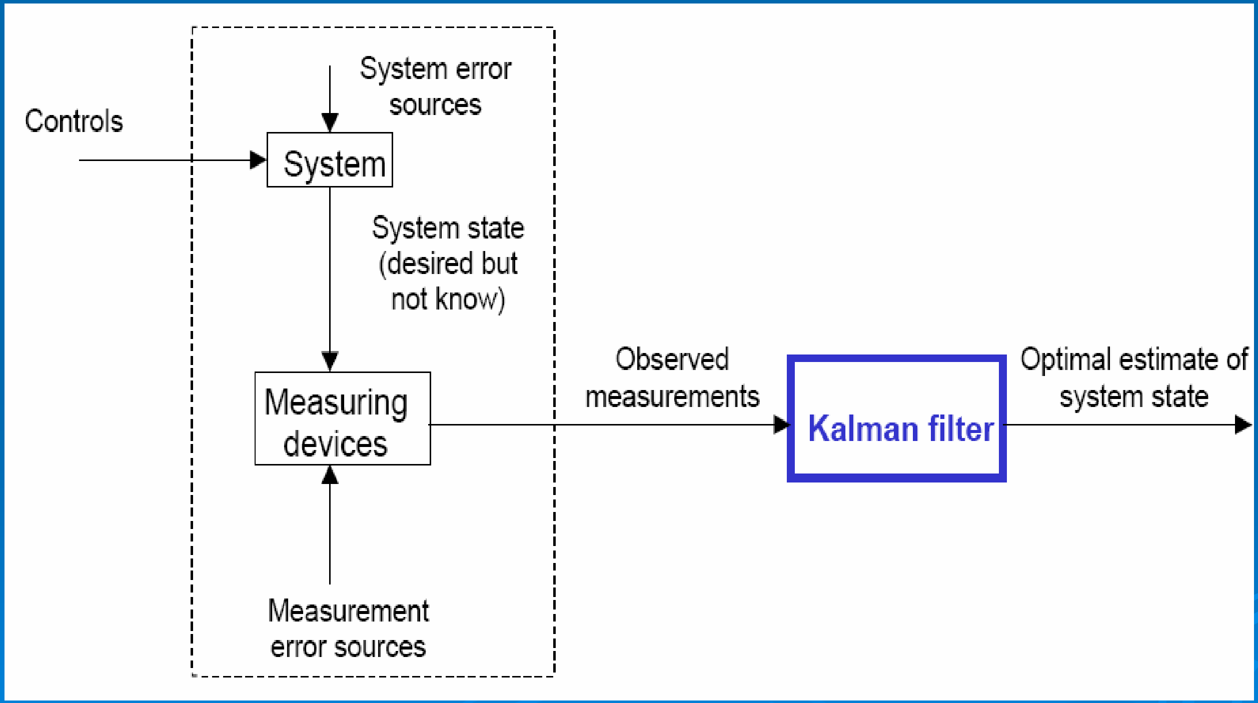
- State Equation :

$$\rho(t+1) = F(t)\rho(t) + v(t)$$

Now, we are all set to use the 'Kalman Filter'

# The Kalman Filter : Basics

- Optimal recursive data processing algorithm
- Typical Kalman Filter application




State cannot be measured directly. Has to be estimated Optimally from measurements

# What Kalman Filter does?

- Generates optimal estimate of desired quantities given the set of measurements

- Optimal : For linear system and white Gaussian errors, Kalman filter is “best” estimate based on all previous measurements

- Of all the possible filters, Kalman Filter minimizes the variance of estimation error i.e. the difference the original state and the estimated state

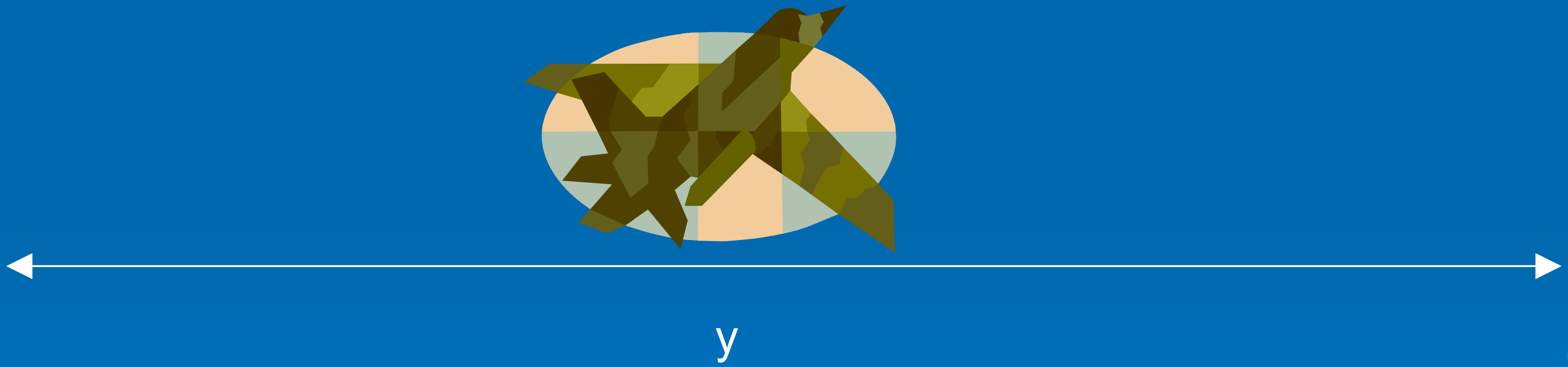
- Recursive : Doesn’t need to store all previous measurements and reprocess all data each time step

# KF: the concepts

- Simple example to motivate the workings of the Kalman Filter
- Theoretical Justification to come later –first the very basic concept
- Important: Prediction and Correction

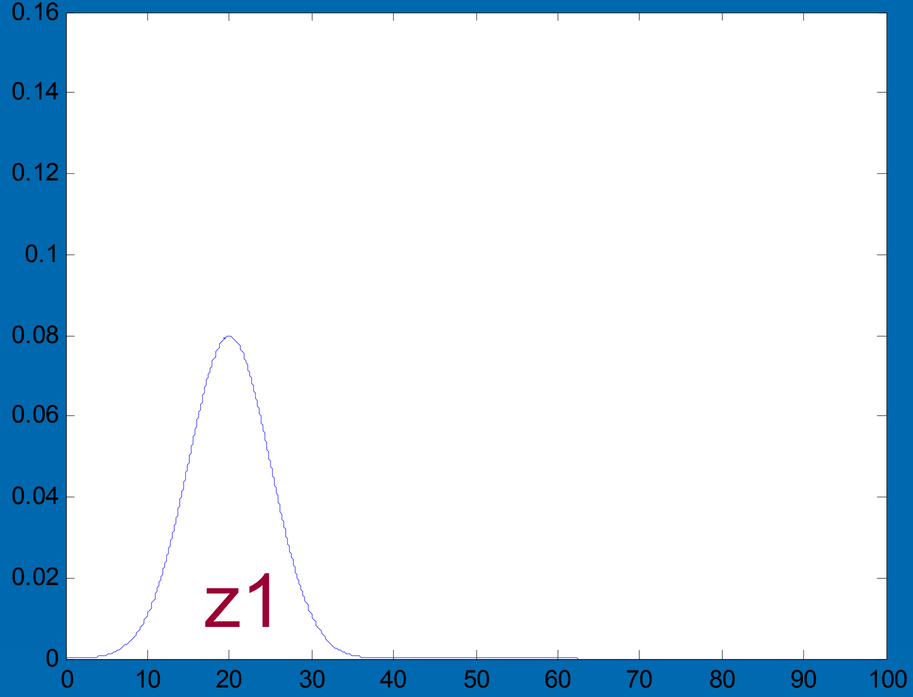
# KF: the concepts

- A simple estimation problem



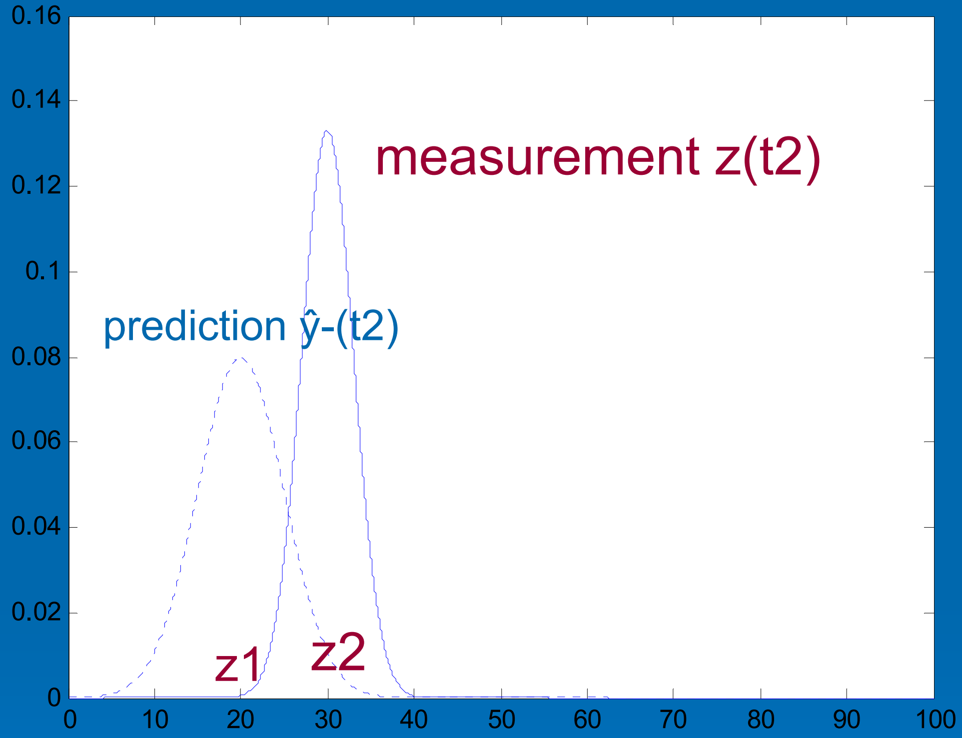

  - Lost on the 1-dimensional line
  - Position : y(t)
  - Assume Gaussian distributed measurements

# KF: the concepts




Sextant Measurement at t1: Mean = z1 and Variance = $\sigma_{z1}^2$
Optimal estimate of position is: ŷ(t1) = z1
Variance of error in estimate: $\sigma_X^2(t1) = \sigma_{z1}^2$
Aircraft in same position at time t2 - Predicted position is z1

# KF: the concepts



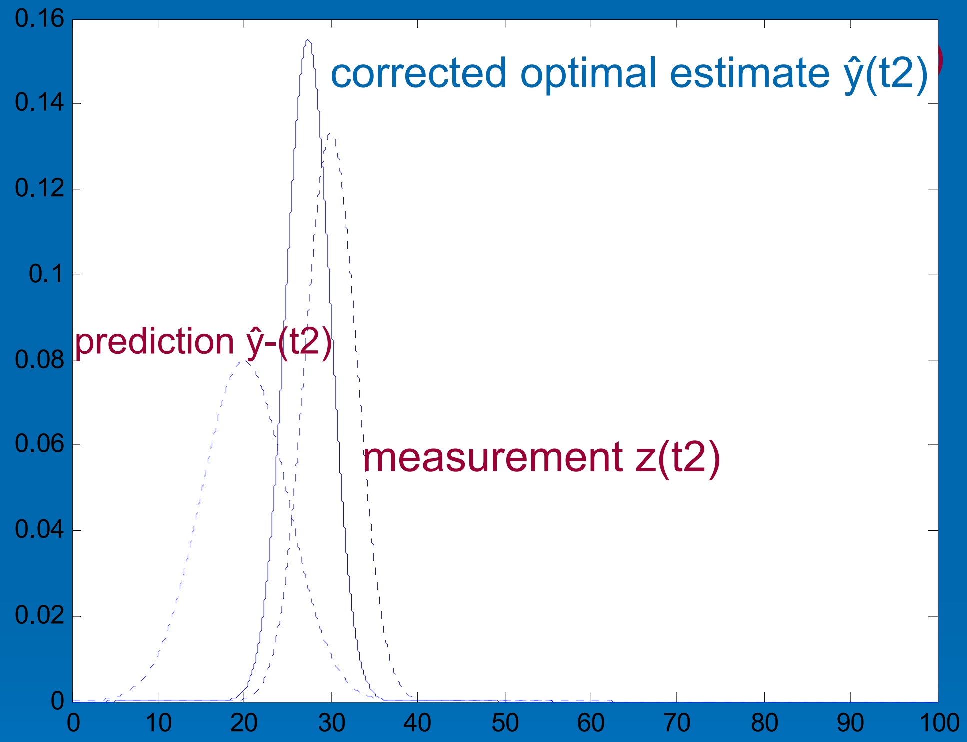

So we have the prediction ŷ-(t2)

GPS Measurement at t2: Mean = $z_2$ and Variance = $\sigma_{z2}^2$

Need to <u>correct</u> the prediction due to measurement to get ŷ(t2)

Closer to more trusted measurement – linear interpolation?

# KF: the concepts




- Corrected mean is the new optimal estimate of position
- New variance is smaller than either of the previous two variances

# KF: the concepts

Make prediction based on previous data - $\hat{y}^-$, $\sigma^-$

↓

Take measurement – $z_k$, $\sigma_z$

↓

Optimal estimate ($\hat{y}$) = Prediction + K* (Measurement - Prediction)

Variance of estimate = Variance of prediction * (1 – K)

**(To be deduced soon!!)**

# KF: the concepts

It turns out that for the simple problem,

K = $\sigma_{z1}^2 / (\sigma_{z1}^2 + \sigma_{z2}^2)$

$$1/\sigma^2 = 1/\sigma_{z1}^2 + 1/\sigma_{z2}^2$$

Where, $\sigma^2$ is the variance of the estimate

Just like merging of two Gaussians…

**( Kalman Filter will give us that as well !)**

# KF: the concepts

So far…

- Initial conditions ($\hat{y}_{k-1}$ and $\sigma_{k-1}$)
- Prediction ($\hat{y}^-_k$ , $\sigma^-_k$)
  - Use initial conditions and model (eg. constant velocity) to make prediction
- Measurement ($z_k$)
  - Take measurement
- Correction ($\hat{y}_k$ , $\sigma_k$)
  - Use measurement to correct prediction by 'blending' prediction and residual – always a case of merging only two Gaussians
  - Optimal estimate with smaller variance

# KF: Optimal Filter design from State-space model

We'll just go ahead with a bunch of equations.

State Process:

$$x_{k+1} = \Phi x_k + w_k$$

Measurement Process:

$$z_k = H x_k + v_k$$

( the control input neglected )

# KF: Optimal Filter design from State-space model

The squared error function,

$$f(e_k) = (x_k - \hat{x}_k)^2$$

MSE( Mean squared Error) Function

$$\epsilon(t) = E(e_k^2)$$

# KF: Optimal Filter design from State-space model

Covariance of two noise models,

$$Q = E\left[w_k w_k^T\right]$$

$$R = E\left[v_k v_k^T\right]$$

Error covariance matrix at k-th instant

$$P_k = E\left[e_k e_k^T\right] = E\left[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T\right]$$

$P_k$ :Trace is the sum of MSEs

# KF: Optimal Filter design from State-space model

Suppose we have a prior estimate of a state at k-th instant so,

$$\hat{x}_k = \hat{x}'_k + K_k(z_k - H\hat{x}'_k)$$

Or,

$$\hat{x}_k = \hat{x}'_k + K_k(Hx_k + v_k - H\hat{x}'_k)$$

**KF design : Find $K_k$ that will give optimal performance i.e. minimum MSE**

# KF: Optimal Filter design from State-space model

How to find $K_k$ for optimal Filter (KF) ?

Solution: Minimize Trace of $P_k$ ( Why? )

Where,

$$P_k = (I - K_k H) P'_k (I - K_k H)^T + K_k R K_k^T$$

# KF: Optimal Filter design from State-space model

Finally,

Kalman Gain ( $K_k$) is given by,

$$K_k = P_k' H^T \left( H P_k' H^T + R \right)^{-1}$$

Mathematical treatment of the equations will give us the other update equations,

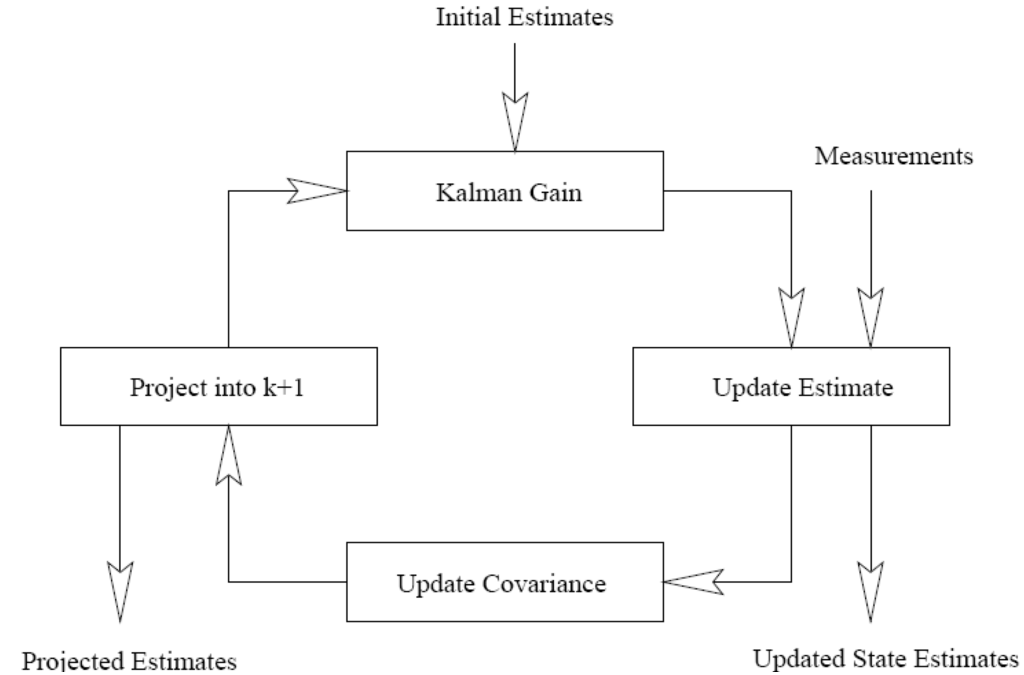
$$P_k = (I - K_k H) P_k'$$

$$\hat{x}_{k+1}' = \Phi \hat{x}_k$$

$$P_{k+1} = \Phi P_k \Phi^T + Q$$

# The Complete KF

The recursive Algorithm,




| Description | Equation |
|---|---|
| Kalman Gain | $K_k = P'_k H^T \left( H P'_k H^T + R \right)^{-1}$ |
| Update Estimate | $\hat{x}_k = \hat{x}'_k + K_k \left( z_k - H \hat{x}'_k \right)$ |
| Update Covariance | $P_k = \left( I - K_k H \right) P'_k$ |
| Project into $k+1$ | $\hat{x}'_{k+1} = \Phi \hat{x}_k$<br>$P_{k+1} = \Phi P_k \Phi^T + Q$ |

# Back to EIT Reconstruction Problem!!

- We can apply the KF recursive algorithm for estimating the states that are evolved with time
- But we need to have the state-space representation in place for KF processing,
- And we have them!!

State process:

$$\rho(t+1) = F(t)\rho(t) + v(t)$$

Measurement process :

$$U(t) = U_{0,k(t)} + J_{k(t)}(\rho(t) - \rho_0) + w(t)$$

# EIT reconstruction with KF

Important: $\rho = \rho(t)$ i.e. the impedance distribution is modeled as state evolved with time. It takes transition from one state to the other with a new current pattern at each instant of time. Transition matrix: F(t)

We have everything in place. Corresponding to our designed KF here we have,

$F \rightarrow \phi$

$J_{k(t)} \rightarrow H$, $X \rightarrow \rho$, $Z \rightarrow U$, R & Q etc…

# EIT reconstruction with KF

- So, estimation of impedance distribution [$\rho$ (t) ], is given by,

$$\hat{\rho}(t+1) = \hat{\rho}_1(t+1) + K_k e(t+1)$$

- subscript k indicates (t+1)th time instant
- $K_k$ can be computed using the Kalman Gain Formula

Measurement residual : e(t) is given by,

$$e(t) = U_t - U_{0,k(t)} - J_{k(t)}(\hat{\rho}_1(t) - \rho_0)$$

# The conventional Reconstruction Algorithm

- Called the NOSER algorithm
- Does not generate impedance images with each current pattern
- Uses a full set of current patterns and Voltage measurements for reconstruction of each distribution
- Performs one step of the regularized Gauss-Newton iteration of associated non-linear least square problem,

$$\min_{\rho} \left\| U - U(\rho) \right\|,$$

# Comparison of conventional vs. KF based reconstruction

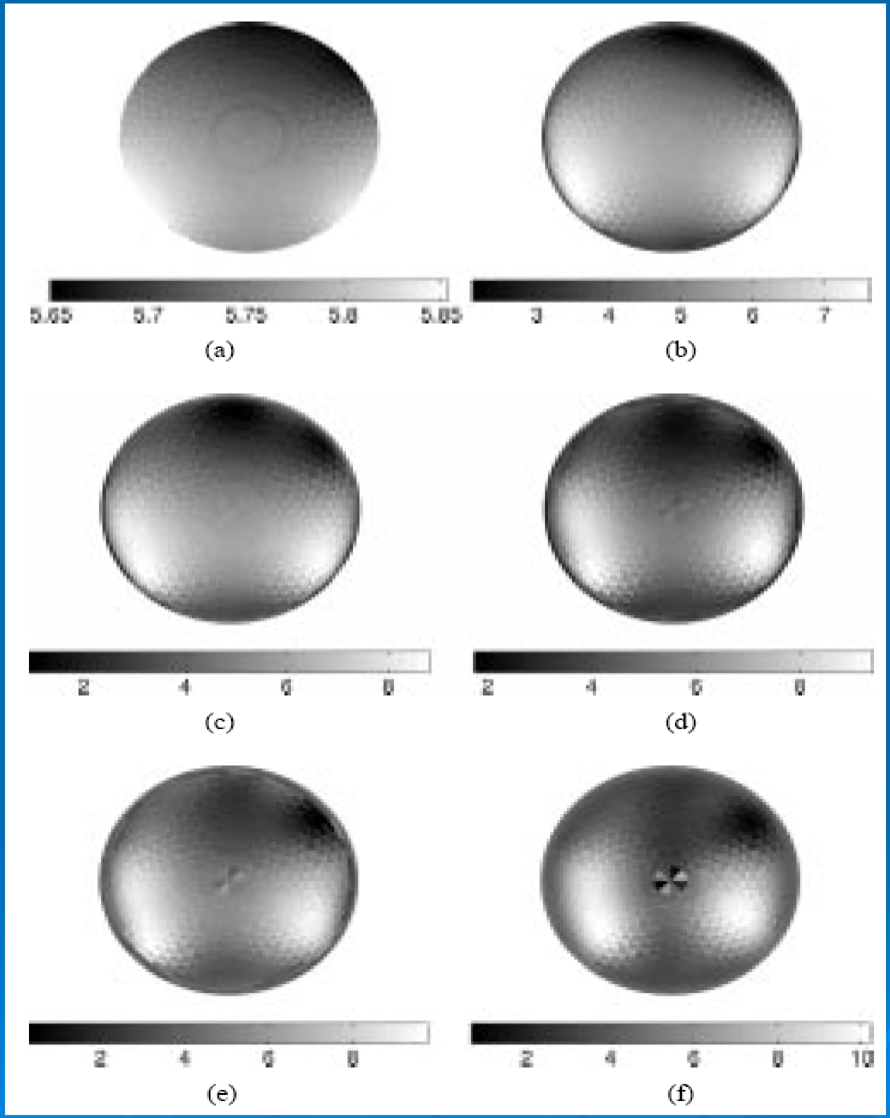
- KF based reconstruction is able to track the time evolution of impedance distribution
- Hence, KF based approach can depict the fast impedance variation inside the body ( e.g.. Cardiac activity and Lungs)
- KF based algorithm is dynamical and faster. For 32 electrodes, all the 31 current patterns is used for a conventional image reconstruction.
- KF based approach reconstructs after each current pattern and hence 31 time faster
- Useful in sports medicine.( 180 Heart beats/min )

# Parameter selection for Simulation : Some Design Issues

- F(t) = I ( the unit matrix ), R=0.2 * I , Q= 0.8* I
- Initial covariance of the estimation error is considered to be 0.1 * I
- Lower the dimension of state vector the better
- Grouping the FEM elements together by preintegration for constructing ROIs and hence lesser dimensional state space
- ROI for lungs, ventricles of heart. Thus dimension of state vector goes down to 3 or 4. Easy to solve.
- We can have average impedance distribution of Lung region (ROI) and can track the variation with cardiac cycle.

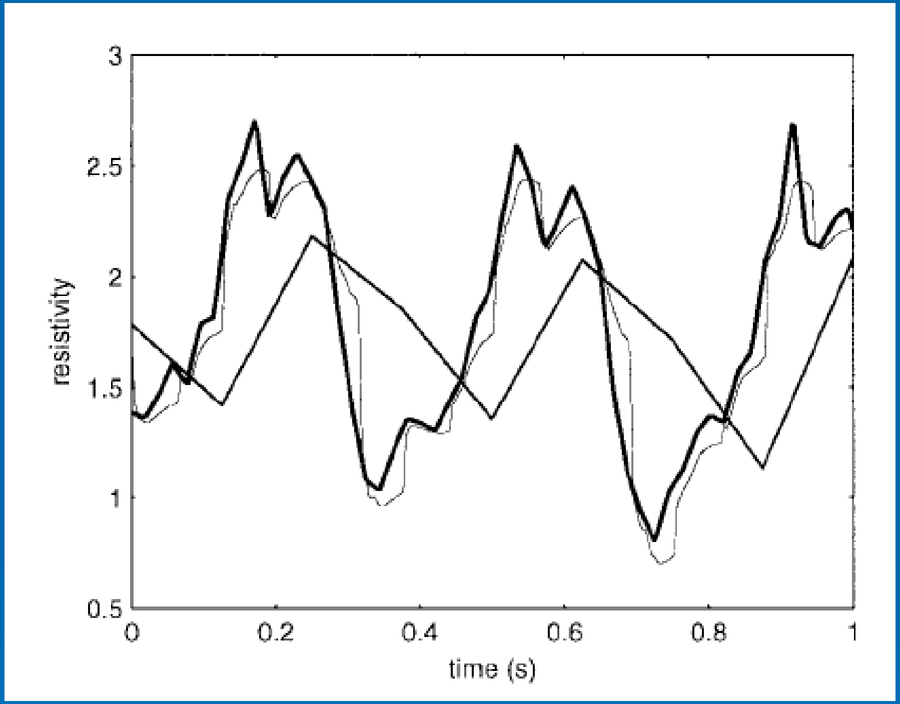
- Trigonometric current patterns across the electrodes

# Simulation Results



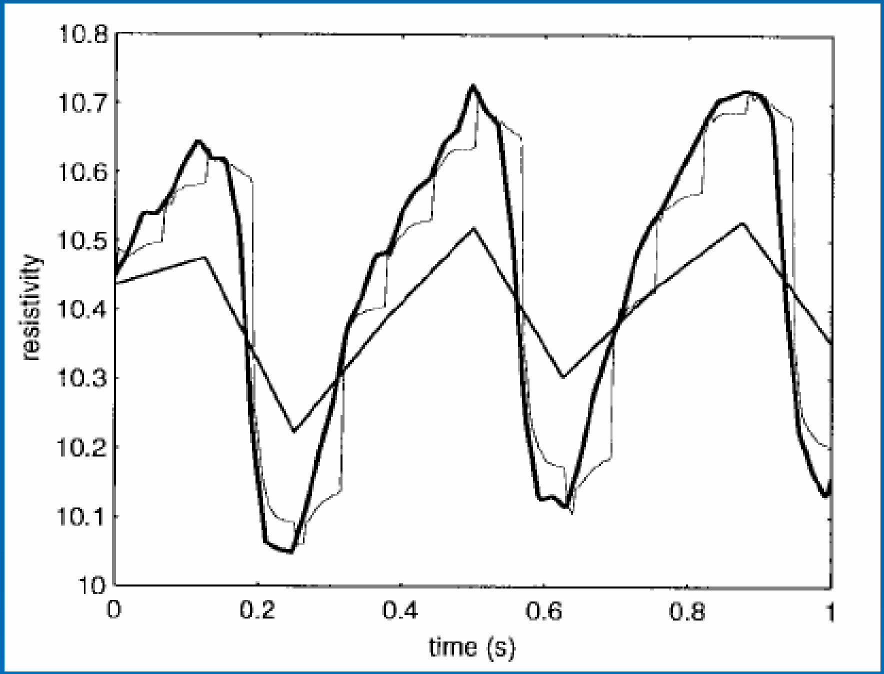

(a) To (e) with KF and (f) with the conventional approach

# Simulation Results



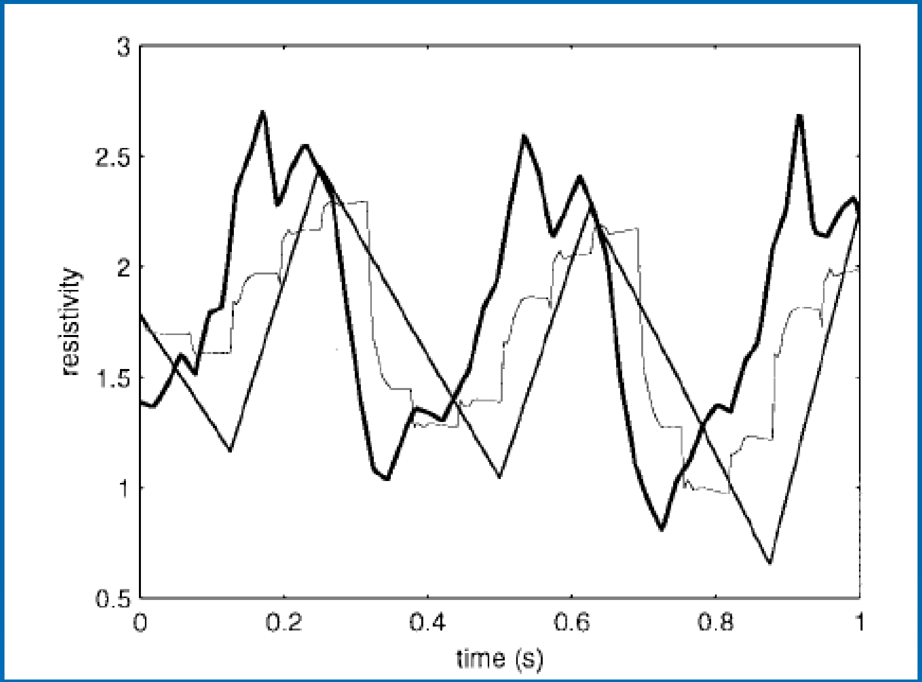

Result from Left Ventricle with 5 ROI parameters and 31 current patterns

# Simulation Results




Result from Right Lung with 5 ROI parameters and 31 current patterns

# Simulation Results




Result from Left Ventricle with 496 FEM parameters
and 31 current patterns

# Future works: A few suggestions

- F(t) = I may not be valid all the time. Appropriate choice of F(t) should be done depending on system under consideration
- With proper modeling we can study the dynamics of blood circulation among various organs
- Take into account of Non-linearity. Use of Extended Kalman Filter (EKF)
- Introduce optimal current pattern for better spatial resolution

# Acknowledgement

- M. Vauhkonen, P.A. Karjalainen, and J.P. Kaipio, "A Kalman fillter approach to track fast impedance changes in electrical impedance tomography," IEEE Trans Biomed Eng, 1998.
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Thank You !

Questions ??